NONLINEAR PROPAGATION OF UNCERTAINTY USING SIMPLIFIED DYNAMICAL MODEL OF NON-KEPLERIAN MOTION

Inkwan Park(1) **and Daniel J. Scheeres**(2)
(1) *University of Colorado at Boulder, 429 UCB, Boulder, CO, 80309, U.S.A., inkwan.park@gmail.com*
(2) *University of Colorado at Boulder, 429 UCB, Boulder, CO, 80309, U.S.A.,scheeres@colorado.edu*

Abstract: *One of the most significant technical discussions in Space Situational Awareness (SSA) is how one can propagate state uncertainty in a way that is consistent with the highly non-linear dynamical environment[1, 2]. The state transition matrix (STM)[3] is the traditional linearized mapping technique and assumes Gaussinity of distributions over time. As demonstrated in Junkins, et al.[1], however, the position and velocity uncertainty of an Earth-orbiting object in the Cartesian space does not preserve normality when propagated over time. Capturing this nonlinearity in a computationally efficient way is a core question in SSA. As such, various mathematical expressions of uncertainty have been applied in the context of SSA and include analytically expressed probability density functions (PDFs), polynomial chaos expansions[4], and Gaussian mixture models[5, 6, 7, 8].*

We study an analytical nonlinear uncertainty propagation method that improves computational efficiency while maintaining acceptable level of accuracy. The salient idea of our suggested method is a combination of a simplified dynamical system (SDS) and state transition tensor (STT). We reduce the nonlinearity of a dynamical system by defining the SDS, and then map a given uncertainty with the STT. The SDS is defined based on a semi-analytic solution[9, 10], which has only the secular and long-period variations, so that the SDS ignores short-period variations when propagating uncertainty. We have defined the semi-analytic solution up to second-order through a Deprit-Lie canonical transformation method[12]. As demonstrated in [12, 13, 14, 15], the canonical transformation derives a solution for each variation, e.g., secular, short-period, and long-period, through several steps by removing angular variables, and thus it is suitable for us to designate as a semi-analytic solution. We have demonstrated the aforementioned approach in [11], and verified an accuracy in propagating uncertainty. For the deterministic dynamical system, the STT can capture nonlinear effect included in the dynamical system as well as propagate any given PDF analytically with respect to the nominal trajectory[16, 17]. In order to capture these nonlinear effects, the STT includes higher-order terms in the Taylor expansion of the dynamical system (or solution flow). In this research, the SDS is expanded with respect to an initial condition when we generate the STT. In brief, the analytical nonlinear uncertainty propagation method is a combination of the SDS and the STT to take advantage of both of their strengths. We have shown that the SDS improves efficiency by factor of 14 times better as compared to a Monte-Carlo simulation for a 10,000 sample case [11]. The new method we suggest though this paper has even better performance than the SDS only approach.

In the paper, the theoretical background of the analytical nonlinear uncertainty propagation method is demonstrated. We propagate uncertainty of orbiting objects perturbed by the Earth oblateness, *direct solar radiation pressure, and gravitational perturbation due to the moon and sun, with the new method. Then, we verify the accuracy of the propagated results and present how much enhancement in the computational efficiency is achieved by comparing the usual Monte-Carlo simulation, SDS, and the combination of the simplified dynamical system (SDS) and state transition tensor (STT) approaches.*

Keywords: *simplified dynamical system, orbit uncertainty, semi-analytic solution, state transition tensor, perturbation theory*

1. References

[1] Junkins, J. L., Akella, M. R., and Alfriend, K. T. “Non-Gaussian Error Propagation in Orbital Mechanics.” Journal of Astronautical Sciences, Vol. 44, No. 4, pp. 541–563, October-December 1996.

[2] Park, R. S. and Scheeres, D. J. “Nonlinear Mapping of Gaussian Statistics: Theory and Applications to Spacecraft Trajectory Design.” Journal of Guidance, Control, and Dynamics, Vol. 29, No. 6, pp. 1367–1375, November-December 2006. doi:10.2514/1.20177.

[3] Tapley, B. D., Schutz, B. E., and Born, G. H. Statistical Orbit Determination. Elsevier Academic Press, Burlington, MA, 1 edn., 2004.

[4] Jones, B. A., Doostan, A., and Born, G. H. “Nonlinear Propagation of Orbit Uncertainty Using Non-Intrusive Polynomial Chaos.” Journal of Guidance, Control, and Dynamics, Vol. 36, No. 2, pp. 430–444, March-April 2013. doi:10.2514/1.57599.

[5] Horwood, J. T., Aragon, N. D., and Poore, A. B. “Gaussian Sum Filters for Space Surveillance: Theory and Simulations.” Journal of Guidance, Control, and Dynamics, Vol. 34, No. 6, pp. 1839–1851, December 2011. doi:10.2514/1.53793.

[6] Terejanu, G., Singla, P., Singh, T., and Scott, P. D. “Uncertainty Propagation for Nonlinear Dynamic Systems Using Gaussian Mixture Models.” Journal of Guidance, Control, and Dynamics, Vol. 31, No. 6, pp. 1623–1633, November-December 2008. doi:10.2514/1.36247.

[7] Giza, D., Singla, P., and Jah, M. “An approach for Nonlinear Uncertainty Propagation: Application to Orbital Mechanics.” AIAA 2009-6082. AIAA, 2009.

[8] DeMars, K. J. Nonlinear Orbit Uncertainty Prediction and Rectification for Space Situational Awareness. thesis, University of Texas at Austin, December 2010.

[9] Metris, G. and Exertier, P. “Semi-analytical theory of the mean orbital motion.” Astronomy and Astrophysics, Vol. 294, pp. 278–286, 1995.

[10] Lara, M., Juan, J. F. S., and Lopez, L. M. “Semianalytic Integration of High-Altitude Orbits under Lunisolar Effects.” Mathematical Problems in Engineering, Vol. 2012, No. Article ID 659396, 2012.

[11] Park, I. and Scheeres, D. J. "The Effect of Dynamical Accuracy for Uncertainty Propagation (submitted)." Journal of Guidance, Control, and Dynamics, 2014.

[12] Deprit, A. "Canonical Transformations Depending on a Small Parameter." Celestial Mechanics, Vol. 1, pp. 12–30, 1969.

[13] Brouwer, D. and Clemence, G. M. Methods of Celestial Mechanics. Academic Press, 1961.

[14] Lyddane, R. H. "Small Eccentricities or Inclinations in the Brouwer Theory of the Artificial Satellite." The Astronomical Journal, Vol. 68, No. 8, pp. 555–558, October 1963.

[15] Hori, G. I. "Theory of general perturbations with unspecified canonical variables." Astronomy Society of Japan, Vol. 18, pp. 287–296, 1966.

[16] Park, R. S. and Scheeres, D. J. "Nonlinear Semi-Analytci Methods for Trajectory Estimation." Journal of Guidance, Control, and Dynamics, Vol. 30, No. 6, 2007. doi:10.2514/1.29106.

[17] Park, S. H. Nonlinear Trajectory Navigation. Ph.D. thesis, University of Michigan, 2007.