MULTI-TETHERED PYRAMIDAL SATELLITE FORMATION: DYNAMICS AND CONTROL

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Abstract: *This paper is devoted to the dynamics of a multi-tethered pyramidal satellite formation rotating about its axis of symmetry in the nominal mode. Whereas the combination of rotation and gravity-gradient forces is insufficient to maintain the mutual positions of satellites, they are assumed to be equipped with low-thrust rocket engines. We propose a control strategy that allows the stabilization of the nominal spin state and demonstrate the system's proper operation by numerically simulating its controlled motion. The discussed multi-tethered formations could be employed, for example, to provide co-location of several satellites at a slot in geostationary orbit.*

Keywords: *Tethered Satellite Systems, Satellite Formations, Dynamics*

1. Introduction

This paper presents a study of the multi-tethered satellite formation's dynamics. This formation consists of a main body connected by means of cables with several deputy satellites. The deputy satellites are only connected with the main body and not with each other. In nominal regime it has a shape of a pyramid with its top directed towards the Earth (Fig. 1). To keep the tethers taut the formation is spinning, but the combination of rotation with gravity-gradient is insufficient for it and therefore the deputy satellites are equipped with low-thrust engines to stabilize the desired dynamics of the system.

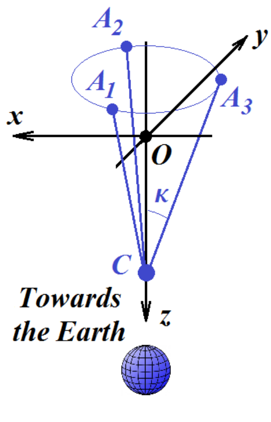

Figure 1. Earth-facing multi-tethered pyramidal satellite formation: *C* – main satellite, A_1, A_2, A_3 - deputy satellites

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If tethers are long enough such a formation can be used to maintain the main body in geosynchronous motion (i.e., in the motion with an orbital period of one sidereal day) below the geostationary orbit (assuming that the mass center O of formation moves in GEO). If tethers are not too close to the formation's mass center O, an "ordinary" geostationary satellite can be placed inside the discussed pyramidal structure. And, of course, any other applications mentioned typically in papers on the dynamics of multi-tethered formations remain relevant (space interferometry, multi-point measurements, etc).

I. Bekey was probably the first to discuss three-dimensional multi-tethered formations, he proposed a double-pyramid configuration [4]. Then the dynamical properties of multi-tethered formations were intensively studied.

To specify formations comprised of a main body and deputy satellites attached to the main body by tethers (as in our case) A. Pizarro-Chong and A.K. Misra introduced the term "hub-and-spoke" [13]. The behavior of such formations was studied in different dynamical environment: in circular orbit [2], in elliptic orbit [3], in halo-orbit [25, 5] and near collinear Lagrangian points [22].

Interesting examples of 3D equilibrium configurations of a chain of four satellites connected by three weightless rods were presented in [7], although at least one rod in these configurations is compressed and thus cannot be replaced with a tether.

Worthy of special mention is a recent series of papers by H. Schaub, C.R. Seubert et al. [15, 16, 12, etc.], where the concept of the Tethered Coulomb Structure (TCS) is introduced. In general TCS is a 3D structure consisting of discrete spacecraft components (nodes) connected by tethers. The discrete components are electrostatically charged to produce repulsive forces between them and to prevent the slack of any cable. The convincing justification of the TCS technical feasibility is provided, several control algorithms are developed to maintain the desirable attitude motion of such a structure. Nevertheless, it appears that the Coulomb repulsive forces are the effective countermeasure against slacks only in the case of relatively close nodes: the discussed length of the tethers in [15, 16, 12] has an order of magnitude of 10 m.

The SPHERES (Synchronized Position Hold Engage and Reorient Experimental Satellite) system, developed by MIT Space Systems Laboratory, NASA, DAPRA and Aurora Flight Sciences, was proposed as a testbed for dynamical experiments with tethered formations [6]. There are three SPHERES satellites currently onboard the International Space Station. Being equipped with twelve carbon dioxide thrusters these satellites can maneuver with great precision in the ISS interior safety for its crew.

As a novel trend one can distinguish the studies on dynamics of space webs composed of large number of spacecrafts connected by tethers (e.g., [11, 24]). Evidently, sophisticated control strategy is required to maintain the desirable shape of a space web and to provide the desired orbital and attitude dynamics. In particular, this strategy should take into account the gravity-gradient effects and centrifugal forces applied to such a very flexible structure. Below we will be dealing with similar requirements.

More references can be found in reviews [8, 20], where special sections are devoted to multi-tethered formations.

Since multi-tethered satellite formation is a mechanical system with a very large number of degrees of freedom, in general its dynamics is rather complicated. Taking into account that the satellites' dimensions are much smaller than length of the tethers the former are usually approximated as point masses. Another commonly used simplification is an assumption that the tethers are weightless. Some authors employ the so-called lumped mass discretization to analyze the dynamics of the system in a more realistic way (e.g., [1, 2, 21]).

In Section 2 we start with the consideration of a simplified dynamical model of multi-tethered formation (point masses + massless tethers). We derive a control strategy allowing to maintain the formation in the uniform rotation around the local vertical and provide the stability conditions of controlled motion. In Section 3 we present the results obtained by numerical simulation of the system's dynamics.

2. Control strategy to maintain the system's rotation about the local vertical

2.1. Basic assumptions and equations of motion

As stated in the Introduction, we will consider a space system composed of the $N+1$ bodies: the main satellite C of mass m_1 and N deputy satellites $A_1,\dots,A_N$ of mass m_2 each. The deputy satellites are linked to the main satellite by tethers; all the tethers are assumed to be identical; unless otherwise stated the tethers' masses are ignored.

The desired nominal mode of motion is a uniform rotation of the system about the local vertical (i.e., a straight line running from the Earth center of mass (CoM) to the system CoM O). In the nominal motion the main satellite is located on the local vertical at a distance d_* from the CoM O and the deputy satellites move around in a circle in the plane normal to the local vertical; the neighboring satellites are located the same distance apart (Fig. 1). To maintain a system in such a rotation the deputy satellites are equipped with low-trust engines.

Developing a control strategy we will use the central field approximation for the Earth gravity field (in Section 3 we present the results of simulations demonstrating the efficiency of the proposed strategy for a more realistic model of the space environment including, in particular, high-degree Earth Gravitational model, Moon and Sun perturbations). The system CoM O moves nominally in circular orbit with the mean motion ω_0.

To write down the motion equations, we introduce a Local Vertical Local Horizontal (LVLH) reference frame, centered on the nominal position of the system CoM in its orbital motion: the Oz axis is aligned with the local vertical, the Ox is running tangentially to the orbit in the direction of the CoM O motion and the Oy axis is directed along the normal line to the orbit plane.

We start with the equations describing the deputy satellite motions:

$$m_2\ddot{\mathbf{r}}_i = \mathbf{F}_i^e + \mathbf{F}_i^{cor} + \mathbf{T}_i + \mathbf{U}_i, \quad i = 1,\dots,N. \tag{1}$$

Here $\mathbf{F}_i^e = (0, -m_2\omega_o^2 y_i, 3m_2\omega_o^2 z_i)^T$ is a sum of the gravity and inertia forces acting in the non-inertial reference frame LVLH, $\mathbf{F}_i^{cor} = (2m_2\omega_0\dot{z}_i, 0, -2m_2\omega_0\dot{x}_i)^T$ is the Coriolis force, $\mathbf{U}_i$ is the control action, and $\mathbf{T}_i$ is the tension force applied to *i*th deputy satellite. We adopt the usual model of the massless visco-elastic tether. In this case

$$\mathbf{T}_i = \begin{cases} [k_{tether}\left(|\mathbf{r}_C - \mathbf{r}_i| - l_0\right) + b_{tether}\dfrac{d}{dt}|\mathbf{r}_C - \mathbf{r}_i|]\dfrac{(\mathbf{r}_C - \mathbf{r}_i)}{|\mathbf{r}_C - \mathbf{r}_i|}, & \text{if } |\mathbf{r}_C - \mathbf{r}_i| > l_0 \\ 0, & \text{otherwise} \end{cases}$$

where k_{tether}, b_{tether} and l_o denote the rigidity of the tether, its viscosity and length in non-deformed state respectively. As follows from the formula for $\mathbf{T}_i$, the tether is subject to tension only.

The main satellite's motion is governed by the equation

$$m_1\ddot{\mathbf{r}}_C = \mathbf{F}_C^e + \mathbf{F}_C^{cor} - \sum_{i=1}^{N}\mathbf{T}_i. \tag{2}$$

Like in the previous case, $\mathbf{F}_C^e$ is the sum of gravity and inertia forces and $\mathbf{F}_C^{cor}$ is the Coriolis force. We recall that no control actions are applied to the main satellite.

2.2. Nominal system motion

In the nominal motion the satellite formation rotates as a rigid body about the local vertical with a relative angular velocity ω_* in the LVLH reference frame:

$$\mathbf{r}_C^* = \begin{pmatrix} 0 \\ 0 \\ -d_* \end{pmatrix}, \quad \mathbf{r}_i^* = \begin{pmatrix} l_*\sin\kappa_*\cos\varphi_i \\ l_*\sin\kappa_*\sin\varphi_i \\ l_*\cos\kappa_* - d_* \end{pmatrix}, \quad \varphi_i = \omega_*(t + t_0) + \frac{2\pi(i-1)}{N} \tag{3}$$

Here l_* is the tether length in nominal motion ($l_* = l_0 + \dfrac{T_*}{k_{tether}}, T_*$ is the tether tension in nominal mode), κ_* is the angle between the tethers and the system CoM radius vector, and d_* is the distance from the main satellite to the system CoM. It is easy to find that

$$d_* = \frac{l_* N m_2 \cos\kappa_*}{m_1 + Nm_2}, \quad T_* = \frac{|\mathbf{F}_C^e|}{N\cos\kappa_*} = \frac{3\omega_0^2 l_* m_1 m_2}{m_1 + Nm_2}, \quad l_* = l_0\left(1 - \frac{3\omega_0^2 m_1 m_2}{(m_1 + Nm_2)k_{tether}}\right)^{-1}$$

By differentiating relations (3), we can find $\dot{\mathbf{r}}_i^*$ and $\ddot{\mathbf{r}}_i^*$ ($i = 1, \ldots, N$). We insert the resulting expressions in motion equations (1) and obtain the thrust, which should be provided by the engines to maintain the nominal motion of the system:

$$\mathbf{U}_i^* = \begin{pmatrix} \left(-m_2\omega_*^2 l_* + \dfrac{3\omega_0^2 l_* m_1 m_2}{m_1 + N m_2}\right)\sin\kappa_* \cos\varphi_i \\ \left((\omega_0^2 - \omega_*^2) l_* m_2 + \dfrac{3\omega_0^2 l_* m_1 m_2}{m_1 + N m_2}\right)\sin\kappa_* \sin\varphi_i \\ -2m_2\omega_0\omega_* l_* \sin\kappa_* \cos\varphi_i \end{pmatrix} \tag{4}$$

The control law (4) can be simplified if the choice of the nominal motion rotation velocity is such that the first or second $\mathbf{U}_i^*$ components equals zero. In particular, the first component equals zero if

$$\omega_* = \sqrt{\frac{3m_1}{m_1 + Nm_2}}\omega_0 = \sqrt{\frac{3\alpha}{1+\alpha}}\omega_0, \;\; \alpha = \frac{m_1}{Nm_2}. \tag{5}$$

If the system's rotation velocity in the LVLH frame meets condition (5), its nominal motion can be maintained by the following control actions:

$$\mathbf{U}_i^* = \begin{pmatrix} 0 \\ \omega_0^2 l_* m_2 \sin\kappa_* \sin\varphi_i \\ -2\omega_0\omega_* l_* m_2 \sin\kappa_* \sin\varphi_i \end{pmatrix} \tag{6}$$

When constructing the nominal control strategy using equation (6), it is worth noting that the nominal motion must meet the constraints:

$$d_* \geq d_{\min}, \;\; R_* = d_* \sin\kappa_* \geq R_{\min}. \tag{7}$$

The first condition in (7) constrains the distance from the main satellite to the system CoM and the second constrains the distance between the CoM and the tethers.

2.3. Small perturbations compensation

For the system to stay in the nominal motion mode, the control actions $\mathbf{U}_i^*$ should be supplemented with the actions $\mathbf{U}_i^{**}$ ensuring that the motion remains stable, at least in the numerical simulations. Let us note that $\mathbf{U}_i^{**}$ is to compensate any perturbations not accounted for in (2) (e.g. luni-solar gravity, solar radiation pressure etc.).

We start constructing $\mathbf{U}_i^{**}$ by introducing a simple penalty function describing the deviation from the nominal mode:

$$\Phi = \frac{k}{2}\sum_{i=1}^{N} |\, \mathbf{r}_i - \mathbf{r}_i^* \,|^2$$

The control action

$$\mathbf{U}_{i,1}^{**} = -\frac{\partial \Phi}{\partial \mathbf{r}_i} = -k(\mathbf{r}_i - \mathbf{r}_i^*) \tag{8}$$

attempts to bring the system back to the nominal motion region but does not fully cope with the task, for it incites perturbations which will be dealt with by "damping":

$$\mathbf{U}_{i,2}^{**} = -b(\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_i^*)\,. \tag{9}$$

The numerical simulation helps select the values for the coefficients k and b so that the formation's motion remains stable if subjected to the control actions:

$$\mathbf{U}_i = \mathbf{U}_i^* + \mathbf{U}_{i,1}^{**} + \mathbf{U}_{i,2}^{**}\,.$$

Other variants of the corrective (feedback) control were considered also. To obtain the optimal strategy, one needs to linearize the relative motion equations in the proximity of reference motion and solve the linear-quadratic optimization problem with continuous time and infinite horizon [10]. The functional to be optimized is Φ. The optimal corrective control is a linear combination of deviations (both positions and velocities) and satisfies a Riccati matrix equation, which was solved numerically. Wide range of tether stiffness and viscosity coefficients were tested. Numerical estimation has shown, that each deputy satellite should respond only to its own disturbance. Hence, relative motion corrective control may be given by (8) and (9).

2.4. An estimate of the fuel consumption in the case of unperturbed nominal motion

It follows from (6) that the force keeping the system in the nominal configuration is:

$$\mathbf{U}_i^* = \begin{pmatrix} 0 \\ \omega_0^2 l_* m_2 \sin\kappa_* \sin\varphi_i \\ -2\omega_0 \omega_* l_* m_2 \sin\kappa_* \sin\varphi_i \end{pmatrix}$$

This control action should be provided by the reaction force $\mathbf{F}_i^* = \dot{m}_i \mathbf{u}_i$, where $\dot{m}_i$ is the fuel consumption rate for the deputy satellite *i*, and $\mathbf{u}_i$ is the flow velocity vector. Since the satellites are arranged into a symmetrical configuration, the subscript *i* will hereafter be omitted.

We calculate the consumption assuming that the control action is created by the change in the flow rate $\dot{m}$ and direction of the vector $\mathbf{u}$ and the fuel flow velocity has a constant absolute value: $u = |\mathbf{u}| = \text{const}$. The relations $\mathbf{F}_i^* = \mathbf{U}_i^*$ yields

$$\dot{m} = \frac{|\,\mathbf{U}\,|}{u} = \frac{m_2 \omega_0^2 l_* \sin\kappa_* |\cos\varphi|}{u} \sqrt{\frac{13m_1 + Nm_2}{m_1 + Nm_2}}\,. \tag{10}$$

Thus the fuel consumption over the time $\Delta = t_1 - t_0$ is found as:

$$m = \frac{m_2 \omega_0^2 l_* \sin \kappa_*}{u} \sqrt{\frac{13m_1 + Nm_2}{m_1 + Nm_2}} \int_{t_0}^{t_1} |\cos \varphi(t)| dt \ . \tag{11}$$

For a typical mission largely exceeding the configuration's rotation period $P_* = \frac{2\pi}{\omega_*}$ (i.e. $\Delta \gg P_*$), we can assume, with a relative error of about P_* / Δ, that:

$$\int_{t_0}^{t_1} |\cos \varphi(t)| dt \approx \frac{2}{\pi} \Delta$$

and, consequently,

$$m = \frac{2 m_2 \omega_0^2 l_* \sin \kappa_*}{\pi u} \Delta \sqrt{\frac{13m_1 + Nm_2}{m_1 + Nm_2}} \ . \tag{12}$$

For the numerical estimates, we take note of the fact that the particles' velocity in the electric thruster has the order of $u \approx 30$ km/s and use values $m_1 = 5000kg$, $m_2 = 1000kg$, $l_* = 10000m$, $\omega_0 = 7.29 \cdot 10^{-5} s^{-1}$, $\kappa_* = 10^0$, $\Delta = 1$ year, $N = 3$. Inserting these values in (12) yields $m = 18.01$ kg/year. The Table 1 features yearly propellant consumptions for other angles κ_* and the number *N* of the deputy satellites. The low propellant consumption justifies the satellite mass change neglect in (1).

Table 1. Annual fuel consumption in the nominal motion

Number of the deputy satellites	$\kappa_* = 5^0$	$\kappa_* = 10^0$
N = 3	9.04 kg	18.01 kg
N = 4	8.58 kg	17.11 kg

2.5. Stability of controlled motion

Stability studies of periodic regimes in the dynamics of the tethered satellite systems are based usually on Floquet theory (e.g. [9, 17, 19]). The complexity of the system at hand (multiple components, non-trivial free motion) strongly impedes the theoretical analysis of its stability. Nonetheless, there exists an extreme case when the stability conditions can be derived analytically - the case of the corrective control $\mathbf{U}_i^{**} = \mathbf{U}_{i,1}^{**} + \mathbf{U}_{i,2}^{**}$ (see (8) and (9)) with fairly high values of the coefficients k, b.

Combining various perturbation techniques with the main ideas of Floquet theory [23], after cumbersome calculations (which are omitted here) we obtain the following result: assuming that the corrective control action is defined by the parameters $k = \eta \tilde{k}$, $b = \eta \tilde{b}$ with $\tilde{k} > 0$, $\tilde{b} > 0$, at least for $N \geq 3$ deputy satellites the asymptotic stability of the vertical system rotation at the angle $\kappa > 0$ and large η is governed by the condition:

$$\frac{l_* - l_0 \sin^2 \kappa}{l_* - l_0} > 1 + \alpha . \qquad (13)$$

It is worth noting that the stability condition (13) is a rather week one - it can be violated only if the tether is strongly stretched, the mass ratio α is high, and/or the rotation angle κ is close to $\pi / 2$.

3. Numerical simulation of controlled multi-tethered satellite formation

The theoretical model of a rigidly rotating pyramidal configuration was implemented in Matlab/Simulink. The implementation supports:

- *An arbitrary number of deputy satellites with massive or weightless tethers;*
- *Models of environment (high-degree Earth Gravitational model, Moon and Sun perturbations, solar radiation pressure, etc.);*
- *Nominal and corrective control actions, the latter using the feedback on the current positions and velocities of the satellites.*

Stability is the main challenge in implementing a multi-tethered satellite formation. The proposed rotation and nominal motion control laws do not take into consideration the environmental perturbation effects and tether mass. Despite the corrective control actions, the system may turn out to be unstable.

We are giving below one example of configuration that showed no signs of instability in the numerical simulation and one example of unstable configuration. All simulations were performed under the following conditions:

- *A model of a heavy tether with 1 point mass;*
- *Non-central Earth gravitational field;*
- *Sun and Moon gravity;*
- *Solar radiation pressure.*

The first example pertains to the vertically oriented satellite case. Its most significant parameter values are listed in the Table 2. The simulation covered 3 months of the system operation and revealed no signs of instability. The main satellite's orientation vector with respect to the CoM diverged from the local vertical by no more than 0.1 degrees (Fig. 2). The expense of characteristic velocity by every deputy satellite is about 940 m/s over three months.

Table 2. The values of the system's parameters in the numerical Example #1

κ_*, [deg]	l_*, [m]	α, [-]	N, [-]	m_1, [kg]	m_2, [kg]	Tether mass, [kg]	k, [N/m]	b, [kg/s]
30	20000	3	3	5000	555	50	0.01	1

In Fig. 3 we plot the magnitudes of the nominal control $\mathbf{U}^{*}$ and the corrective control $\mathbf{U}^{**}$ for one of the deputy satellites. As expected, the corrective control is less regular than the nominal one. We cannot exactly divide the propellant consumption budget into “nominal” and “corrective” parts since we deal with a vector sum of the nominal and corrective control forces. Let us note, however, that the amount of propellant required to create the total control force is less than the sum of amounts used to create both ingredients separately.

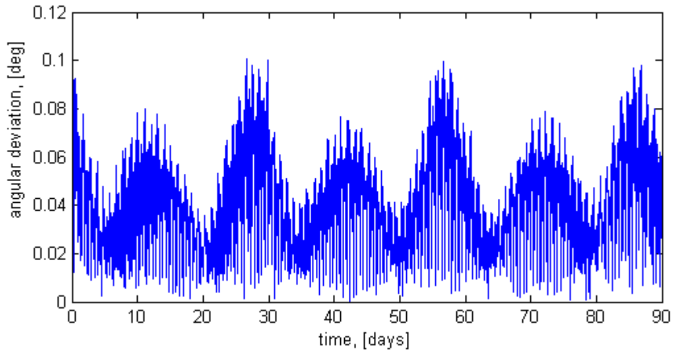

Figure 2. Angular deviation of the main satellite from the nominal position

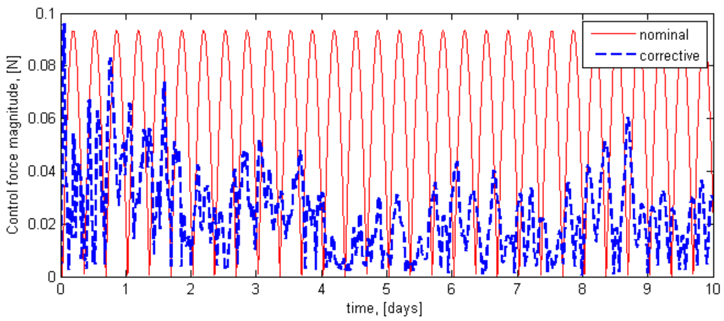

Figure 3. Magnitudes of the nominal and corrective trust in the case of rotation around local vertical

The second example demonstrates the effect of making the system smaller in size. We start from the stable vertical configuration considered in the first example, and decrease nominal tether length l_* from 20000 to 5000 m. In this scenario, we observe a remarkable phenomenon of the system overturn related to the instability of the nominal rotation pattern. Specifically, in the first few days of system functioning, it rotates more or less as a rigid structure, but shows some signs of instability. Then, it suddenly loses stability and rigid structure, and starts to move in a seemingly chaotic manner. Finally, it regains its rigid rotating structure, but now is oriented upside down. This new rotating state is stable. The time line of main satellite angular deviation is shown in Fig. 4.

This phenomenon can be described as a transition from an unstable to a stable equilibrium under environmental perturbations. Note that with short linear sizes of the system, the force required to keep the main satellite at a fixed position relative to CoM of the system is small (it scales linearly with the system size). In our test case this force equals 0.08 N and is comparable in magnitude to the lunar and solar gravity perturbations acting on the main satellite. These perturbations force the main satellite to move to the stable position beyond the nominal orbit. That this new upside down position is indeed stable is easy to see from the much larger values of tether tensions – in the new configuration the distance from the main satellite to the orbit is much larger, and the force needed to keep it there is much higher. This, and the fact that the tethers no longer do their job of balancing the system about the orbit, obviously means that the upkeep of the new configuration requires much stronger engine forces and hence more fuel.

The described phenomenon is only observed if the environmental effects (lunar/solar gravities) are included in simulation – otherwise the system rotates in a stable manner. The phenomenon does not depend on whether the tethers are massive or weightless.

Finally, we point out that this phenomenon is related to our definition of the correcting control: the deputy satellites move as if they were a part of a system whose CoM moves along a strictly circular orbit. The other control strategy might produce different results.

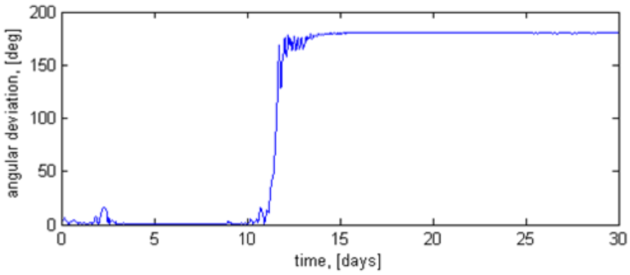

Figure 4. Angular deviation of the main satellite from the nominal position with respect to the system CoM

4. A parametric study

Let us summarize the observed effects of various parameters on the system performance for the typical set of their values. Regarding parametric optimization of our tethered satellite formation, we note that one might consider several natural objectives, in particular maximizing distances between the system's components - tethers and satellites - to prevent collisions, minimizing control action to decrease fuel consumption, minimizing the mass of deputy satellites relative to the main satellite (assuming deputy satellites are less useful than the main satellite). These objectives generally conflict with one another, so that Pareto-optimal configurations (see [14, 18]) are not unique and form an infinite set (in fact, a two-dimensional set if we consider the above three objectives formulated as three scalar-valued functions).

The ratio α of the main satellite mass to the total mass of the deputy satellites. On the one hand, it is desirable to minimize the mass of the deputy satellites and, therefore, to maximize the ratio α. On the other hand, the theoretical estimates and simulations show that an increase in α leads to an increase in the characteristic acceleration of the deputy satellites and, consequently, higher propellant consumption. Besides, the simulation testifies to lower stability of the systems with a larger α.

Number of deputy satellites N. Neither theoretical analysis nor simulations have revealed any clear preferences as to the number of the deputy satellites.

Angle κ_* between the tethers and the system axis. At small κ_* the required nominal control action force decreases but the system becomes more difficult to control and, as a result, more stringent requirements are imposed on the corrective control action. Therefore, a decrease in the angle κ_* may not necessarily lead to propellant savings. Moreover, at small κ_* the distance between the tethers and the main satellite becomes shorter, thus raising the chances of collisions.

5. Conclusion

Our numerical and analytical results demonstrate that the multi-tethered pyramidal satellite formation is implementable, stable and capable of maintaining the main satellite in a specified (vertical or other) position with respect to CoM of the system. The configuration requires, however, that the deputy satellites' engines stay in continuous operation. Various system optimality criteria (distance between the system and the central satellite, deputy satellite masses, fuel consumption, degree of stability, etc.) conflict with one another, so it appears impossible to specify a unique optimal set of parameters.

Natural extensions of the current work include the alternative motion configurations of the multi-tethered system, which could be maintained with significant reduction of the propellant consumption. For example, a continuous and real time control of the length/tension of all the tethers between the satellites could be implemented using controlled winches for instance. Other shapes of the composite assembly could also be envisaged such as a polyhedral composition or any other shape where tethers would connect nodes. We propose to describe such configurations in the subsequent publications.

References

[1] Aslanov, V.S., Ledkov, A.S.: Dynamics of the tethered satellite systems. Woodhead Publishing, Cambridge, UK (2012)

[2] Avanzini, G., Fedi, M.: Refined dynamical analysis of multi-tethered satellite formations. Acta Astronautica. **84**, 36-48 (2013).

[3] Avanzini, G., Fedi, M.: Effects of eccentricity of the reference orbit on multi-tethered satellite formations. Acta Astronautica. **94**, 338-350 (2014).

[4] Bekey, I.: Tethers open new space options. Astronautics and Aeronautics, **21**, 23 – 40 (1983).

[5] Cai, Z., Zhao, J., Peng, H., Qi, Z.: Nonlinear control of rotating multi-tethered formations in halo orbits. Int. J. Comput. Methods, **11**, 1344008 [26 pages], (2014)

[6] Chung, S.J., Kong, E.M.C., Miller, D.W.: Dynamics and Control of Tethered Formation Flight Spacecraft Using the SPHERES Testbed. AIAA Guidance, Navigation and Control Conference, San Francisco, CA (2005)
[7] Guerman, A.D., Smirnov, G., Paglione, P., et al.: Stationary configurations of a tetrahedral tethered satellite formation. J. Guidance Control Dyn., **31**, 424-428 (2008).
[8] Kumar, K.D.: Review of dynamics and control of nonelectrodynamic tethered satellite systems. J. of Spacecraft and Rockets, **43**, 705-720 (2006).
[9] Levin, E.M.: Dynamic analysis of space tether missions. American Astronautical Society, Washington, DC, USA (2007).
[10] Lewis, F.L., Syrmos, V.L.: Optimal Control. Wiley, New York (1995).
[11] Palmerini G B, Sgubini S, Sabatini M. Space webs based on rotating tethered formations. Acta Astronautica, **65**, 131–145 (2009)
[12] Panosian, S., Seubert, C.R., Schaub, H.: Tethered Coulomb Structure Applied to Close Proximity Situational Awareness, AIAA Journal of Spacecraft and Rockets, **49**, 1183–1193 (2012).
[13] Pizarro-Chong, A., Misra, A.K.: Dynamics of multi-tethered satellite formations containing a parent body. Acta Astronautica, **63**, 1188-1202 (2008).
[14] Sawaragi, Y., Nakayama, H., Tanino, T.: Theory of Multiobjective Optimization. Academic Press, Orlando (1985).
[15] Seubert, C.R., Schaub, H.: Tethered Coulomb Structures: Prospects and Challenges, Journal of Astronautical Sciences, **57**, 347-348 (2009)
[16] Seubert, C.R., Panosian, S., Schaub, H.: Attitude and Power Analysis of Multi-Tethered, Two-Node Tethered Coulomb Structures, AIAA Journal of Spacecraft and Rockets, **48**, 1033–1045 (2011)
[17] Sidorenko, V.V., Celletti, A.: A "Spring–mass" model of tethered satellite systems: properties of planar periodic motions. Cel. Mech. Dyn. Astron., **107**, 209-231 (2010)
[18] Steuer, R.E.: Multiple Criteria Optimization: Theory, Computations, and Application. Wiley, New York (1986).
[19] Tragesser, S.G., Tuncay, A.: Orbital Design of Earth-Oriented Tethered Satellite Formations. J. of Astronautical Sci., **53**, 51- 64 (2005).
[20] Wen, H., Jin, D.P., Hu, H.Y.: Advances in dynamics and control of tethered satellite systems. Acta Mech. Sin, **24**, 229-241 (2008)
[21] Williams, P.: Optimal control of a spinning double-pyramid Earth-pointing tethered formation. Acta Astronautica, **64**, 1191-1223 (2009).
[22] Wong, B., Misra, A.: Planar dynamics of variable length multi-tethered spacecraft near collinear Lagrangian points. Acta Astronautica. **63**, 1178-1187 (2008).
[23] Yakubovich, V.A., Starzhinskii, V.M.: Linear Differential Equations with Periodic Coefficients. Wiley, New York (1975)
[24] Yu, Y., Baoyin, H., Li, J.: Dynamic modelling and analysis of space webs. Science China. Physics, Mechanics and Astronomy, **54**, 783-791 (2011)
[25] Zhao, J., Cai, Z.: Nonlinear dynamics and simulation of multi-tethered satellite formations in Halo orbits. Acta Astronautica, **63**, 673-681 (2008).