The applicability of semi-analytical method for different orbits in long term prediction

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Abstract: *We hope to give some useful suggestions when dealing with the long term prediction for a lot of targets, especially retired satellites and space debris. Generally, we want to give the suggestions in two aspects, the integrating method and the dynamic models.*

Semi-analytical method is a common method for long term orbit prediction, especially in the evolution of the space debris and solar system dynamics. Orbit prediction can be written in the following ordinary differential equation as shown in Eq (1)

$$\begin{cases} \dot{\vec{X}} = \vec{F}(t_0, t, \vec{X}) \\ \vec{X}(t_0) = \vec{X_0} \end{cases} \tag{1}$$

where $\vec{X}$ *represents the six dimensional orbit state which can be position-velocity or orbital elements.In this paper, we will adopt orbital elements whose acceleration is perturbation, which may lead to high integrating speed.*

The key idea of semi-analytical method is to eliminate the short period terms of perturbation functions. Then, integrating the remaining averaged system by numerical integrator. The averaged system can be shown as Eq (2)

$$\frac{d\bar{\sigma}}{dt} = \bar{f}_{\varepsilon}(t_0, t, \sigma) \tag{2}$$

where $\bar{\sigma}$ *represent averaged orbital elements,* σ *represent osculating elements and* $\bar{f}_{\varepsilon}(t_0,t,\sigma)$ *are averaged perturbation functions. After averaged, the system only has secular and long period variation. Usually, the averaged system can get 1-2 orbital period as an integrating step, which leads to high integrating speed. We will try both analytical and numerical method to get averaged perturbation value, and analytical expressions to convert averaged orbital elements to osculating orbital elements.*

Since we want the semi-analytical method be applicable to long term prediction (100 years time scale) according to different types of orbit, these details will be discussed in the following:

(1) How to choose numerical integrators to get best integrating speed and accuracy when integrating averaged system. Single or multi-step method, low or high order and so on.

(2) When need averaged perturbation value as right hand functions,we should choose numerical or analytical method. The former will cost much time on numerical integration to calculate averaged value, but get large step size and more accurate in exchange. The later will also have large step size and even quicker, but sacrifices some accuracy. How to keep balance?

(3) If we use numerical method to give averaged right hand functions value, how to choose numerical method in appropriate way. Here we choose from two kinds of typical numerical integral formula, Guass and Newton-cotes, both of them have different orders which mean different accuracy.

(4) How to choose proper analytical expressions to satisfy the accuracy when converting averaged orbital elements to osculating orbital elements. Since we can't retain all the analytical expressions, we must make choice which analytical expressions play the key role(The osculating elements will be used in numerical method to calculate averaged right hand value).

(5) Since it is a long term prediction, for different orbit type, some perturbation should be ignored, but how to make the best strategy.

Obviously, these details above should be discussed according to different types of orbits respectively. Finally, the result will be used in 100 years time scale orbit prediction to test the performance in LEO, MEO, GEO, IGSO compared with the pure numerical integral method.

We use J2000 epoch equator coordinate system, JGM-3(20×20) non spherical gravitation model, DE406 numerical ephemerides, average atmospheric density model and Cylindrical shadow solar radiation pressure.

Keyword :*long term prediction; semi-analytical method*