Trajectory Design Using the Center Manifold Theory in the Circular Restricted Three-Body Problem: Applications to Spacecraft Formation Flying

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A spacecraft is forced various perturbations including gravities of planets during missions. In the two-body problem, there exist the optimization theories of the trajectory in the sense of minimizing the energy consumption in the literature. For the n-body problem, however, optimization theories have not been established yet. The three-body problem is the representative n-body problem. For the circular restricted three-body problem (CR3BP), there are five liberation points, called Lagrangian points, which are led from geometric relationship. Among them, collinear liberation points are located in the saddle points of the potential surface. Moreover, there is known to existence of three different kinds of periodic or quasi-periodic orbits in the vicinity of them: Lyapunov, Lissajous and halo orbits. In addition, the orbits are associated with stable and unstable manifolds. Therefore, it is possible to inject and eject the spacecraft into the halo orbit by using these manifolds without any energy consumption.

Up to now, great deals of research about formation flying or stationkeeping around the Lagrangian point have studied. An approach of formation flying by using the center manifold of halo orbits was suggested by Barden and Howell[1]. Moreover, the formation flying along the halo orbits are realized by applying the simple control law which stabilizes unstable orbits and creates the additional center manifold by Scheeres[2].

Determination of reference halo orbit is fundamental problem for CR3BP. In computing halo orbits, differential correction is often used. However this method cannot avoid numerical and trial-and-error procedures. On the other hand, an analytical method known as Lindstedt-Poincaré method provides a good initial condition of halo and Lissajous orbits. However, ad hoc algebraic manipulations and further numerical adjustment are necessary to obtain accurate halo orbits. Therefore the effective method to design halo orbits has not been established yet.

The main purpose of this study is the design of the trajectory of spacecrafts and formation flying by using the center manifold theory in the circular restricted three-body problem. Especially, a new semi-analytical theory [3] to approximate a solution of ordinal differential equation is applied to design periodic or quasi-periodic orbits around the Lagrangian points. In general, a dynamics is intermixed the center, unstable and stable manifolds. Consequently, the dynamics described the motion in the CR3BP should be transformed into the diagonal form Eq. (1) to apply the successive approximation method for center manifold.

$$\begin{aligned} \boldsymbol{v}' &= \boldsymbol{P}\boldsymbol{v} + \boldsymbol{f}(\boldsymbol{v},s,u) \\ s' &= Qs + g(\boldsymbol{v},s,u) \\ u' &= Ru + h(\boldsymbol{v},s,u) \end{aligned} \tag{1}$$

where $(\boldsymbol{v},s,u) \in \boldsymbol{R}^4 \times \boldsymbol{R}^1 \times \boldsymbol{R}^1$. Matrices $\boldsymbol{P},Q,R$ have eigenvalues with zero, negative and positive real parts, respectively. $\boldsymbol{f},g,h$ are nonlinear and continuous functions. In computing periodic orbits, all we need is to give initial values $\boldsymbol{\xi} = \boldsymbol{v}(t_0)$ to the first equation of Eq. (1). The rest of initial values are obtained by applying the successive approximation method. Here let consider the initial values as $\boldsymbol{\xi} = [0,\xi_2,\xi_3,0]^T$ which is perpendicular to the x-z plane of the original coordinate. This solution will also be symmetric with respect to the x-z plane. Thus only two variables are needed to compute the periodic or quasi-periodic orbits. The proposed method is different from differential correction method and Lindstedt-Poincaré method. The main difference is that the approach is semi-analytical method to compute periodic or quasi-periodic orbits while differential correction method is numerical and Lindstedt-Poincaré method relies on perturbation theory. Consequently, the approach offers a way to design not only periodic and quasi-periodic orbits but also formation flying or single spacecraft missions. In addition, the spacecraft which is on the center manifolds can fly around Lagrangian points without any control. Therefore, it might be possible to realize more fuel efficient missions operated around the Lagrangian points.

Now let consider Sun-Earth CR3BP around L_2. Figure 1 and 2 show the periodic and quasi-periodic orbits around the Lagrangian point obtained by the proposed method, respectively. All the orbits is symmetric with respect to the x-z plane. Figure 3 shows the spacecraft formation flying. In this case, all $\xi_{3,n}$ $(n = 1, \cdots, 5)$ is set as same value but $\xi_{2,n} = n\xi_{2,1}$. These initial conditions correspond to be arranged at the equal interval of z. All the periods of the obtained orbits is close to each other.

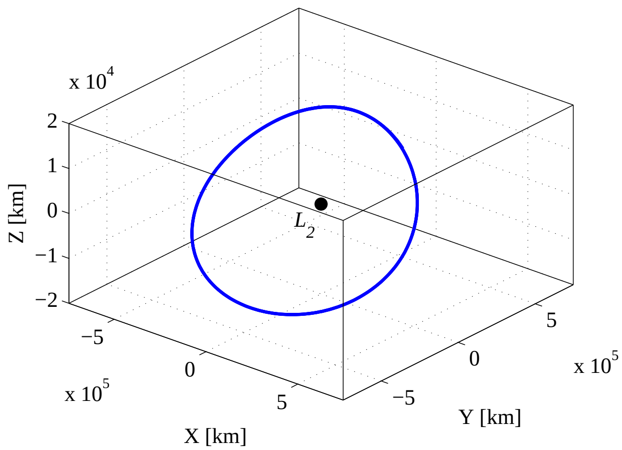


Fig. 1: Periodic orbit (Halo orbit)

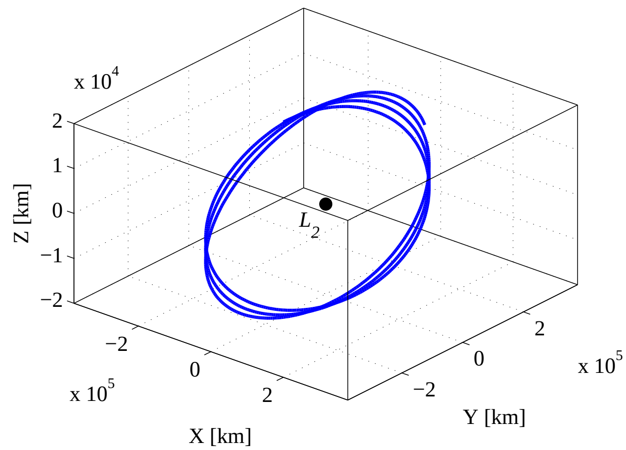
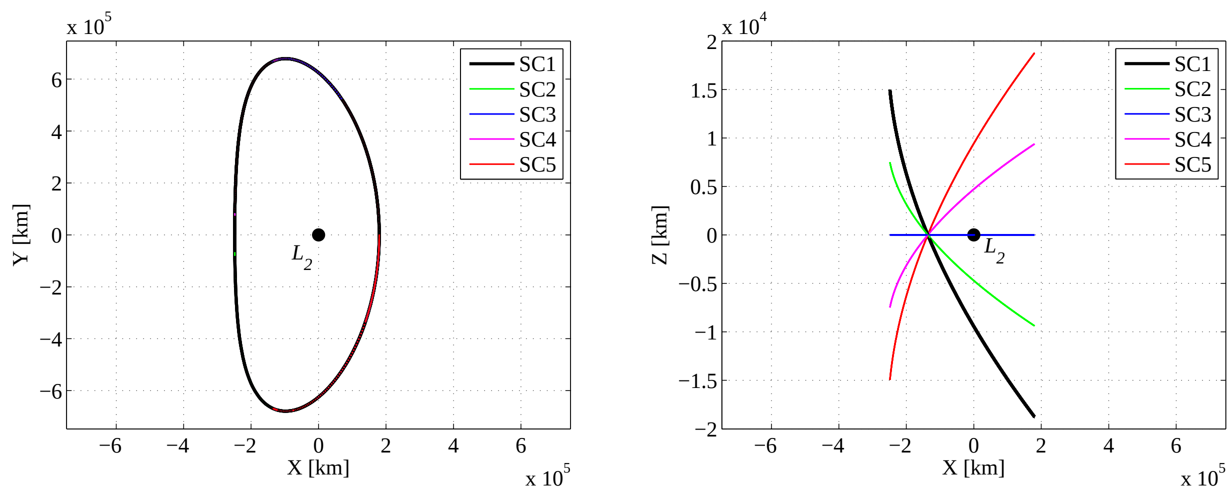


Fig. 2: Quasi-periodic orbit (Lissajous orbit)



(a) x-y projection

(b) x-z projection

Fig. 3: Spacecraft formation flying

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