STATION-KEEPING AND FORMATION FLYING FOR PERIODIC ORBIT AROUND LAGRANGIAN POINTS BY FOURIER SERIES

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Abstract: *This paper studies a control law to inject into and maintain the desired periodic or quasi-periodic orbit in the vicinity of unstable equilibrium points in the circular restricted three-body problem. For ideal case, since these orbits are natural periodic orbits, no control input is required for station-keeping. From this point of view, the tracking problem of a periodic and quasi-periodic orbit is considered. Utilizing a Fourier series, reference orbits can be expressed in analytical form and the tracking problem can be formulated by the output regulation theory. Solving the output regulation problem, the fuel efficient controller to realize the tracking trajectory is constructed. To validate the controller, simulation studies are conducted for the periodic orbit around the Sun-Earth* L_2 *point. The velocity change necessary for the maintenance of the periodic orbit is calculated for one period as a function of the Fourier series approximation order. Simulation results show that the proposed controller realize the injection to and maintenance of the periodic orbit with small fuel consumption. Moreover the higher Fourier series approximation order is, the smaller the velocity change for station-keeping is.*

Keywords: *Circular Restricted Three-Body Problem, Fourier Series, Periodic Orbit, Output Regulation Problem.*

1. Introduction

Recently unstable orbits in the vicinity of the libration points, so-called Lagrangian points, in the circular restricted three-body problem have attracted much attention, and station-keeping on them has been studied by many authors [1, 2, 3, 4, 5]. According to Dunham, there are mainly two strategies to implement station-keeping and formation flying : “tight” and “loose” strategies [1]. Tight methods based on the Lyapunov stability were proposed in [2, 4] to stabilize it in the tighter bounds of the nominal orbit, and loose methods based on Floquet and invariant manifolds theories were proposed in [3, 5] for the stabilization of spacecraft trajectory in the neighborhood of the nominal orbit. For an another “loose” approach, the use of the eigenstructure of the system was first investigated by Barden and Howell [6] to categorize the types of motion near the collinear libration points based on the center manifold for flying a constellation of spacecraft. However, the center manifold is itself unstable since it is strongly affected from the unstable manifold. To overcome the problem, a simple feedback law were proposed by Scheeres [7] for the formation flying along the halo orbits by stabilizing unstable orbits and creating the additional center manifold. Based on the output regulation theory of the linear system [8] and by utilizing the small periodic orbit in the vicinity of collinear libration points studied by Wie [9], a new strategy to realize the formation flying and station-keeping was proposed by Bando and Ichikawa [10].

In this paper, station-keeping and formation flying for periodic orbit around the collinear Lagrangian points are considered [11, 12]. The objective of this paper is to propose a “tight” control strategy for station-keeping and formation flying for periodic orbit around any collinear Lagrangian point for any state of the spacecraft which is stable and robust. The proposed strategy is similar to that of [10], but they derive the output regulation theory along the reference orbit and LQR is applied for station-keeping while our strategy formulates around the libration points and the output regulation theory is applied for station-keeping. Reference orbits or paths are generated by a simple mathematical model referred to as an exosystem. Note that periodic orbits around the Lagrangian points cannot be expressed by an analytic function due to non-integrability of the three-body problem, and these are only numerically available. Thus to obtain analytical expression, they have to be approximated by any function. Therefore it is natural to use Fourier series to approximate periodic and quasi-periodic orbits. Assuming that exosystem generates reference orbits expressed in the form of Fourier series, the output regulation problem in three-body problem can be implement. Based on this idea, the control law for the injection to and maintenance of reference orbits is constructed by the output regulation theory for linear system [8]. The algebraic Riccati equation of the linear quadratic regulator (LQR) thoery is imployed to design stabilizing feedback. As a performance index, the velocity change (L_1 norm of the input) necessary for the maintenance of the periodic orbit is calculated. The advantages of the proposed method are that the control method is simple and stable for any initial state, and that since the frequencies and amplitudes of the reference orbits are arbitrary, formation flying such as shifted orbits or integer multiple frequency orbits can be designed easily.

To show the applicability of the control law, the Sun-Earth circular restricted three-body problem (CR3BP) is considered. Lagrangian point is fixed to L_2. The halo orbit is specified as the reference orbit, and the numerical solution of a halo orbit is represented by an approximate function based on interpolation. Tracking problem of the halo orbit is demonstrated and the relation between approximation order and the velocity change required for station-keeping is revealed.

2. Equations of Motion in CR3BP

In the circular restricted three-body problem (CR3BP), the equations of motion in the non-dimensional form [9] are expressed as

$$\begin{aligned} X'' - 2Y' - X &= -\frac{1-\rho}{r_1^3}(X+\rho) - \frac{\rho}{r_2^3}(X-1+\rho) + u_x \\ Y'' + 2X' - Y &= -\frac{1-\rho}{r_1^3}Y - \frac{\rho}{r_2^3}Y + u_y \\ Z'' &= -\frac{1-\rho}{r_1^3}Z - \frac{\rho}{r_2^3}Z + u_z \end{aligned} \tag{1}$$

where $\{X,Y,Z\}$ is the rotating frame whose origin is the barycenter of the system, the coordinates are normalized by the distance between two main bodies and time by the period of the circular orbit, $\rho = M_2/(M_1+M_2)$, M_1 and M_2 are the masses of the two main bodies with $M_1 > M_2$, and

$$\begin{aligned} r_1 &= [(X+\rho)^2 + Y^2 + Z^2]^{1/2} \\ r_2 &= [(X-1+\rho)^2 + Y^2 + Z^2]^{1/2} \end{aligned}$$

The differentiation with respect to the non-dimensional time is denoted by $'$. Eq. (1) has stationary points known as Lagrangian points L_i satisfying [13, 14]

$$\begin{aligned} X &= \frac{1-\rho}{r_1^3}(X+\rho)+\frac{\rho}{r_2^3}(X-1+\rho) \\ Y &= \frac{1-\rho}{r_1^3}Y+\frac{\rho}{r_2^3}Y \\ Z &= 0 \end{aligned} \tag{2}$$

and

$$\begin{aligned} L_1 &= (l_1(\rho),0,0), \quad L_2=(l_2(\rho),0,0), \quad L_3=(l_3(\rho),0,0) \\ L_4 &= (1/2-\rho,\sqrt{3}/2,0), \quad L_5=(1/2-\rho,-\sqrt{3}/2,0) \end{aligned}$$

where $l_i(\rho)$ are determined by setting $Y=0$ and solving the first equation of Eq. (2). To describe the motion near a collinear equilibrium point L_i ($i=1,2,3$), it is convenient to use the coordinate system with the origin at L_i. Replacing $\{X,Y,Z\}$ by $\{x+l_i,y,z\}$, Eq. (1) can be rewritten as

$$\begin{aligned} x''-2y'-x &= -\frac{1-\rho}{r_1^3}(x+l_i+\rho)-\frac{\rho}{r_2^3}(x+l_i-1+\rho)+l_i+u_x \\ y''+2x'-y &= -\frac{1-\rho}{r_1^3}y-\frac{\rho}{r_2^3}y+u_y \\ z'' &= -\frac{1-\rho}{r_1^3}z-\frac{\rho}{r_2^3}z+u_z \end{aligned} \tag{3}$$

where

$$\begin{aligned} r_1 &= [(x+l_i+\rho)^2+y^2+z^2]^{1/2} \\ r_2 &= [(x+l_i-1+\rho)^2+y^2+z^2]^{1/2} \end{aligned}$$

The state space form of Eq. (3) can be represented as

$$\boldsymbol{x}' = \boldsymbol{A}\boldsymbol{x}+\boldsymbol{B}\boldsymbol{f}(\boldsymbol{x})+\boldsymbol{B}\boldsymbol{u} \tag{4}$$

where $\boldsymbol{x}=[x,y,z,x',y',z']^T$, $\boldsymbol{u}=[u_x,u_y,u_z]^T$, and

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 2\sigma_i+1 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1-\sigma_i & 0 & -2 & 0 & 0 \\ 0 & 0 & -\sigma_i & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} -\frac{1-\rho}{r_1^3}(x+l_i+\rho)-\frac{\rho}{r_2^3}(x+l_i-1+\rho)+l_i-2\sigma_i x \\ -\frac{1-\rho}{r_1^3}y-\frac{\rho}{r_2^3}y+\sigma_i y \\ -\frac{1-\rho}{r_1^3}z-\frac{\rho}{r_2^3}z+\sigma_i z \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}$$

$$\sigma_i = \frac{1-\rho}{|l_i(\rho)+\rho|^3}+\frac{\rho}{|l_i(\rho)-1+\rho|^3}$$

3. Constructing controller

In this section, the controller to achieve transition to and maintenance of periodic and quasi-periodic orbits is constructed. Since tracking these orbits can be satisfied the requirement of the desired controller, an output regulation problem which can deal with the tracking problem is applied. In solving an output regulation problem, an exosystem generating the desired reference signals must be described in a space state form as an autonomous system (*i.e.*the system without inputs). However, since the analysis solution for the periodic or quasi-periodic orbit in CR3BP does not exist, it is impossible to construct the *exact* exosystem generating the periodic or quasi-periodic orbit. Therefore by approximating these orbits with Fourier series, the *approximate* exosystem need to be designed. Here, $(x_{ref}, y_{ref}, z_{ref})$ is a reference orbit approximated in n^{th} order Fourier series and is given by

$$\begin{aligned} x_{ref}(t) &\approx a_0^x + \sum_{k=1}^{n}(a_k^x \cos k\omega_x t + b_k^x \sin k\omega_x t) \\ y_{ref}(t) &\approx a_0^y + \sum_{k=1}^{n}(a_k^y \cos k\omega_y t + b_k^y \sin k\omega_y t) \\ z_{ref}(t) &\approx a_0^z + \sum_{k=1}^{n}(a_k^z \cos k\omega_z t + b_k^z \sin k\omega_z t) \end{aligned} \tag{5}$$

This is a quasi-periodic orbit in general and becomes periodic orbit as a special case if $\omega_x = \omega_y = \omega_z$. The orbit described by Eq. (5) is generated by the following exosystem:

$$\dot{\boldsymbol{w}} = \boldsymbol{S}\boldsymbol{w} \ , \ \boldsymbol{w}(0) = \boldsymbol{w}_0 \tag{6}$$

where $\boldsymbol{w}$ is a $3n$ vector, $\boldsymbol{S}$ is a $3n \times 3n$ matrix, and

$$\begin{aligned} \boldsymbol{w} &= [\boldsymbol{w}^x, \boldsymbol{w}^y, \boldsymbol{w}^z]^T \ , \ \boldsymbol{w}^i = [w_0^i, w_1^i, \overline{w}_1^i, w_2^i, \overline{w}_2^i, \cdots, w_n^i, \overline{w}_n^i]^T \\ \boldsymbol{w}_0 &= [\boldsymbol{w}_0^x, \boldsymbol{w}_0^y, \boldsymbol{w}_0^z]^T \ , \ \boldsymbol{w}_0^i = [a_0^i, a_1^i, b_1^i, a_2^i, b_2^i, \cdots, a_n^i, b_n^i]^T \\ \boldsymbol{S} &= diag[0, \boldsymbol{D}_1^x, \boldsymbol{D}_2^x, \cdots, \boldsymbol{D}_n^x, 0, \boldsymbol{D}_1^y, \boldsymbol{D}_2^y, \cdots, \boldsymbol{D}_n^y, 0, \boldsymbol{D}_1^z, \boldsymbol{D}_2^z, \cdots, \boldsymbol{D}_n^z] \\ \boldsymbol{D}_n^i &= \begin{bmatrix} 0 & d_n^i \\ \overline{d}_n^i & 0 \end{bmatrix} \ , \ d_n^i \overline{d}_n^i = -(n\omega_i)^2 \ , \ (i = x, y, z) \end{aligned}$$

By using the solution of Eq. (6), the reference orbit Eq. (5) are expressed as

$$x_{ref}(t) = \sum_{k=0}^{n} w_k^x = \boldsymbol{L}\boldsymbol{w}^x \ , \ y_{ref}(t) = \sum_{k=0}^{n} w_k^y = \boldsymbol{L}\boldsymbol{w}^y \ , \ z_{ref}(t) = \sum_{k=0}^{n} w_k^z = \boldsymbol{L}\boldsymbol{w}^z$$

where $\boldsymbol{L} = [1, 1, 0, 1, 0, \cdots, 1, 0]$.

In station-keeping and formation flying, the spacecraft must track the periodic or quasi-periodic orbit. This tracking problem can be solved by the linear output regulation problem. Recall that the output regulation problem for a general linear system

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{B}_1\boldsymbol{w} \\ \boldsymbol{e} &= \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} + \boldsymbol{D}_1\boldsymbol{w} \\ \dot{\boldsymbol{w}} &= \boldsymbol{S}\boldsymbol{w} \ , \ \boldsymbol{w}(0) = \boldsymbol{w}_0 \end{aligned}$$

is to find a stabilizing feedback control law such that $\boldsymbol{e} \to 0$ as time goes to infinity for any initial conditions $x(0)$ and $w(0)$ [8]. This considered problem is solvable if and only if there exist matrices $(\boldsymbol{\Gamma}, \boldsymbol{\Pi})$ which satisfied the following regulator equations

$$\begin{aligned} \boldsymbol{A}\Pi - \Pi\boldsymbol{S} + \boldsymbol{B}\Gamma + \boldsymbol{B}_1 &= 0 \\ \boldsymbol{C}\Pi + \boldsymbol{D}\Gamma + \boldsymbol{D}_1 &= 0 \end{aligned}$$

An admissible feedback control law is given by

$$\boldsymbol{u} = -\boldsymbol{F}\boldsymbol{x} + (\Gamma + \boldsymbol{F}\Pi)\boldsymbol{w}$$

where $\boldsymbol{F}$ is an arbitrary matrix such that $\boldsymbol{A} - \boldsymbol{B}\boldsymbol{F}$ is stable. In addition, Γ and Π specify the asymptotic behavior of the input and the state, respectively. In fact, the following equations are satisfied as $t \to \infty$:

$$\begin{aligned} \boldsymbol{u}(t) &\to \Gamma\boldsymbol{w}(t) \\ \boldsymbol{x}(t) &\to \Pi\boldsymbol{w}(t) \end{aligned}$$

Though Eq. (4) includes nonlinear term, the linear output regulation problem can be applied by employing the input feedback linearization. Applying the nonlinear feedback control $\bar{\boldsymbol{u}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{u}$, Eq. (4) can be rewritten as

$$\boldsymbol{x}' = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\bar{\boldsymbol{u}} \tag{7}$$

Based on Eq. (7), the controller to tracking the periodic and quasi-periodic orbits is designed. In our tracking problem, the spacecraft position (x, y, z) must track $(x_{ref}, y_{ref}, z_{ref})$ asymptotically. The error $\boldsymbol{e}$ is expressed as

$$\boldsymbol{e} = \begin{pmatrix} x - x_{ref} \\ y - y_{ref} \\ z - z_{ref} \end{pmatrix} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}_1\boldsymbol{w} \tag{8}$$

where $\boldsymbol{C} = [\boldsymbol{I}_3, \boldsymbol{O}_3]$ and

$$\boldsymbol{D}_1 = -diag[\boldsymbol{L}, \boldsymbol{L}, \boldsymbol{L}]$$

From Eqs. (6), (7) and (8), the regulator equations are given by

$$\boldsymbol{A}\Pi - \Pi\boldsymbol{S} + \boldsymbol{B}\Gamma = 0 \tag{9}$$

$$\boldsymbol{C}\Pi + \boldsymbol{D}_1 = 0 \tag{10}$$

By solving these regulator equations, the solution $(\boldsymbol{\Gamma}, \boldsymbol{\Pi})$ are obtained as

$$\boldsymbol{\Pi} = \begin{bmatrix} -\boldsymbol{D}_1 \\ \boldsymbol{\Omega} \end{bmatrix} \quad , \quad \boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\Gamma}^x & -2\boldsymbol{\Omega}^y & \boldsymbol{0} \\ 2\boldsymbol{\Omega}^x & \boldsymbol{\Gamma}^y & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\Gamma}^z \end{bmatrix} \tag{11}$$

where

$$\begin{aligned}
\Omega &= diag[\Omega^x,\Omega^y,\Omega^z] \\
\Omega^i &= [0,0,\omega_i,0,2\omega_i,\cdots,0,n\omega_i] \quad , \quad (i=x,y,z) \\
\Gamma^x &= -[2\sigma_i+1,\omega_x^2+2\sigma_i+1,0,4\omega_x^2+2\sigma_i+1,0,\cdots,n^2\omega_x^2+2\sigma_i+1,0] \\
\Gamma^y &= -[1-\sigma_i,\omega_y^2-\sigma_i+1,0,4\omega_y^2-\sigma_i+1,0,\cdots,n^2\omega_y^2-\sigma_i+1,0] \\
\Gamma^z &= -[-\sigma_i,\omega_z^2-\sigma_i,0,4\omega_z^2-\sigma_i,0,\cdots,n^2\omega_z^2-\sigma_i,0]
\end{aligned}$$

Hence, the controller to track the periodic or quasi-periodic orbit is obtained as

$$\boldsymbol{u} \;=\; -\boldsymbol{F}\boldsymbol{x}-\boldsymbol{f}(\boldsymbol{x})+(\Gamma+\boldsymbol{F}\Pi)\boldsymbol{w} \tag{12}$$

where $\boldsymbol{F}$ is an arbitrary matrix such that $\boldsymbol{A}-\boldsymbol{B}\boldsymbol{F}$ is stable.

In fact, by applying the feedback control (12), the state (x,y,z) tracks the reference periodic or quasi-periodic orbit asymptotically for any initial position (x_0,y_0,z_0). Especially when $\omega_x=\omega_y=\omega_z$ or $\omega_x=\omega_y\neq\omega_z$, the spacecraft converges to a halo orbit or a Lissajous orbit, respectively. The reference orbit gets more accurate approximation of the periodic or quasi-periodic orbit as approximation order n is higher. Since a halo orbit and Lissajous orbit are natural orbits, no control is required on these orbit to maintain them for ideal case. Therefore, the necessary input ΔV for station-keeping is expected to be reduced significantly for sufficiently large n. This prediction is discussed in Sec. 3.

4. Simulation Results for Sun-Earth CR3BP

In this section, the control law (12) is applied to the nonlinear equations of motion (1) for the Sun-Earth CR3BP. First, the problem tracking to the halo orbit is considered to verify the controller. Then, it is verified that the necessary input ΔV decreases as approximation order n increases. The Lagrangian point is specified as L_2. The period and the radius of the Sun-Earth system are assumed to be $T_0=365.26$ days and $R_0=1.4960\times10^8$ km, respectively. Then, the parameters are $\rho=3.0542\times10^{-6}$, $l_2=1.0101$, $\sigma_2=3.9393$. Initial condition for the halo orbit is given by the normalized units

$$\bar{\boldsymbol{x}}_0=[-1.6623\times10^{-3},0.0000,1.0000\times10^{-3},0.0000,9.8104\times10^{-3}]^T \tag{13}$$

and its period is $T=3.1026$ (180.36 days). Note that for the numerical integration, the ode113 solver of the MATLAB R2013a is used. Parameters such as ρ,σ are given in 15 digits.

4.1. Tracking to the halo orbit

The initial condition of the halo orbit cannot be described in analytical form due to the non-integrability of the three-body problem. Therefore, the differential correction method is often used to obtain the initial condition of the halo orbit numerically [11, 15]. The trajectory with the initial condition (13) is not exactly periodic, and it diverges away within four periods. Thus the reference orbit is modified to be periodic by the extension of the trajectory of the first period. That is, the

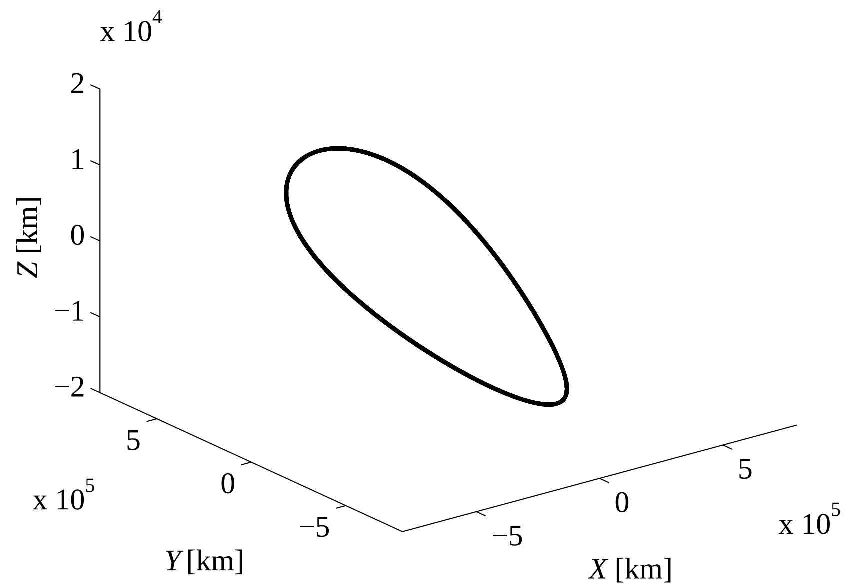

Figure 1. Pseudo-periodic orbit

Table 1. Coefficients and frequencies of Fourier series($n = 3$)

ω_x	2.0251	ω_y	2.0251	ω_z	2.0251
a_0^x	-4.1560e-04	a_0^y	-6.4446e-13	a_0^z	-1.8866e-05
a_1^x	-1.4555e-03	a_1^y	4.3523e-07	a_1^z	1.1172e-04
b_1^x	-2.1072e-08	b_1^y	4.5589e-03	b_1^z	-4.9359e-09
a_2^x	1.8091e-04	a_2^y	1.8197e-08	a_2^z	5.9955e-06
b_2^x	5.2378e-09	b_2^y	9.5306e-05	b_2^z	-5.2979e-10
a_3^x	2.2104e-05	a_3^y	6.8997e-09	a_3^z	9.5299e-07
b_3^x	9.5983e-10	b_3^y	2.4094e-05	b_3^z	-1.2632e-10

state of the reference orbit after each period is corrected to the initial condition (13). The modified reference orbit, referred to as "*pseudo-periodic orbit*", is shown in Fig. 1. By approximating the time history of the position of the pseudo-periodic orbit in Fourier series by MATLAB Curve Fitting Toolbox, the coefficients and frequencies with the approximation order $n = 3$ are shown in Tab. 1.

As a preliminary step, this feedback gain $\boldsymbol{F}$ is designed by the linear quadratic regulator (LQR) theory to minimize the cost function and is given by $\boldsymbol{F} = \boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}$, where $\boldsymbol{P}$ is the solution of the algebraic Ricatti equation

$$\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} + \boldsymbol{Q} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P} = \boldsymbol{0}$$

and the weighting matrices $\boldsymbol{Q} = \boldsymbol{I}_6$ and $\boldsymbol{R} = \boldsymbol{I}_3$.

Here, the nonlinear feedback control (12) is applied to the nonlinear equations of motion (1). Setting the initial state $\boldsymbol{x}_0 = 2 \times \bar{\boldsymbol{x}}_0$ and the coefficients and frequencies shown in Tab. (1) are used

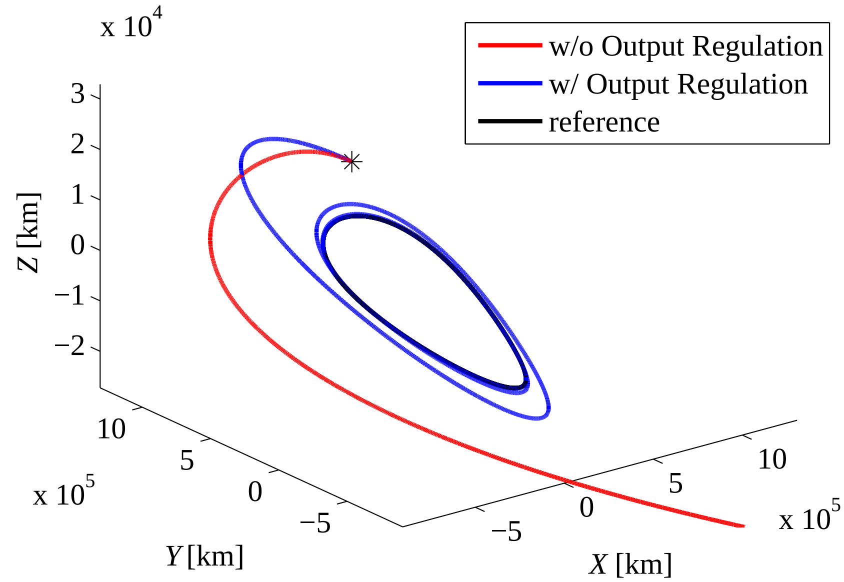

Figure 2. Trajectories ($n = 3$)

to describe the initial state of exosystem w_0, the controlled trajectory is shown in Fig. 2. The blue and red lines are controlled and uncontrolled trajectory respectively, black line is reference orbit, and $*$ is the initial position. The time history of the input is shown in Fig. 3, and Fig. 4 is the enlargement of Fig. 3 .

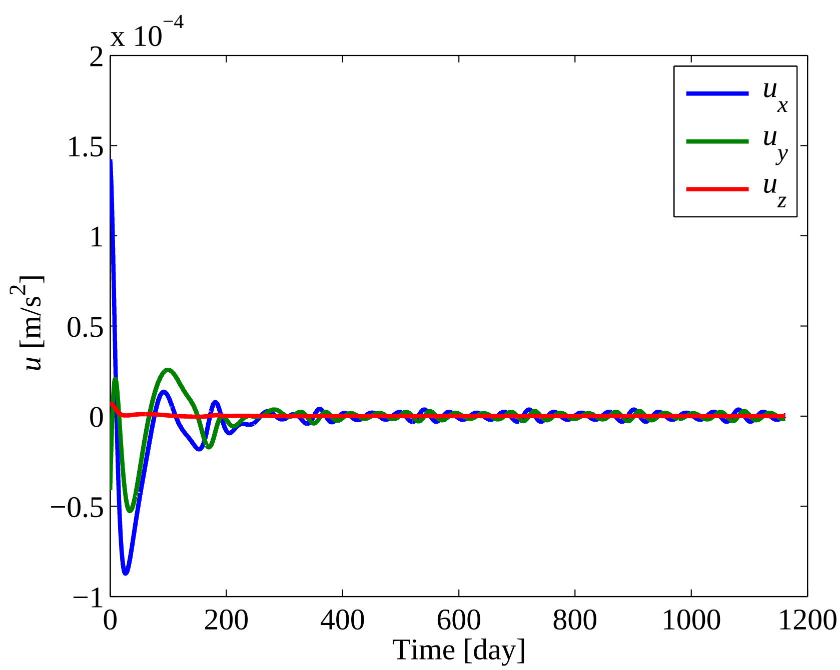

Figure 3. Time history of input ($n = 3$)

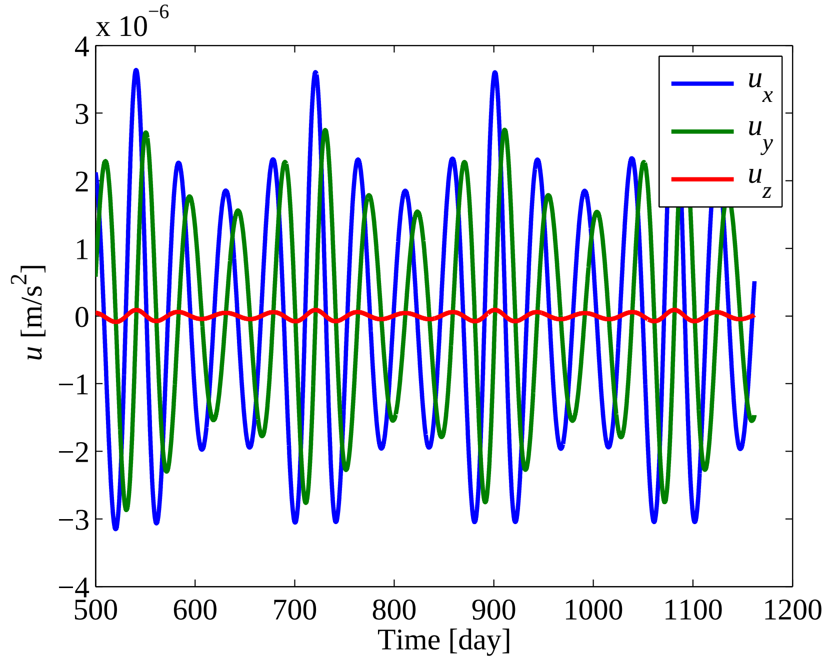

Figure 4. Time history of input ($n = 3$) (enlargement)

Figure 2 shows that by applying the control (12), the state asymptotically tracks the pseudo-periodic orbit while the initial condition diverging away without any control. It can be seen from Figs. 3 and 4 that the tracking orbit is realized with very small input and the input gets smaller as time passes. Moreover the input becomes periodic after sufficiently long period. This is because that the input $\boldsymbol{u}$ converges to $\Pi \boldsymbol{w} - \boldsymbol{f}(\boldsymbol{x})$ as time passes. After $T_c = 3 \times T = 9.3078$ (541.09 days), the trajectory is regarded as on the reference orbit with the convergence level $C_{error} = \sqrt{e_x^2 + e_y^2 + e_z^2} = 1e-5$. Therefore, in the following discussion, the necessary input ΔV for station-keeping is calculated at the fourth period.

4.2. The relation between approximation order and ΔV

In the previous section, the prediction that the necessary input ΔV for station-keeping becomes smaller as n increase is raised. Here, ΔV (the absolute integral (L_1 norm) of the inputs) for the in-plane motion and out-plane motion as a function of the approximation order n are calculated respectively for the fourth period and are shown in Fig. 5 with $n = 1, 2, \cdots, 8$.

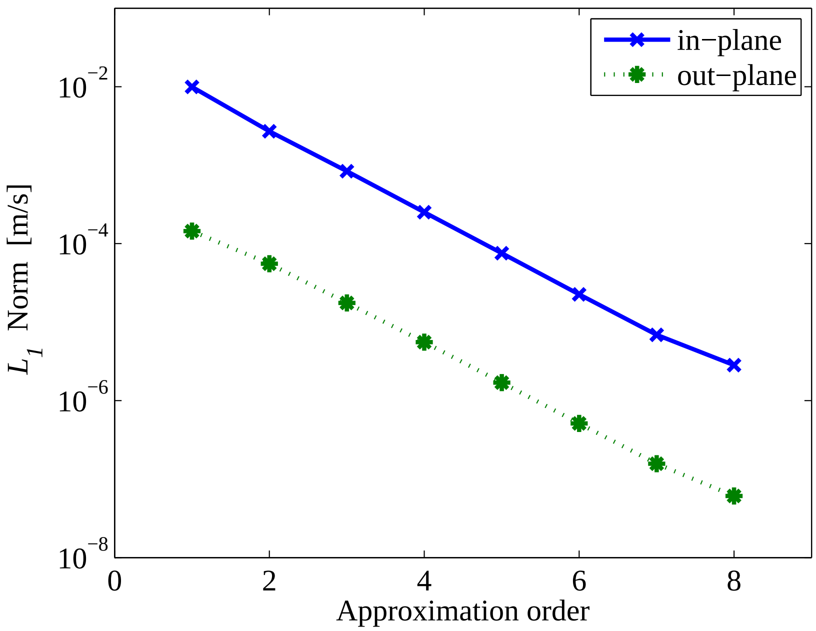

Figure 5. ΔV **vs** n

Figure 5 demonstrates that the higher the order of the approximation is, the smaller ΔV for station-keeping is. This is reasonable result because the higher order approximation produce the closer orbit to the reference orbit, and no control input is required on the natural periodic orbit.

5. Conclusion

A new control strategy based on output regulation theory is proposed for asymptotic tracking of the periodic and quasi-periodic orbit in the vicinity of the Lagrangian points in the circular restricted three-body problem. To overcome the difficulty of no analysis solution for these orbits, a Fourier series approximation is employed. The exosystem to generate the desired periodic or quasi-periodic orbit is formulated, and then the linear output regulation theory is applied. First the proposed control method is applied to the halo orbit and the usefulness of the proposed method is verified. Then the relation between the cost for station-keeping and approximation order of Fourier series is discussed, and reveals that the cost becomes significantly smaller as the approximation order increases.

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