# DESIGN OF CIRCLE-TO-CIRCLE OPTIMAL LOW-THRUST TRANSFER TRAJECTORIES USING COORDINATE TRANSFORMATION

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## ABSTRACT

The purpose of this paper is to study the method to design optimal low thrust trajectories without numerical optimal calculation. Generally, some numerical optimal calculations are used in order to design optimal interplanetary trajectories with continuous thrust such as Direct Collocation with Nonlinear Programming (DCNLP), Quasi Linearization Sequential Conjugate Gradient Restoration Algorithm (SCGRA) and so on. These numerical methods are indirect method. In these methods the problem to minimize Hamiltonian using calculus of variations and this problem is changed to the problem to solve the two boundary problem that adjoint variables are innovated. Although these indirect methods are practical because these converge faster than other optimal numerical calculation (direct method), if a large number of low-thrust trajectories to many celestial bodies candidate have to be calculated in mission design, the calculation cost is high exceedingly.

In this study, therefore, the new method of designing the approximate optimal low thrust trajectories is proposed. In this method optimal trajectories are designed without numerical optimal calculation and the new coordinate transformation is innovated. Optimal calculation based on calculus of variations is dominated by manipulating the adjoint variables and the orbits of adjoint variables get quite simple like linear function in this new coordinate. In this new method the adjoint variables orbits are designed firstly in the new coordinate and after that, adjoint variables are described again in inertial coordinate. The history of thrust can be obtained by the derived adjoint variables and the trajectories of the spacecraft are designed by them. Numerical optimal calculations produce the trajectories of the spacecraft and the history of thrust simultaneously. On the other hand, this new method produces the history of thrust earlier and trajectories subsequently. Although previous study proposed the trajectory design method using coordinate transformation which design the optimal low thrust trajectories without numerical calculations and permit to derive the trajectories and the history of thrust separately, obtained trajectories doesn’t guarantee optimality theoretically [1]. However, the new method of designing optimal low thrust trajectories in this paper guarantees the optimality because it is based on calculus of variations.

In this paper, circle-to-circle (Earth-to-Mars) transfer trajectories are designed using this new trajectories design method. The two body problem is considered. At first, it is presumed that the mass change of the spacecraft is negligible and the radius of terminal position is near the radius of initial position of the spacecraft. The differential equations of the equation of motion and adjoint variables in polar coordinate are linearized and the coordinate transformation which makes the orbits of adjoint variables simple is derived. Objective function is minimizing fuel consumption and the results of low thrust trajectories which is designed by using the coordinate transformation are shown. The results are compared to optimal trajectories using numerical optimal calculation (DCNLP) and the validity of this trajectory design method is indicated. This method may enable to find the new low thrust trajectories which are not found by numerical calculation possibly.

## References


[1] Jun, M. and Junichiro, K. “ORBIT EXPRESSIONS IN THE CONTINUOUS POLAR COORDINATES SYSTEM.” International Space Conference of Pacific-basin Societies, 2012.