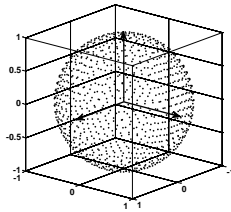


# TRAJECTORY OPTIMIZATION USING DISCRETE SETS OF PSEUDOIMPULSES: A REVIEW OF ADVANTAGES AND DIFFICULTIES

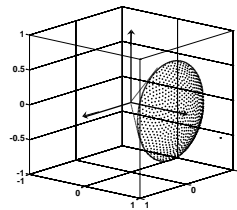
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New methods for spacecraft trajectory optimization using discrete sets of pseudoimpulses was proposed in last years [1-9]. The methods are based on two ideas. The first is a well-known trajectory discretization of time, in the general case, on non-uniform segments. The second is the key idea, based on a near-uniform discrete approximation of a control space (i.e. thrust direction and magnitude) by a set of pseudoimpulses with an inequality constraint for each segment. The controls are defined as the thrust direction and thrust level. The optimization problem formulation is to minimize a performance index that is the total characteristic velocity with terminal conditions. Depending on the problem formulation, the thrust direction can be arbitrary (Fig. 1a) or inside of a spherical segment (Fig. 1b) or in a 3D subspace. All of the possible thrust directions can be presented as a set of pseudo-impulses within the unit sphere with a small angle between them. The optimal impulse in each segment can be presented by the sum of non-zero pseudoimpulses with a constraint on the total characteristic velocities of the pseudoimpulses.



a)



b)

**Figure. 1 Near uniform distributions of pseudoimpulses in 3D space**

The optimal vector approximation of the optimal impulse by the pseudo-impulses for each segment is a vector sum of nearest neighbor pseudo-impulses [1]. Terminal conditions are presented as a linear matrix equation. The elements of the matrix are the partial derivatives of the terminal conditions for all of the pseudoimpulses in each segment. A matrix inequality on the sum of the characteristic velocities for the pseudo-impulses is used to transform the problem into a large-scale linear programming form. An additional post-processing is required for the linear programming solutions. It is necessary to find all of the segments corresponding to the non-zero decision variables. The adjacent segments among these should be joined in maneuvers.

The main part of the report is a critical review of advantages and difficulties of the methods which provide flexible opportunities for the trajectory computation in complex missions with various requirements and constraints. Major advantages are automatic determination of an optimal number of the maneuvers for the nonsingular trajectories, wide possibilities for accounting of interior-point boundary conditions or inequalities, constraints or preferences for some thrust directions, maneuver intervals, possibilities for trajectory optimization with a multi-mode propulsion system with a combination of high-, medium-, and/or low-thrust, computation

of a required thrust-to-weight ratio and other operational constraints. Systematic mathematical representations for the problems are described.

The use of the pseudoimpulse set methods significantly increased the problem dimension (tens or hundreds of thousands of unknown variables). Modern scientific software have effective algorithms for large-scale linear programming (the interior-point method) and for sparse matrix computations.

The spacecraft maneuvers can be divided into two categories: trajectory station-keeping (or corrections) and orbit transfers. The station-keeping uses nearly periodic maneuvers to maintain a reference orbit, which has continuous perturbations. As a rule, the station-keeping maneuvers and corrections performed in a neighborhood of the reference trajectory and the partial derivatives can be computed analytically or numerically. By contrast to the station-keeping problem, intermediate transfer orbits and, respectively, the partial derivatives, are usually not known a priori. For this case, an iterative technique with a refinement of the partial derivatives at each iteration can be used. Features of such solutions are discussed.

A summary of examples for long-term stationkeeping of Earth's high elliptical [1] and lunar halo orbits [8-9], orbit transfers [1,7], rendezvous [2,5], proximity trajectories [6], lunar landing [3] and ascent trajectories is presented. There is an extension to the concept of discrete pseudoimpulse sets for a more general optimal control problem using pseudocontrol sets [4].

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