Lunar Landing Trajectory and Abort Trajectory Integrated Optimization Design

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Abstract: *During the trajectory design for manned lunar landing mission, abort trajectory design for returning to earth in case of any emergency shall be taken into consideration in order to find the optimal trajectory meeting the requirements of emergency lifesaving. Based on the present researches, this paper focused on the abort return in case of any emergency by analyzing the trajectory for manned lunar landing mission – free return trajectory and hybrid trajectory with a hybrid optimization method of genetic algorithm and sequential quadratic programming, in order to find the lunar landing trajectory meeting the abort requirements and with optimal energy. The effectiveness of this method had been verified by the simulation results.*

Keywords: *Trajectory Design, Integrated Optimization, Genetic Algorithm, Sequential Quadratic Programming.*

1. Introduction

There are two categories of trajectories for manned lunar landing mission – free return trajectory and hybrid trajectory[1]. In case there is no braking at perilune, the free return trajectory is able to send the spaceship back to earth after the mission, so the free return trajectory is capable of aborting mission and making the spaceship return to earth in case of emergency (such as explosion of oxygen tank of Apollo 13). However, the accessible lunar surface of free return trajectory is limited to the lunar equator[1], and such free return trajectory will lose abort return ability after braking at perilune. The hybrid trajectory combines free return trajectory and non-free return trajectory. Such trajectory is widely used due to the unlimited accessible lunar surface in theory. However, the hybrid trajectory doesn't have the ability of auto return during the flight other than in the initial free return trajectory. Therefore, abort trajectory must be taken into consideration during the trajectory design.

Scholars all over the world had made numerous analysis and design for the trajectory for manned lunar landing mission and abort trajectory. Among them, Berry summarized the planning and control problems of launch window, earth-moon transfer trajectory, lunar orbit and moon-earth transfer trajectory of Apollo 11[1]. Hyle et al summarized the mission abort planning for each flight phase of Apollo[2]. In 1969, Babb put forward a mission abort plan of using the lunar module as the rescue capsule after analyzing the emergencies similar to those of Apollo 13 in April 1970[3]. Anselmo analyzed the mission abort of Apollo 14 in earth-moon transfer stage and lunar orbit[4]. Kelly analyzed the identification of abort point and thrust for earth-moon transfer stage[5]. Merrick put forward a solution of two-impulse abort with the help of on-board computer[6]. These methods lay the foundation for the successful implementation of Apollo Mission.

In recent years, Ocampo put forward the initial trajectory model for a multi-impulse moon-earth abort[7]. Huang Wende made a series of study on the trajectory design for manned lunar landing mission[8~11]. He not only summarized the characteristics of manned lunar landing trajectories and abort trajectories but also discussed the optimization design of trajectories for manned lunar mission, control of abort trajectories and verification of stimulation results.

This paper focused on the integration optimization design of manned lunar landing trajectories and its abort trajectories under perturbation, in order to find the manned lunar landing trajectories meeting the abort requirements, with the optimization energy and closer to the project reality. In order to accelerate the design of convergence rate, this paper regarded the unperturbed solution of the manned lunar landing trajectories and abort trajectories as the initial value of perturbation questions. Therefore, this paper firstly introduced the unperturbed models of manned lunar landing trajectories and their abort trajectories, then considered the perturbation model and analyzed the major constraint conditions. After that, this paper displayed the hybrid optimization method combining the genetic algorithm and sequential quadratic programming (SQP). Finally, this optimization method was used in the simulating calculation to verify the creativeness and effectiveness of this method.

2. Perturbation Model for Trajectory Design

2.1. Trajectory Calculation Model[12]

The following perturbation calculation model applies to the earth-moon transfer trajectory and moon-earth transfer trajectory (including abort trajectory) regardless of the different flight stages:

$$\ddot{R} = -\frac{\mu_e}{R^3} R + a_e + a_s + a_M + a_R \tag{1}$$

Where, R is the position vector of spaceship according to geometric inertial coordinate system; μ_e is gravitational coefficient of the earth; a_e is the nonspherical perturbation; a_S is solar perturbation; a_M is lunar perturbation; a_R is solar radiation pressure perturbation. The gravity field of the earth is JGM3 Model; the positions of solar and lunar are calculated according to JPL Planetary Ephemeris DE405. This paper focused on the design for free return trajectory, hybrid trajectory and abort trajectory, therefore, only the two-body gravitation with the lunar as the central body was considered for lunar orbit.

In case of no maneuver, the trajectory at any time can be calculated with numerical integrator. The impulse hypothesis under maneuver adopted by this paper is as follows:

$$\begin{cases} r^+ = r^- \\ v^+ = v^- + \Delta v \\ t^+ = t^- \end{cases} \tag{2}$$

Where, the superscript “-” and “+” is the condition before orbital transfer and after orbital transfer respectively. In general $r^{+}=r^{-}=r, t^{+}=t^{-}=t$ are assumed in the following discussion.

2.2. Constraint Analysis

The major focuses of hybrid trajectory are earth-moon transfer maneuver, hybrid maneuver and perilune braking maneuver, the required speed impulses of which are referred to as $\Delta v_{TLI}, \Delta v_{H}, \Delta v_{LOI}$ respectively. As the purpose of abort is to return earth, the required abort speed impulse is regarded as Δv_{ab} in the following discussion with no differences. The initial condition, terminal constraint and other constraints of these maneuvers are analyzed in the following discussion.

(1) Earth-Moon Transfer Maneuver

The earth-moon transfer maneuver speeds up the spaceship at the earth parking orbit to the free return trajectory. With the acceleration moment t_{EA} and required speed impulse Δv_{TLI} as design variable, and hypothesizing the parking orbit is circular orbit, expressing the geocentric distance with R_0 and expressing the acceleration point with latitude argument u, the initial condition of earth-moon transfer maneuver is determined by R_0 and u, the function is shown as follows:

$$X_0 = X(t_{EA}) = f(R_0, u) \tag{3}$$

The terminal constraint is determined by geocentric distance under vacuum R_p and flight path angle of re-entry point Θ_{re}, and is expressed with the following function:

$$X_f = X(t_{rep}) = g(R_p, \Theta_{re}) \tag{4}$$

Where, the t_{rep} is the re-entry time. In order to save energy, the geocentric phase of free return trajectory shall be elliptical; in order to ensure the safe re-entry, Θ_{re} shall follow $-7.5^{\circ} \le \Theta_{re} \le -5.5^{\circ}$ and the R_p shall be about $6430 \pm 10km$. Besides, the fight time T constraint (the earth-moon transfer time and moon-earth transfer time shall be 3 days respectively), lunar distance r_p, lunar orbit inclination i_L and return trajectory inclination i_{re} constraints (which are determined by latitude returning to landing ground) shall be met.

$$\psi_{L0} \le \psi(T, r_p, i_L, i_{re}, \sigma_0) \le \psi_{U0} \tag{5}$$

Where, the σ_0 is the free return trajectory; ψ is the functions with various constraint conditions calculated with σ_0, which are nonlinear functions generally; ψ_{U0}, ψ_{L0} is the upper limit and lower limit of the constraint condition respectively.

(2) Hybrid Maneuver

The purpose of hybrid maneuver is to change the orbit inclination and near-lunar distance in order to make the spaceship entry the lunar orbit meeting the lunar landing requirements. With the orbital transfer time t_H and impulse Δv_H as the design variables, the initial condition of hybrid orbital transfer is the condition of orbital transfer point. Terminal constraint applies to the variations of near-lunar distance and lunar orbit inclination.

$$X_f = \begin{pmatrix} \Delta r_p \\ \Delta i_L \end{pmatrix} = \begin{pmatrix} r_{pH} - r_{p0} \\ i_{LH} - i_{L0} \end{pmatrix} \tag{6}$$

Where, i_{L0} and r_{p0} is the inclination and near-lunar distance of the initial free return trajectory respectively; i_{LH} and r_{PH} is the inclination and near-lunar distance of the non-free return trajectory respectively. Where, i_{LH} and r_{PH} are determined by lunar orbit which is determined by lunar landing point, exploration mission, etc. The constraint is that the flight time and the lunar orbit generated after braking at perilune shall pass the lunar landing point.

$$\psi_{LH} \le \psi(T_H, \lambda_L, \varphi_L, \sigma_H) \le \psi_{UH} \tag{7}$$

Where, σ_H is the orbit after hybrid orbital transfer; T_H is the flight time to the near-lunar point after orbital transfer; λ_L, φ_L is the longitude and altitude of lunar landing point respectively. ψ_{UH}, ψ_{LH} is the upper limit and lower limit of the constraint condition respectively.

(3) Braking at Perilune

The article adopted the strategy of realizing braking at perilune by multiple-impulse maneuver. Compared with braking at perilune by single-impulse maneuver, the multiple-impulse maneuver is able to avoid the problem of significant error arising from premature shutdown of single-impulse maneuver. At first, the spaceship turns its orbit from hyperbolic to elliptic when it has reached the perilune for the first time, represented with LOI-1, then if a change to orbital plane is necessary to pass the moon-landing point (the landing usually follows a few courses of spaceship on the lunar orbit), which can be carried out when the spaceship reaches the apolune, represented with LOI-PC; at last, after 2 courses on the elliptic orbit, the orbit of spaceship is turned to be a circle when it reaches the perilune, represented with LOI-2. The whole process is shown as below:

On the condition that the free return trajectory or hybrid trajectory is set, the velocity impulse that LOI-1 requires is:

$$\Delta V_{LOI-1} = \sqrt{\frac{2\mu_L}{r_p} - \frac{\mu_L}{a_p}} - \sqrt{\frac{2\mu_L r_a}{r_p(r_a + r_p)}} \tag{8}$$

Where, a_p represents the semi-major axis of hyperbolic orbit at perilune and r_a the selenocentric distance of apolune on the elliptic orbit.

If the change to orbital plane is necessary, then the velocity impulse required is:

$$\Delta V_{LOI-PC} = 2v_a \sin\frac{\beta_1}{2} \tag{9}$$

Where, v_a represents the velocity of spaceship at apolune on the elliptic orbit and β_1 the angle that the change to orbital plane requires. In order to save energy, the conditions for spaceship passing through the moon-landing point shall be taken into account when the earth-moon transfer trajectory is designed, that is, under normal conditions, the optimal lunar landing trajectory with least energy consumed shall meet the formula $\beta_1 = 0$, which is also one of factors for optimization design.

The velocity that the orbit turned to be a circle requires:

$$\Delta V_{LOI-2} = \sqrt{\frac{2\mu_L r_a}{r_p(r_a + r_p)}} - \sqrt{\frac{\mu_L}{r_p}} \tag{10}$$

Thus, the total velocity impulse for braking at perilune is:

$$\Delta V_{LOI} = \Delta V_{LOI-1} + \Delta V_{LOI-PC} + \Delta V_{LOI-2} \tag{11}$$

(4) Abort Maneuver

First of all, allowing for the visibility constraint of ground tracking telemetry control station, the abort maneuver shall be conducted on the condition that the visibility of control station is guaranteed. If the time of abort maneuver t_a and velocity impulse required Δv_{ab} are design variables, then the initial state is the state of abort trajectory-transfer point and the terminal constraint is as same as that of free return trajectory. Moreover, such constraints as flight time T_{ab} and inclination of return trajectory i_{ab} (determined by the latitude of landing ground when the spaceship returns) shall be met:

$$\psi_{Lab} \le \psi(T_{ab}, i_{ab}, \sigma_{ab}) \le \psi_{Uab} \tag{12}$$

Where, σ_{ab} represents abort trajectory, ψ_{Uab}, ψ_{Lab} represent upper and lower limits of constraint conditions respectively.

3. Integrated Optimization Algorithm Based on Hybrid Method

3.1. Integrated Design Model

If the energy consumption is taken as an indicator of global optimization design and the hybrid trajectory as flight trajectory for earth-moon transfer, then main energy consumption includes ΔV_{TLI} for earth-moon transfer, ΔV_H for hybrid maneuver, ΔV_{LOI} for braking at perilune and ΔV_{ab} for abort. The global optimization problem taking energy consumption as optimization indicator is thus described as follows:

$$\begin{cases} \min \ \ J = K_1 \cdot \Delta V_{TLI} + K_2 \cdot \Delta V_H + K_3 \cdot \Delta V_{LOI} + K_4 \cdot \Delta V_{ab} \\ s.t. \ \ H_i(p,x) = 0 \quad i = 1, \cdots, m \\ \qquad \psi_j(q,x) \le 0 \quad j = 1, \cdots, n \\ \qquad x_L \le x \le x_U \end{cases} \tag{13}$$

Where, $K_i, i = 1, \cdots, 4$ represents the weight of each maneuver; $H(p,x)$ is an equality constraint, $\psi_j(q,x)$ an inequality constraint, $p \ and \ q$ the constraint parameters such as flight time, lunar distance, lunar orbit inclination and re-entry point flight path angle as mentioned in the last section; x is the design variable, $x_U \ and \ x_L$ the upper and lower limits. The design variable x is composed of all maneuver time and velocity impulses required. In addition, in order to obtain the global optimization trajectory, it is allowed to carry out the optimization inside the launch window.

$$t'_{EA} \le t_{EA} \le t''_{EA} \tag{14}$$

The inequality constraint $\psi_j(q,x)$ is determined by (5), (7) and (12):

$$\begin{pmatrix} \psi_{L0} \\ \psi_{LH} \\ \psi_{Lab} \end{pmatrix} \le \psi(q,x) \le \begin{pmatrix} \psi_{U0} \\ \psi_{UH} \\ \psi_{Uab} \end{pmatrix} \tag{15}$$

The equality constraint $H(p,x)$ that the article refers to mainly includes earth-moon transfer trajectory inclination i_{EA} and the selenocentric distance of lunar orbit r_L, namely,

$$H(i_{EA}, r_L, x) = \begin{pmatrix} i_{EA} - i_{EP0} \\ r_L - r_{L0} \end{pmatrix} = 0 \tag{16}$$

In which, i_{EP0} represents parking orbit and r_{L0} the design value of r_L.

3.2. Hybrid Optimization Algorithm

The optimization problem composed of (13) to (16) is a typical constrained nonlinear optimization problem which may be solved with a constrained nonlinear optimization method. Sequential quadratic programming (SQP), an effective method to solve such problems now, possesses global convergence as well as local convergence at least once. But the method is sensitive to initial value so that a good initial point shall be provided. In order to obtain the globally optimal solution, the optimization process shall fall into two steps: Step 1, carry out an optimization design under the condition without perturbation to obtain a globally convergent solution by conducting global search with genetic algorithm; step 2, draw the optimal solution under perturbation condition by using the optimal solution drawn from genetic algorithm as the initial value of SQP.

4. Simulations

The paper took the year of 2025 as an example so as to calculate the launch window of free return trajectory after injection from the near-earth orbit and design a mission trajectory and an abort trajectory which meet the constraint conditions. The input conditions for calculation and simulation results drawn are as follows:

4.1 Basic Input

The following basic input conditions shall be taken into account: 1) The lunar interface point with a moon longitude and latitude of $(-21.0^{\circ}, -18.0^{\circ})$ selected to be the moon-landing point; 2) the lunar orbit with a height of 350km and the circular orbit with an inclination of 30°; 3) the circular lunar orbit with a height of 100km; in the first manned moon-landing mission, the abort can only be required once at most.

4.2 Simulation Results and Analysis

If the integrated design is carried out in the moon-landing point window with a minimum earth-moon distance and the flight time of both earth-moon transfer and moon-earth transfer is within 3 days, then the design results of mission trajectories can be drawn. The mission trajectories are, in turn, free return trajectory, hybrid trajectory, lunar orbit and abort trajectory, in which the earth-moon transfer trajectory, hybrid trajectory and abort trajectory are given under J2000 geocentric inertial coordinate and the lunar orbit given under selenocentric inertial coordinate system. The flight time when perturbation is considered is about 66.229h. The results can be seen in Tab. 1 and Fig. 1.

Table 1. Design results

Track epoch	a	e	i	Ω	w	f
2460705.5959721	260461.510	0.974532	28.7456	359.5612	186.0453	0.0000
2460706.0512347	249932.012	0.973232	29.1456	359.2563	187.4203	160.1562
2460708.4933917	1838.000	0.000000	134.1925	153.2004	0.0000	166.0311
2460708.4203122	308561.566	0.978600	80.3124	176.1264	352.9055	190.4780

When STK is inputted into the manned moon-landing trajectory and abort trajectory towards which the perturbation is considered, the demonstration results can be drawn, proving the effectiveness of integrated design method of hybrid trajectory and abort trajectory.

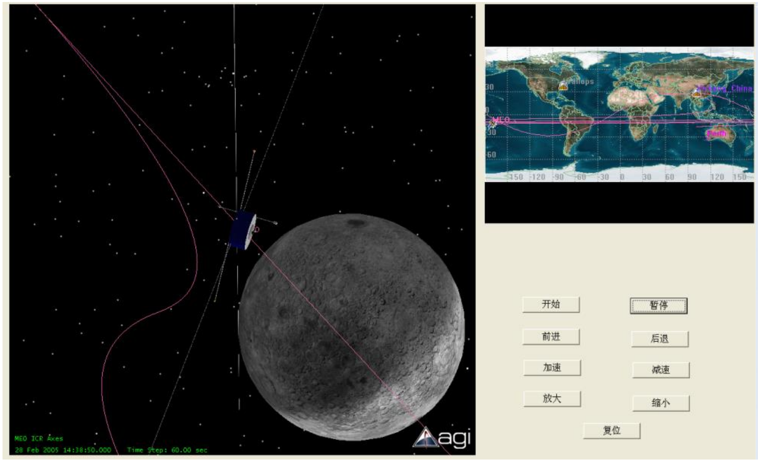

Figure 1. STK simulation

5 Conclusions

Conclusions can be drawn from the design results and simulation verification that the desirable hybrid/abort trajectory for moon exploration can be obtained by the integrated design method proposed in the article starting from free return trajectory. The article focused on the analysis of integrated design of joint trajectory with abort constraint considered and it turned out that the hybrid trajectory is more complicated than free return trajectory with more design variables and difficulties in the optimization on one hand and the abort near lunar orbit more complicated on the other hand. Relatively satisfactory results had been achieved with respect to solving the problem by adopting the genetic algorithm and SQP hybrid algorithm in the article. The current results are just preliminary simulation exploration and next step we will take more actual constraints into accounts, making the design more practical.

6. References

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