STUDY OF SMALL FORCES FOR MISSION ANALYSIS

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ABSTRACT

The study of the effect of perturbations is central in space flight dynamics mission design, for Earth orbits in particular: it impacts the station keeping cost (and the station keeping window size) or the disposal orbit lifetime for instance.

This paper focusses on those forces whose effects are often neglected in early design phases: solid tides, albedo, apparent acceleration are examples of such forces.

The paper includes useful analytical expressions of the averaged effects of the forces, when they can be computed, as it is the case for solid tides or apparent acceleration.

For instance for solid tides, the averaged effects over one orbit on the Keplerian elements are:

$\dot{a} = 0;$

$\dot{e} = 0;$

$\dot{i} = 6KXZ;$

$\dot{\Omega} = \dfrac{6K}{\sin i} YZ;$

$\dot{\omega} = 3K(1 - 3\,Z^2) - 6K\dfrac{\cos i}{\sin i} ZY;$

$\dot{M} = 3K\sqrt{1 - e^2}(1 - 3Z^2)$

where K is constant (function of gravitation constant…), and X, Y, Z are the components of the central body to perturbing body unit vector in a particular frame tied to the orbit.
In addition to enabling a better understanding of the effects of the force, one can derive conclusions regarding the amplitude of the effect compared to that originating from the 3rd body perturbation.
It can be shown that the ratio of inclination time derivative (solid tide compared to 3rd body) for Sun-synchronous orbits is given by:

$\dfrac{Solid\ tide\ effect}{3rd\ body\ effect} = k_2 \left(\dfrac{R_E}{a}\right)^5$

with k_2, R_E, and a stand for the degree 2 Love coefficient, the Earth radius and the orbit semi-major axis respectively.

This result, obtained by double averaging is very useful for mission analysis. By the way it appears that the effect is not always negligible: the ratio is up to 20% at an altitude of 500 km.

All these results have been confirmed by comparison with real data from operational satellites.

The same kind of analysis has been conducted for apparent acceleration.
Averaged equations are detailed in the paper, as well as the method used to derive them.

From these equations one can for instance obtain a simple model for GEO objects for the change in inclination in CIRF frame :

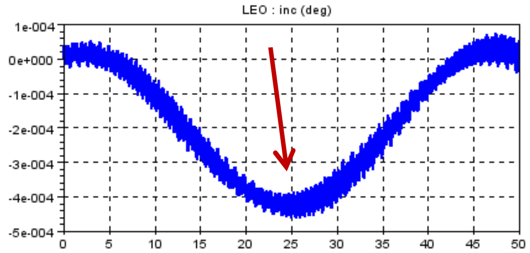
$$\Delta i = \frac{-\omega_X}{\dot{\Omega}_{J_2}} \sin \Omega + \frac{\omega_Y}{\dot{\Omega}_{J_2}} \cos \Omega + \Delta i_0$$

ω_X and ω_Y are the components of the angular rotation vector of CIRF with respect to ICRF (which is inertial).

The computation of the angular velocity vector may be quite time intensive because of the numerous number of nutations terms that have to be computed.

The paper details an analysis aimed at finding the most simple, yet accurate enough model for the angular velocity vector, that is with the smallest number of expansion terms.
This can be of interest if the computation had to be done onboard for instance.

Results for typical orbits (LEO, MEO, GEO, GTO) are given.
In particular, if the angular velocity is not accurate enough, or rather if some frequencies are missing in the expansion series, some kind of long-term resonance effect appear, as shown below for LEO on inclination over 50 years (comparison the the most accurate model available) :



The model that resulted from this analysis has been implemented in STELA, tool used at CNES for orbit long-term propagation (and used to support the French Space Operations Act).

Other force effects will be analysed in the paper as Albedo. In that case, no analytical calculations exist (excepte for particular cases), but some conclusions regarding the amplitude of the perturbation effect can still be drawn.

As a synthesis, the paper shows various results related to small forces whose effects are not always as negligible as one may think.

Some aspects of what is presented in the paper (the force models in particular) are also implemented in CelestLab, open-source SpaceFlight Dynamics library used at CNES for mission design.