DIFFERENTIAL DYNAMIC PROGRAMMING APPROACH FOR ROBUST-OPTIMAL LOW-THRUST TRAJECTORY DESIGN CONSIDERING UNCERTAINY

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ABSTRACT

This study proposes a fast computation technique to solve robust optimal control based on two techniques: Robust Dynamic Programming [1] and Differential Dynamic Programming [2] to design a low-thrust robust optimal trajectory considering uncertainty to minimize the worst-case objective function.

In recent years, low-thrust propulsion systems have been increasingly used in the interplanetary missions, because these systems have high specific impulse and can achieve large delta-V to travel far from the Earth. Due to this background, various low-thrust trajectory design methods have been developed [3]. These methods assume that the spacecraft is perfectly guided to the predesigned trajectory as planned; however, the assumption is violated in realistic operation because of various uncertainties including missed-thrust [4] (i.e. contingent coasting period due to temporary operational troubles). Because the low-thrust propulsion systems require long-time operation to achieve a certain delta-V in exchange for high specific impulse and the spacecraft always have the risk to enter the safe mode that the propulsion system is not operated, the occurrence of missed-thrust, or uncertainty, caused by operational anomaly is inevitable. Therefore, the low-thrust trajectory for actual mission must improve robustness against the uncertainties.

To improve robustness of low-thrust trajectory, we conventionally impose *forced coast period* [4], where we force coast periods during long thrusting periods (e.g. 4 days every 6 months), and *duty cycle* [4] on the trajectory, where the duty cycle is defined to be the rate of time available for low-thrust propulsion system thrusting at the maximum achievable throttle level in planned thrusting period. The experienced specialists have empirically determined the forced coast period and the duty cycle on a regular basis all along the trajectory; however this empirical method does not give *robust optimal* solution. Robust optimal solution is the optimal solution within the solutions that have certain robustness, and it is obtained by minimizing the worst-case objective function. In other words, the conventional solution is unnecessarily too conservative and it has room for improving the optimality, shown in Fig.1.

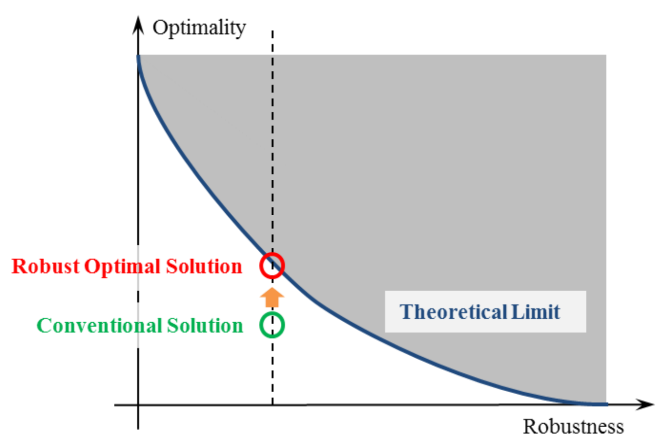

Fig. 1 Comparison of robust optimal solution and conventional solution with respect to robustness and optimality

Most of the previous works to design robust low-thrust trajectory considering uncertainty (especially missed-thrust) have been based on sampling methods (i.e. Monte-Carlo method)[4][5]; however it is not only a large burden on the computation because of large number of samples but also difficult to treat infeasible solutions in optimization. To overcome this difficulty, Olympio[6] has suggest the sophisticated method by two-stage stochastic programming with indirect method; however this method is only focused on one temporary engine failure (because of two-stage) that is not proved as the worst case. Therefore, we need to invent an innovative multi-stage robust optimization method to attack the low-thrust trajectory design considering uncertainty.

Multi-stage robust optimization problem can be achieved by Robust Dynamic Programming (RDP)[1], which has recently received substantial attentions in the field of Model Predictive Control (MPC) as Robust Model Predictive Control (RMPC)[1]. MPC has been historically developed for on-board feedback controller that is effective for linear dynamic system; however there are difficulties to apply MPC or RMPC to the low-thrust problem, because the low-thrust trajectory optimization is a highly nonlinear problem and is usually computed off-line. Instead of MPC, this study proposes a new algorithm to solve RDP based on Differential Dynamic Programming (DDP)[2].

DDP has recently been re-developed for low-thrust problem. To apply dynamic programming to continuous state system numerically, we notice that it has inherent difficulty called "curse of dimensionality" since the dimension of the state variables becomes incredibly large. To overcome this fundamental difficulty, DDP was created based on expanding the Principle of Optimality by second order around the reference trajectory. The classical DDP was only effective for smooth unconstrained problems; on the other hands low-thrust problems fundamentally have constrained bang-bang structure (i,e, DDP may converge slowly or may not converge at all for the low-thrust problems). Recent works for DDP[2] have been improving the applicability to the low-thrust problem by incorporating well-developed Nonlinear Programming (NLP) techniques to DDP. The advantages of DDP-based low-thrust optimization methods against conventional trajectory design methods are:

(1) Robustness to poor initial guess like conventionally dominated direct collocation methods.
(2) Good applicability to large-scale problems such as multi-revolution transfer, because the computational effort per iteration of DDP increases only linearly with the number of stages, whereas that of the common NLP-based methods increases exponentially.
(3) Optimal linear feedback control law around reference trajectory can be retrieved.

A proposed Robust Differential Dynamics Programming (RDDP) has the following advantages.

(1) *Robust Optimal Controller*: The conventional design method manually adds the robustness to the controller by non-optimal and empirical strategy; on the other hand, RDDP provides a robust-optimal controller, which is one of Pareto optimal solution of optimality and robustness.
(2) *Fast Computation*: The conventional design method provides open-loop controller; on the other hand, DDP and RDDP provide both open-loop controller and closed-loop controller around reference trajectory. Therefore, the unnecessity of the trajectory redesign process gives fast re-computation.

This paper presents the theoretical derivation of a proposed RDDP and shows the numerical examples of interplanetary mission considering missed-thrust and thrust error.

REFERENCES

[1] Diehl, M. and Bjornberg, J. "Robust Dynamic Programming for Min-Max Model Predictive Control of Constrained Uncertain Systems" IEEE Transactions on Automatic Control, Vol.49, No.12, pp.2253-2257, 2004.

[2] Lantoine, G. and Russell, R.P. "A Hybrid Differential Dynamic Programming Algorithm for Constrained Optimal Control Problems. Part 1: Theory" Journal of Optimization Theory and Applications, Vol.154, No.2, pp.382-417, 2012.

[3] Betts, J. "Survey of Numerical Methods for Trajectory Optimization" Journal of Guidance, Control, and Dynamics, Vol.21, No.2, pp.193-207.

[4] Rayman, M.D., Fraschetti, T.C., et. al. "Coupling of System Resource Margins through the Use of Electric Propulsion: Implications in Preparing for the Dawn Mission to Ceres and Vest." Acta Astronautica, Vol.60, Nos.10-11, pp.930-938, 2007.

[5] Olympio, J.T. and Yam, C.H. "Deterministic Method for Space Trajectory Design with Mission Margin Constraints" 61th International Astronautical Congress, 2010.

[6] Olympio, J.T. "Designing Robust Low-Thrust Interplanetary Trajectories Subject to One Temporary Engine Failure" 20th AAS/AIAA Space Flight Mechanics, AAS10-171, San Diego, 2010.