OBSERVABILITY OF NON-COOPERATIVE SPACE OBJECT'S TRACKING AND CHARACTERIZATION

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Abstract: *One of the major problems in the observation and characterization of space objects is that in general, not all quantities of interest are observable. In standard observations, e.g. by radar or optical, only a subset of the full state (position and velocity) are available. Although observability of the full state can be achieved when combining several observations, two problems arise. One, normally it is assumed that observations are error free when observability is discussed, utilizing classical observability concepts. Secondly, the quantities beyond the state are relevant for the dynamical evolution of the object. Non-conservative force depend also on the surface properties and shape of the object. Dedicated characterization measurements are not always available and time consuming and of course they are error affected, too. In this paper, the classical concepts of observability of space objects is discussed based on optical measurements. Astrometric measurements for the determination of the state and light curve measurements for characterization are taken into account. A new concept of observability calculations is proposed that is able to include measurement noise. A realistic model for the measurement noise is shown. It is shown that measurement noise does not only serve to degrade the measurements but is also an additional source of information.*

Keywords: *space debris, astrodynamics, optical sensors*

1. Introduction

Observability considerations are common in the context of control theory, where the concepts of observability and controllability are juxtaposed. In the situation of independent observations of space objects, the observability is put in a different context, in which the controllability is not of interest. This is because either the objects are uncontrolled to begin with or provide a mean of independent observation that are not directly coupled to their control, which can belong to a different entity.

Observations of satellites are mostly ground based, utilizing radars and telescopes, and to a lesser extent space based. Specialized observations of space debris objects, have been established and methods have been refined over the past 20 years. In general it is distinguished between two different kinds of observations, observations that target to determine the state of the object, in observing directly a subset of it, .e.g. angles, or range and range rate.[1,2,3,4,5,6,7,8,9,10,11,12] Or so called, characterization measurements, that aim to extract features of the object, such as shape, materials or attitude state.[13,14,15,16,17,18,19,20]

Although characterization features are of course of interest as such, they also significantly influence the trajectory of an object via non-conservative forces. The question arises, what the observability of all features of interest is, the state as well as shape, attitude and surface properties and how that observability is influenced by different kind of sensors that allow to observe in different bandwidth and have different strength and weaknesses. Different sensors and observation scenarios differ by the observed quantities, angles, range, range-rate, brightness variations, spectra etc, but also by the accuracy of the data that can be extracted and hence the measurement noise levels.

In the established observability theory as known from control theory, eventual measurement noise does not play a role. On the one level this makes sense, as the general information about observability in the best possible case is sought. On the other hand it is also clear, that there could be so much measurement noise that the original measurement information cannot be extracted any more, so although a quantity is observable as such, is not observable for a high measurement noise.

In the current paper the classical method of observability of time varying systems is reviewed and extended to include measurement noise.[21,22] Furthermore, measurement noise is modeled physically realistically, showing that it also bears valuable information. Incorporating measurement noise in the observability Gramian, together with the realistic representation of the noise, allows to determine the observability in dependence of specific sensor characteristics sensor and observation scenarios. The example of astrometric optical observations and observations of brightness variations, so-called light curves are shown. Preliminary work on this topic has done by the author in.[23,24,25]

2. Dynamics

The dynamics of the problem can be summarized as the following from a modeling perspective:

$$\ddot{\mathbf{x}}(t) = \mathbf{a}_{\text{body-indep.}}(\mathbf{x}(t)) + \mathbf{a}_{\text{body-dep.}}(\mathbf{x}(\mathbf{t}), \mathbf{b}, \mathbf{q}(\mathbf{t})) + \mathbf{a}_{\text{unmodeled}}(\mathbf{x}(\mathbf{t}), \mathbf{b}, \mathbf{q}(\mathbf{t}), \mathbf{p}(\mathbf{t})) \tag{1}$$

were $\mathbf{x}$ is the geocentric object state (position and velocity), $a_{\text{body-indep.}}$ are the accelerations that only depend on the center of mass, the accelerations $a_{\text{body-dep.}}$ are the non-conservative accelerations that depend on the body parameters $\mathbf{b}$, the body orientation $\mathbf{q}(t)$ and the state of the object $(\mathbf{x}(t)$. $a_{\text{unmodeled}}$ are the accelerations that remain unmodeled. They are either higher orders of magnitudes or fidelity than the force models that are included, or physical effects that have been ignored, or inaccuracies and mismodeling in the force models, body parameters etc.

In the concrete realization of the dynamics in Eq1, the Earth gravitational field and third body perturbations by the Sun and Moon are chosen as conservative forces, and solar radiation pressure as non-conservative perturbation that depends on the state and the body and orientation parameters.

$$\ddot{\mathbf{x}}(t) \quad = \quad -\mu \nabla V(\mathbf{x}(t)) - G \sum_{k=1,2} M_k \left[\frac{\mathbf{x}(t) - \mathbf{x}_k(t)}{|\mathbf{x}(t) - \mathbf{x}_k(t)|^3} + \frac{\mathbf{x}_k(t)}{\mathbf{x}_k^3(t)} \right] + \mathbf{a}_{\text{SRP}}(t), \tag{2}$$

where μ gravitational parameter and $V(\mathbf{x})$ the Earth gravitational potential. The third body gravitational perturbations of the Sun and Moon (k=1,2) with the states $\mathbf{x}_k$ have also been taken into account. The radiation pressure can be expressed as the following:

$$\begin{aligned} \mathbf{a}_{\text{SRP}}(t) \quad &= \quad \sum_{i=1}^{n} \frac{E}{c}\frac{\mathfrak{A}_i}{m}\frac{A^2}{|\mathbf{x}-\mathbf{x}|^2}\hat{\mathbf{S}}\,\hat{\mathbf{N}}_i[(1-C_{\text{s},i})\,\hat{\mathbf{S}}+2(C_{\text{s},i}\cdot\,\hat{\mathbf{S}}\,\hat{\mathbf{N}}_i+\frac{1}{3}C_{\text{d},i})\,\hat{\mathbf{N}}_i] \\ \text{for:} \qquad & 0<\arccos(\,\hat{\mathbf{S}}\,\hat{\mathbf{N}}_i)<\pi/2 \quad \text{with:} \quad C_{\text{s},i}+C_{\text{d},i}=1-C_{\text{a},i} \end{aligned} \tag{3}$$

m is the total mass of the satellite, E is the solar flux, A the astronomical unit, $\mathbf{x}$ the geocentric position of the sun, c velocity of light, $\mathbf{S}$ the direction of the radiation source, $\mathfrak{A}_i$ is the area of the i-th sub-area and $\mathbf{N}_i$ the normal vector of it. $C_{\text{s,d,a},i}$ are the coefficients for specular, diffuse reflection and absorption for the i-th sub-area facet. The object is assumed to made up of n flat facets. The $m=5$ body parameters are hence the following set:

$$\mathbf{b}=[\rho,\mathbf{N},\mathfrak{A},\mathbf{C}_\text{d},\mathbf{C}_\text{s}]^T, \qquad \mathbf{b}\in\mathbb{R}^{3\times5\times n} \tag{4}$$

Note: The normal vectors are represented in the absolute frame here. For a normal vector representation in the body frame, additional body rotation parameters would need to be incorporated. Only the latter offers complete knowledge of the body. If we do not take the torques on the object into the account but only the orbit, four body parameters are sufficient to describe the orbit dynamic problem, and the vector ρ can be neglected.

For a more complete representation of the dynamics, an extended state is proposed, the traditional state extended by the body parameters. The extended state could then be formulated as the following:

$$\mathbf{x}_{\text{ext}}(t)=[x_1(t),x_2(t),x_3(t),\dot{x}_1(t),\dot{x}_2(t),N_1(t),....,N_n(t),\mathfrak{A}_1,....,\mathfrak{A}_n]^T \tag{5}$$

for $\rho << x$, and $\rho << |\mathbf{x}_{\text{topo}}|$, where $\mathbf{x}_{\text{topo}}$ is the ECI vector from the center of the earth to the topocenteric position of the observe, the center body distance ρ is not taken into account as part of the sensor extension. This of course would be necessary if attitude dynamics would be included. The reflection coefficients are not explicitly displayed here because an explicit representation for the body reflection has been chosen and is deeply buried in the differential equation itself, see Eq.2 and 3.

In order to evaluate the observability, the linearized dynamics are needed. The state transition matrix is approximated directly via the partials in order to preserve an analytic expression. For the state transition matrix the two body dynamics and the solar radiation pressure has been taken into account. Third body perturbations or higher orders of the Earth gravitational field have been neglected because they do not add to the observability investigation. Conservative perturbations just depend on the state, not even a direct coupling with the velocity is given, further terms with the same kind of coupling do not add observability. Of course, if the aim is a propagation of the orbit, and not just the determination of the observability condition, these perturbations need to be included.

3. Observability

The observability of the time-varying non-linear system can be defined via a linearization of both the measurement function as defined in the previous sections and the linearization of the dynamics itself (Eq.2). Defining the matrix G:

$$G(t_1,t_2) = \int_{t_1}^{t_2} T^T(\tau,t_1)H^T(\tau)H(\tau)T(\tau,t_1)d\tau, \tag{6}$$

with the transition matrix T and the observation matrix H, superscript T denotes transpose of the matrix. The system is completely observable at time t_1, if there exists a finite time t_2, such that either of the following equivalent conditions hold: $G(t_1,t_2)$ is either positive semi-definite, none of its eigenvalues is zero, and/or its determinant is different from zero.

Besides the fact that numeric evaluation of values that need to be exactly zero or non-zero is generally plagued with the difficulty to define what small number numerically equals an exact zero, the definition bears a number of problems. The transition matrix T could be defined based on the two body assumption.

Besides the more or less reliable definition of the linearized transition matrix, it has to be noted that the observability is independent of the measurement noise. This means, observability is evaluated assuming perfect measurements. This raises the question what the actual merit of such an observability statement actually bears. Even if we have observability in perfect measurements, does not mean we would have it, in actual noise affected measurements. Different observation scenarios and sensors result in different noise levels, and hence should be reflected, e.g. there should be a threshold at which the measurement is buried to such an extend in the noise that the information cannot be reliably extracted or reliably estimated any more and observability in a strict sense is thwarted. Secondly, in the next two sections it is shown that the measurement noise actually is not only a burden, but also bears additional information that remains currently unexplored. In the following two sections we realistically model and define measurement noise for the astrometric and the light curve measurements. Those measurement noise models are then incorporated in a new definition of the observation matrix. With this new observation matrix, the measurement noise can be incorporated directly in the evaluation of the observability condition.

4. Observability with Measurement Noise

It is possible to quantify the measurement noise for both types of measurements. Once these definitions are available, a redefinition of the observation matrix is proposed. If it is assumed that in Eq.11 a zero mean measurement noise covariance is defined, for decorrelated measurements, a so-called prewhitening process can be applied. Then the measurement covariance can be decomposed in the following way:

$$K_{x_o,y_0} = L \cdot L^T \tag{7}$$

This allows to redefine the measurement function as the following:

$$\mathbf{z} = \mathbf{L}^{-1}\mathbf{H}\mathbf{x} + r, \tag{8}$$

where r represents the measurement noise with zero mean and covariance identity. This means the new measurement matrix is:

$$\tilde{H} = \mathbf{L}^{-1}\mathbf{H}, \tag{9}$$

which directly incorporates the measurement noise. This allows to also adapt the observability accordingly:

$$\tilde{G}(t_1,t_2) = \int_{t_1}^{t_2} T^T(\tau,t_1)\tilde{H}^T(\tau)H(\tau)T(\tau,t_1)d\tau, \tag{10}$$

This means, a twofold is achieved. For one, an observability measure is found that is realistic and not relies on perfect measurements only. It allows to investigate the observability for realistic measurements and in dependence of the influence of sensors and the specific measurement scenario. Furthermore, the additional information on the extended state and the body parameters that are buried in the measurement noise can be directly explored as a mean to increase observability.

5. Measurements

In general, the measurements in the vector $\mathbf{z}$ can be expressed via the measurement function $h(\mathbf{x},t)$ and the measurement noise $\nu(t)$ for the r measured quantities:

$$\mathbf{z}(t) = [z_1(t), z_2(t), ...z_r(t)]^T = h(\mathbf{x}(t),t) + \nu(t) \approx \mathbf{H}(t)\cdot\mathbf{x}(t) + \nu(t), \tag{11}$$

where $\mathbf{H}$ is the linearized measurement matrix.
For this study an optical sensor is assumed. Two different kinds of measurements are considered. Astrometric position measurements and brightness measurements, so- called light curves.

5.1. Astrometric Optical Measurements

For the astrometric position measurements, right ascension α and declination δ, the relation between the state and the measured values is the following:

$$z_1 = \alpha = \arctan(\frac{-x_1}{x_2}) \qquad z_2 = \delta = \arcsin(\frac{x_3}{\sqrt{x_1^2+x_2^2+x_3^2}}) \tag{12}$$

If not two subsequent images are combined or long duration exposures are used, no angular velocity information is available. This leads to the following linearized observation matrix:

$$\mathbf{H} = \begin{pmatrix} \frac{\partial\alpha}{\partial x_1} & \frac{\partial\alpha}{\partial x_2} & \frac{\partial\alpha}{\partial x_3} & \frac{\partial\alpha}{\partial \dot{x}_1} & \frac{\partial\alpha}{\partial \dot{x}_2} & \frac{\partial\alpha}{\partial \dot{x}_3} \\ \frac{\partial\delta}{\partial x_1} & \frac{\partial\delta}{\partial x_2} & \frac{\partial\delta}{\partial x_3} & \frac{\partial\delta}{\partial \dot{x}_1} & \frac{\partial\delta}{\partial \dot{x}_2} & \frac{\partial\delta}{\partial \dot{x}_3} \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} & H_{13} & 0 & 0 & 0 \\ H_{21} & H_{22} & H_{23} & 0 & 0 & 0 \end{pmatrix} \tag{13}$$

with

$$H_{11} = \frac{-x_2}{x_1^2 + x_2^2} \tag{14}$$

$$H_{12} = \frac{x_1}{x_1^2 + x_2^2} \tag{15}$$

$$H_{13} = 0 \tag{16}$$

$$H_{21} = \frac{-x_1 x_3}{\sqrt{x_1^2 + x_2^2} \cdot (x_1^2 + x_2^2 + x_3^2)} \tag{17}$$

$$H_{22} = \frac{x_2 x_3}{\sqrt{x_1^2 + x_2^2} \cdot (x_1^2 + x_2^2 + x_3^2)} \tag{18}$$

$$H_{23} = \frac{\sqrt{x_1^2 + x_2^2}}{x_1^2 + x_2^2 + x_3^2} \tag{19}$$

As said before, the state and the observability matrix does not entail the necessary information needed to evaluate body dependent accelerations. Hence, the observation function and matrix are sought to be evaluated for the extended state, as defined in Eq.5. The extension of the observation matrix is rather trivial, because no direct dependence on the measured quantities is given:

$$\mathbf{H}_{\text{ext}} = \begin{pmatrix} H_{11} & H_{12} & H_{13} & 0 & 0 & 0 & 0 & ... & 0 & 0 & ... & 0 \\ H_{21} & H_{22} & H_{23} & 0 & 0 & 0 & 0 & ... & 0 & 0 & ... & 0 \end{pmatrix} \tag{20}$$

$$\tag{21}$$

A first step is to determine the measurement noise and the second one to explore ways to incorporate it into the observation matrix. A CCD sensor is assumed. Under the assumption of zero mean Gaussian measurement noise (an assumption that might not be justified), we are seeking to determine the covariance on the extracted astrometric position. For now, only the confidence of the extracted pixel position of the object trace itself is evaluated and directly translated into the variance on the right ascension and declination. In the processing of actual measurements, the pixel positions are set in relation to the extracted pixel positions of the background stars and then compared with an external star catalog. This, theoretically very easily can introduce a bias, which is neglected here. However, it is save to assume that the uncertainties in the extracted star positions are small as only very bright star images are used and no additional covariances besides a potential bias are introduced by the star catalog comparison. One tricky part is of course always the conversion between the actual position that is observed and averaged aberration star catalogs.

A standard procedure is to fit a two dimensional Gaussian surface to the object image that is spread over the pixel grid. If a Gaussian fitting is via e.g. Newton-Raphson method is applied a lower bound can be found via Fisher information gain.[24,26,27,28] If the shape of the Gaussian and the noise level is known, the lower bound can be expressed as the following function:[24]

$$K_{x_0,y_0} = \frac{N}{64 \cdot A^2 \cdot \delta^2} \begin{bmatrix} \frac{5c_2\delta^2 c_1 + \delta^2 c_2{}^2 + 128c_2 + 2\delta^2 c_2 c_3 - 4\delta^2 c_3{}^2}{\sqrt{D}\pi} & \frac{4\delta^2 c_1 c_2 - \delta^2 c_1 c_3 - \delta^2 c_3 c_2 - 6\delta^2 c_3{}^2 - 128c_3}{\sqrt{D}\pi} \\ \frac{4\delta^2 c_1 c_2 - \delta^2 c_1 c_3 - \delta^2 c_3 c_2 - 6\delta^2 c_3{}^2 - 128c_3}{\sqrt{D}\pi} & \frac{\delta^2 c_1{}^2 + 5\delta^2 c_1 c_2 + 128c_1 + 2\delta^2 c_1 c_3 - 4\delta^2 c_3{}^2}{\sqrt{D}\pi} \end{bmatrix}, \tag{22}$$

where $c_{1,2,3}$ are the Gaussian parameters, as defined below, $D = c_1c_2 - c_3^2$, δ is the pixel width, N is the image noise. It can be assumed that the estimated Gaussian is reasonable close to the true Airy disk. The two-dimensional Gaussian that is fitted is defined as the following,

$$P = A\exp\left(-\frac{1}{2}\left(c_1(x-x_0)^2 + 2c_3(x-x_0)(y-y_0) + c_2(y-y_0)^2\right)\right), \tag{23}$$

A detailed derivation of the variance in the above stated form can be found in.[24] In order to find the parameters, the noise and the fitted Gaussian in a general manner, the complete detection process is modeled.

5.1.1. Signal

Optical observations rely on the sunlight being reflected off the object in the direction of the observer. Therefore, first of all, the visibility of the object needs to be ensured, see details in.[25] The light actually has to be reflected in the direction of the observer. The phase function determines the amount of reflected light relative to the incoming one under a specific observation geometry. The phase function is defined in the following expression (here in spherical coordinates):

$$\bar{\Psi} = \frac{I(\alpha)}{I(0)} = \frac{\int_{\alpha-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\pi} f_r \mu_0 \mu \sin\theta d\theta d\phi}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{\pi} f_r \mu_0 \mu \sin\theta d\theta d\phi} \tag{24}$$

Under the simple assumption of a Lambertian reflection, the reflection flux is uniform in all directions, hence $f_r = \frac{1}{\pi}$, α is the phase angle between the incoming flux and the direction of the observer. For a spherical object, it can be assumed, without loss of generality, that the observer can be placed in the xy plane of the sphere, the incoming radiation direction can be expressed as $\mu_0 = \sin\theta\cos\phi$ and the outgoing observer direction as $\mu = \sin\theta\cos(\alpha - \phi)$. This leads to the following phase reflection function for a spherical object:

$$\Psi_{\text{sphere}}(\lambda) = \frac{C_{\text{d}}(\lambda)}{\pi}(\sin(\alpha) + (\pi - \alpha)\cos\alpha), \tag{25}$$

where the function is the wavelength λ dependent scaled with the diffuse reflection coefficient $C_{\text{d}}(\lambda)$. Specular reflection on a sphere is not taken into account here as classical glints cannot be caught from real spherical space objects, but only in so far as they have small flat reflectors on their surface. Spherical surfaces are always visible to the observer as long as the phase angle is smaller than π and always appear the brightest at the smallest phase angle.

For a flat plate, the integrals over the hemisphere has to be replaced by the integral over the illuminated surface, and are hence very simple. The reflection flux becomes equal to one, the one phase angle expression needs to be replaced by two angles β, the angle between the radiation direction and the normal vector, and γ, the angle between the observer and the normal vector. This leads to $\mu_0 = \cos\beta$ and $\mu = \cos\gamma$ This leads to the following phase reflection function:

$$\Psi_{\text{flat}}(\lambda) = \cos\beta\left[\frac{C_{\text{d}}(\lambda)}{\pi}\cos\gamma + \frac{\tau\cdot C_{\text{s}}(\lambda)\cdot x^2}{a^2}\right], \quad \tau = \begin{cases} 1 & \text{for } \cos(0.25\,\text{deg}) \leq \cos\delta \\ 0 & \text{else} \end{cases} \tag{26}$$

$\delta = \arccos\left(\frac{\vec{O}+\vec{S}}{|\vec{O}+\vec{S}|}\cdot\vec{N}\right)$, whereas $\vec{O}$ is the direction of the observer, $\vec{S}$ is the direction to the Sun, and $\vec{N}$ is the normal vector. τ is the specular reflection parameter, where $0 < \arccos(\hat{\vec{S}}\hat{\vec{O}}) < \pi/2$. $C_{\text{s}}(\lambda)$ is the wavelength dependent specular reflection parameter. The dimension of the sun has been taken into account, where a is the radius of the Sun, and x is the distance from the object to the Sun. No limb darkening effects have been accounted for. A cylinder barrel can be constructed from the equations above, leading to the following reflection function:

$$\begin{aligned} \Psi_{\text{cyl}}(\lambda) &= \sin(\beta')\sin(\gamma')\left[\frac{C_{\text{d}}(\lambda)}{\pi}\Big(\sin(\alpha') + (\pi-\alpha')\cos(\alpha')\Big) + \frac{0.5}{180.0}\cdot\frac{\tau\cdot C_{\text{s}}(\lambda)\cdot x^2}{R^2}\right] \\ \tau &= \begin{cases} 1 & \text{for } 89.75\text{deg} \leq \delta' \leq 90.25\text{deg} \\ 0 & \text{else} \end{cases} \end{aligned} \tag{27}$$

β' is the angle between the sun and the symmetry axis, γ' the angle between the observer and the symmetry axis in the plane containing the symmetry axis in the body fixed coordinate system, and α' is the projected phase angle in the same plane. θ is the angle between the observation direction in the plane orthogonal to the symmetry axis of the cylinder.

For a cylindrical or flat object, it can happen that no light is reflected in the direction of the observer, even though the object is illuminated and phase angle is not 180 degrees. Additionally, instead of the one phase angle α, the double angles, β and γ need to be used to determine the expected reflection in a specific object orientation and observation geometry.

The spectral radiation $I_{\text{obj}}(\lambda)$, and (total) irradiation $\mathfrak{I}_{\text{obj}}$ at the position of the observer is determined as:

$$I_{\text{obj}}(\lambda) = I(\lambda)\frac{A}{x^2_{\text{topo}}}\cdot\Psi(\lambda) \qquad \mathfrak{I}_{\text{obj}} = \int I(\lambda)\frac{A}{x^2_{\text{topo}}}\cdot\Psi(\lambda)d\lambda \approx \bar{\mathfrak{I}}_{\text{obj}} = E\frac{A}{x^2_{\text{topo}}}\bar{\Psi}, \tag{28}$$

$$\text{mag}_{\text{obj}} = \text{mag} - 2.5\log\left(\frac{\mathfrak{I}_{\text{obj}}}{E}\right) \approx \text{mag} - 2.5\log\left(\frac{A}{x^2_{\text{topo}}}\cdot\bar{\Psi}\right), \tag{29}$$

$I(\lambda)$ is the spectral radiation of the Sun, the integral $\int_\lambda I(\lambda)d\lambda = E$ is the total radiation, the solar constant. A is the area of the exposed surface, x_{topo} the distance between the topocentric observation position and the object. $\bar{\Psi}$ is the wavelength averaged reflection function assuming reflection parameters (C_i), which are constant over all wavelengths. The magnitude of the object is determined via the reference to the absolute Sun magnitude, which is known. Apparent magnitudes, based on measurements, differ from the absolute magnitudes. For absolute magnitudes, reference values are needed.

When the irradiation is passing through the optics, we acquire the following signal function:

$$S = \int (D-d)\frac{\lambda}{hc} I(\lambda)\cdot \exp(-\tau(\lambda)R(\zeta))d\lambda \approx (D-d)\frac{\bar{\lambda}}{hc}\exp(-\tau(\lambda)R(\zeta))\cdot I \tag{30}$$

with the speed of light, c, the Planck's constant h, and the area of the aperture D and the obstruction of the aperture d, ζ is the elevation and τ the atmospheric extinction coefficient and R the atmospheric function. The simples atmospheric model is $R = \frac{1}{cos\zeta}$. The count rate is derived from the signal via the time integration dt, during with the sensor is able to catch photons:

$$C = \int S(\lambda)\cdot Q(\lambda)\cdot dt \approx S(\bar{\lambda})Q(\bar{\lambda})\Delta t, \tag{31}$$

where Q is the quantum efficiency. The approximation neglects the shutter function itself and assumes that the integration time is the same over all the field of view of the sensor. The signal however, is spread over several pixels, which report their count rates independently.

The above equations define the overall signal. In order to determine the shape of the signal on the CCD image, the diffraction has to be modeled. The size of the diffraction pattern is, without atmosphere, given by the size of the Airy disk. The Airy refraction pattern consists of one main maximum and several side maxima, which have drastically reduced intensities. The Airy disk is defined as the extension of the first maximum of the diffraction pattern on the detector, the higher order maxima are merged into the background. The diffraction at a round aperture of a telescope is expressed via the Bessel functions:

$$I(\theta) = I_0\Big(\frac{2B_1(k\cdot D\sin\theta)}{k\cdot D\sin\theta}\Big)^2 \quad \rightarrow \quad \xi = \arcsin(1.22\frac{\lambda f}{D}) \tag{32}$$

with $k = \frac{2\pi}{\lambda}$, D is the aperture, B_1 are the first Bessel functions, f is the focal distance, and ξ is the distance between the center of the Airy disk and the first minimum at the detector. The larger the aperture, the smaller is the Airy disk and more light is concentrated on fewer pixels, and the better two different object images can be resolved. However, ground based telescopes do suffer from the effects of atmosphere. Atmosphere does not only attenuate the signal, but the turbulent mixing of the atmosphere breaks up the Airy disk in speckle pattern. The superimposed signal in the integration time during an observation interval leads to an effectively broadened signal at the detector. This so-called seeing leads to the fact that the direct correlation of the observed signal disk with the aperture is broken. In general seeing is expressed in the full width of half maximum (FWHM) that the signal disk has on the detector.

$$FWHM_{\text{airy}} = \frac{1.03\lambda}{D} \quad \rightarrow \quad FWHM_{\text{seeing}} = const. \tag{33}$$

Depending on the size of the telescope and the specific seeing conditions at the observing site, the FWHM can be dominated by the telescope aperture or limited by seeing. Seeing is usually limited to around one arcsecond in best cases.

The total encircled energy can be calculated by integrating Eq.32:

$$E = \int_0^\infty 2\pi \cdot I(\theta)\theta d\theta = I_0 \cdot 4\pi \cdot (1 - B_0^2(k \cdot D \sin\phi) - B_1^2(k \cdot D \sin\phi)) \tag{34}$$

The amount of energy in the first Airy ring, is 83.8 percent of the overall energy stemming from the object. This means only 83.8 percent of the total time integrated irradiation, hence the signal, are subsumed in the pattern that is usually detected. The amplitude irradiation A_0 is determined by the overall irradiation:

$$A_0 = \frac{I \cdot \pi D^2}{\lambda^2 f} \tag{35}$$

where f is the focal length. However, this does not take into account that the signal is smeared by the seeing conditions to a wider FWHM. To take this effect into account, a Gaussian is fitted to the Airy pattern. The FWHM is used to determine the variance of the Gaussian and the signal integrated in the Airy disk to determine the amplitude. This allows to give credit to the possibly wider FWHM by seeing and still delivers the best estimate on the signal that is available. This leads to:

$$FWHM = 2\sqrt{2\ln 2}\sigma \tag{36}$$

where σ is the variance of the (non-normalized) Gaussian. The total signal of the object corresponds to the area underneath the two-dimensional Gaussian and scales its amplitude. The full FWHM fit is only performed in one dimension, perpendicular to the relative velocity of the object to the observer. Full width at half maximum in the velocity direction might be extended due to the observation scenario:

$$FWHM_\perp = FWHM_{\text{seeing}} \qquad FWHM_{||} = FWHM_{\text{seeing}} + \int v_{||,\text{obj}} - v_{||,\text{tel}} dt \tag{37}$$

The velocity of the telescope $v_{||,\text{tel}}$ compared to the object $v_{||,\text{obj}}$ is dependent on the specific observation scenario that is chosen. The amount of signal that is contained in a single pixel depends on the relation between the variance σ of the Gaussian fit and the pixel scale P:

$$[\sigma_x, \sigma_y]^T = \frac{R_3(\psi)}{2\sqrt{2\ln 2}} \cdot [FWHM_{||}, FWHM_\perp]^T, \tag{38}$$

where, ψ is the angle between the pixel grid (x,y) and the velocity direction of the object on the image ($||, \perp$), and R_3 the rotation matrix.

The signal of the brightest pixel is determined in integrating the non-normalized Gaussian in the following way:

$$S_b = \int_{-s/2+k_x}^{s/2+k_x} \int_{-s/2+k_y}^{s/2+k_y} I_0 \cdot \exp(-c_1(x-x_0) + c_2(x-x_0)(y-y_0) + c_3(y-y_0)))dxdy, \tag{39}$$

$$c_1 = \frac{\cos^2\psi \cdot 2\ln 2}{FWHM_{||}^2} + \frac{\sin^2\psi \cdot 2\ln 2}{FWHM_\perp^2}, \quad c_2 = \frac{\sin 2\psi \cdot \ln 2}{FWHM_{||}^2} + \frac{\cos 2\psi \cdot \ln 2}{FWHM_\perp^2},$$

$$c_3 = \frac{\sin^2\psi \cdot 2\ln 2}{FWHM_{||}^2} + \frac{\cos^2\psi \cdot 2\ln 2}{FWHM_\perp^2},$$

where s is the pixel scale (rad per pixel) and x_0 and y_0 are the offsets (in radian) from the center of the pixel wrt to the center of the Gaussian. This defines all parameters except the image noise that are needed to evaluate Eq.22, to characterize the measurement noise.

5.1.2. Image Noise Sources

The irradiation of the object is not the only light that is reflected towards the observing sensor. In optical observations several background sources need to be taken into account. The moon is treated in a geometrical sense, another option would be to use moon radiation as an additional light source and embed it in the signal to noise calculation. Intensive studies on the background sources, and tabulated values can be found in[29].[30]

In this paper the spectral irradiance is defined in units SI units of $Watt/m^3$, all angles are considered in radians. One of the background sources is the so-called airglow spectral radiation $I_{AG}(\lambda)$, which is the brightness of the atmosphere itself; it is faint glow if the atmosphere itself, which is caused by chemiluminescent reactions occurring between 80 and 100 km. Atmospherically scattered light is the sum of all light that is scattered by the atmosphere, excluding Sun and Moonlight. It is a relatively small contribution but adds to an overall elevated image background and hence should not be neglected.

$$I_{AG,AS}(\lambda) = s^2 \cdot J_i(\lambda) \cdot R(\zeta) \qquad J_i = J_{AG}, J_{AS} \tag{40}$$

where s is the angle under consideration, in case of the telescope, e.g. the field of view, or the angle that is fitted into a single pixel, $R(\zeta)$ is the van Rhijn factor, it can be approximated as $\frac{1}{\cos\zeta}$ in first order and describes the deviation from the zenith by angle ζ and the additional air mass and thickness, that has to be accounted for in low elevations.[29] J_{AG} is the spectral radiance of the zenith unit angle airglow in units of $Watts/m^2 ster \mu m$. J_{AS} is the spectral unit zenith angle radiance due to scattered light. It can be assumed that the faint star spectrum is an adequate representation. The diffuse galactic light is a light source that is concentrated along the galactic plane. Its spectral radiance can be represented as the following:

$$I_{GAL}(\lambda) = s^2 \cdot J_{GAL}(\lambda) \exp(-\beta \cdot 180/(15 \cdot \pi)), \tag{41}$$

where J_{GAL} is the spectral radiance at unit angle zero galactic latitude β. Zodiac light is the sunlight which is scattered by the dust in the ecliptic. It is hence a function of the ecliptic latitude and longitude with the same spectral distribution as the Sun as a first order approximation. Zodiac light is obtained using look-up tables for the white light radiance.

$$I_{ZODI}(\lambda) = s^2 \cdot J_{ZODI}(\gamma, \delta) \cdot \frac{J(\lambda)}{E}, \tag{42}$$

where γ, δ are the longitude and latitude in the ecliptic coordinate system, $J_{ZODI}(\gamma, \delta)$ is the total radiance per unit angle. Another major light source are the stars. One way to include stars is to include them at the exact position as they appear in extensive star catalogs. However, this is a very time consuming procedure. Tables exist with the number of stars of given photographic magnitudes, as displayed in the appendix. Using these, they can be converted to radiance values, assuming the spectral distribution of faint stars, as in the case of the galaxies. The conversion is done in the blue wavelength (440*nm*), to have the best equivalence with the photographic magnitudes *m*. This leads to the spectral star irradiation:

$$I_{STAR}(\lambda) = n \cdot \frac{s^2 \cdot 32400}{pi^2} \cdot 6.76 \cdot 10^{-12-0.4 \cdot m} \frac{J_{GAL}}{\int J_{GAL} d\lambda} \tag{43}$$

where *n* is the number of stars in the assigned bin. The irradiation values correspond to the irradiation without an atmosphere. Sometimes one is not interested in a specific wavelength, but the total radiation, one can integrate or use approximations for the white light, which leads to the following, utilizing also the simple airmass approximation:

$$\begin{aligned}
\mathfrak{I} &= \int I(\lambda) d\lambda \approx \bar{\mathfrak{I}} = I(\bar{\lambda}) \cdot \Delta\lambda && (44)\\
\bar{\mathfrak{I}}_{AG} &= s^2 \cdot \frac{1}{\cos\zeta} \cdot 1.42 \cdot 10^{-14} \qquad [W/m^2] && (45)\\
\bar{\mathfrak{I}}_{AS} &= s^2 \cdot \frac{1}{\cos\zeta} \cdot 1.57 \cdot 10^{-15} \qquad [W/m^2] && (46)\\
\bar{\mathfrak{I}}_{GAL} &= s^2 \exp(-\beta \cdot 180/(15 \cdot \pi)) \cdot 2.12 \cdot 10^{-15} \qquad [W/m^2] && (47)\\
\bar{\mathfrak{I}}_{ZODI} &= s^2 \cdot J_{ZODI}(\gamma, \delta) \cdot 5.0 \cdot 10^{-15} \qquad [W/m^2] && (48)\\
\bar{\mathfrak{I}}_{STAR} &= n \cdot s^2 \cdot 10^{-0.4 \cdot m} \cdot 3.0 \cdot 10^{-16} \qquad [W/m^2] && (49)\\
\bar{T} &= \exp\left(-0.27 \frac{1}{\cos\zeta}\right) \qquad [-] && (50)
\end{aligned}$$

At the detector level, the signal of the object is superimposed by the sky background and the detector noise itself. If we assume the noise from all the different sources is uncorrelated (so no covariance terms are taken into account), the noise sources are added in quadrature:[31]

$$N^2 = \sigma_o^2 + \sigma_{\mathrm{Sky}}^2 + \sigma_B^2 + \sigma_{\mathrm{est}}^2, \tag{51}$$

where σ_o is the variance of the count rate stemming from the object itself, σ_{Sky} is the variance of the sum of all previously discussed external background sources, σ_B is the variance of the count rate from the internal sensor background and, σ_{est} is the variance stemming from the estimation of the background level. The count values are actually derived in a time averaged fashion of indepenent processes, hence it can be assumed that the signal is a Poisson distributed random variable. The zeroth moment then leads to $\sigma_S^2 = S$. This is applied to the signal of the object itself as well as the signal of the sky. The background noise is detector dependent and has the following components:[31]

$$\sigma_B^2 = R^2 + T^2 + P^2, \tag{52}$$

where *R* is the readout noise variance, *T* the truncation noise variance, and *P* the processing noise variance. Truncation is caused by the fact that the signal per pixel is rounded to an integer value. If we assume that this rounding error can be seen as a randomly distributed variable having a a domain equal to the gain for readout and precision element in the rounding process, the variance equals to $T^2 = (g^2 - 1)/12$. Processing noise is introduced by the way the single observation frames are treated to enable detection of objects and comparable results. This includes subtracting raw frames with dark and bias frames. In general, it includes the noise of all pixels that are folded with the pixel in question by prior reduction steps. As a simple estimate, the readout and truncation noise of all previous frames that are added, subtracted or folded with the image are added up in the processing noise term. Hence, it can easily exceed the readout and truncation term of the actual frame. This provides all the necessary information to calculate the image noise on the detector underlying the brightest pixel and Eq.22 can be fully evaluated.

5.2. Light Curve Measurements

If only the brightness (and not the astrometric position) is the measured value, so-called light curve measurements are taken. The measured quantity is only one, the irradiation I that is received at the sensor:

$$I = E \cdot \left(\sum_{i=1}^{n} \frac{\mathfrak{A}_i}{\pi \cdot (x-R)^2} \left[C_{\mathrm{d},i}\, \hat{\mathbf{O}}\, \hat{\mathbf{N}}_i \cdot\, \hat{\mathbf{S}}\, \hat{\mathbf{N}}_i + \frac{\tau_i \cdot C_{\mathrm{s},i} \cdot x^2}{R^2} \right] \right) \tag{53}$$

$$\tau_i = \begin{cases} 1 & \text{for } 1-\cos(0.25\,\mathrm{deg}) \leq \frac{\mathbf{O}+\mathbf{S}}{|\mathbf{O}+\mathbf{S}|} \cdot\, \hat{\mathbf{N}}_i \leq 1+\cos(0.25\,\mathrm{deg}) \\ 0 & \text{else} \end{cases}$$

$$\text{with} \qquad 0 < \arccos(\,\hat{\mathbf{S}}\,\hat{\mathbf{N}}_i) < \pi/2 \qquad 0 < \arccos(\,\hat{\mathbf{S}}\,\hat{\mathbf{O}}) < \pi/2$$

I is a scalar of the received irradiation. $x = |\mathbf{x}|$, R the mean radius of the Earth, $\hat{\mathbf{S}}$ the direction of the Sun, τ_i the specular reflection parameter, determining if specular reflection condition is met, R is the radius of the Sun. No limb darkening effects have been accounted for. $\hat{\mathbf{O}}$ is the viewing direction of the sensor, and the following relation holds:

$$\mathbf{x} = \mathbf{x}_{\mathrm{topo}} + \mathfrak{r} \cdot \hat{\mathbf{O}}, \tag{54}$$

whereas $\mathfrak{r}$ is the topocentric range to the object, $\mathbf{x}_{\mathrm{topo}}$ is the geocentric position vector of the observing station in the inertial space fixed reference frame. For different BRDF models the irradiation function looks differently. However, giving the measurement function, at a specific time of just a specific solar and object position, does not embrace the full problem. If we extend again the state in the fashion as outlined in Eq.5. Again, we are making a decision on a specific object model again. For different object models, the measurement function looks slightly different. As mentioned before as we are not taking into account any torques, the center distance of the facet is not needed, as $\delta << x, |\mathbf{x}_{\mathrm{topo}}|$. The linearized observation matrix of the extended state hence would be the following:

$$\mathbf{H}_{\text{light}} = \begin{pmatrix} \frac{\partial I}{\partial x_1} & \frac{\partial I}{\partial x_2} & \frac{\partial I}{\partial x_3} & \frac{\partial I}{\partial \dot{x}_1} & \frac{\partial I}{\partial \dot{x}_2} & \frac{\partial I}{\partial \dot{x}_3} & \frac{\partial I}{\partial N_1} & .. & \frac{\partial I}{\partial N_n} & \frac{\partial I}{\partial \mathfrak{A}_1} & \cdots & \frac{\partial I}{\partial \mathfrak{A}_n} \end{pmatrix} \tag{55}$$

with

$$\frac{\partial I}{\partial x_j} = E \cdot \Big(\sum_{i=1}^{n} \frac{\mathfrak{A}_i}{\pi \cdot (x-R)^2} \big[\frac{C_{\text{d},i}}{\mathfrak{r}} \, \hat{\mathbf{S}} \, \hat{\mathbf{N}}_i \cdot \, \hat{\mathbf{N}}_{i,j} \big]$$

$$-2 \frac{\mathfrak{A}_i}{\pi \cdot (x-R)^3} \frac{x_j}{x} \big[C_{\text{d},i} \, \hat{\mathbf{O}} \, \hat{\mathbf{N}}_i \cdot \, \hat{\mathbf{S}} \, \hat{\mathbf{N}}_i + \frac{\tau_i \cdot C_{\text{s},i} \cdot x^2}{R^2} \big] \Big)$$

$$\frac{\partial I}{\partial \dot{x}_j} = 0$$

$$\frac{\partial I}{\partial N_i} = E \cdot \left(\sum_{i=1}^{n} \frac{\mathfrak{A}_i}{\pi \cdot (x-R)^2} \big[C_{\text{d},i} \, \hat{\mathbf{O}} \, \hat{\mathbf{N}}_i \cdot \, \hat{\mathbf{S}}_j + C_{\text{d},i} \, \hat{\mathbf{S}} \, \hat{\mathbf{N}}_i \cdot \, \hat{\mathbf{O}}_j \big] \right)$$

$$\frac{\partial I}{\partial \mathfrak{A}_i} = E \cdot \left(\sum_{i=1}^{n} \frac{1}{\pi \cdot (x-R)^2} \big[C_{\text{d},i} \, \hat{\mathbf{O}} \, \hat{\mathbf{N}}_i \cdot \, \hat{\mathbf{S}} \, \hat{\mathbf{N}}_i + \frac{\tau_i \cdot C_{\text{s},i} \cdot x^2}{R^2} \big] \right)$$

for $j = 1..3$ and $i = 1,..,n$. Unlike in the astrometric case, you already have an almost fully populated measurement matrix, which means the dependencies on almost all parts of the extended state vector are explicit even in the linearization.

In the light curve measurements the noise function is much easier, as the brightness is the directly measured quantity, rather than the derived quantity for the center extraction for the astrometric measurements. Again it is assumed a zero mean noise function, then the variance K_{light} is directly given by the inverse of the signal to noise ratio S/N:[32,33]

$$K_{\text{light}} = \frac{N}{S} \tag{56}$$

The signal and noise follows the same relations as in Eq.40,51. The same noise decomposition can be performed as in the astrometric measurements.

6. Observability Analysis

Evaluation of the matrix G for the observation matrix of the astrometric observations, Eq.19, reveals observability of the state. It also readily reveals that we have no observability on the extended state, which holds the quantities of interest, as defined in Eq.5. Which is not surprising: Observability can only be gained when the parameters have impact on the measurements. When we assume the simplification to the two body problem, the body parameters have no impact at all, and should not be observable.

Without the two body approximation, the extended state still does not become observable as the parameters are coupled and are not singled out in the measurement. Only the traditional state and one "fudge" parameter, which soaks in all effects from all other perturbations, can be made observable. Similar issues have to be faced in the case of the light curve measurements.

With the redefinition of the observability, the measurement noise has been taken into account, and directly influences the observability integral.A stronger coupling with the extended state is the case as given in the measurement noise. Further investigations are needed to determine all implications of the extended framework.

7. Conclusions

The independent observability of space objects has been investigated. The focus has been laid on optical ground based observations. Observability in the traditional sense of the state can be achieved, under the assumption of perfect measurements. However, in order to reliably predict the future state of a space object of interest not only the position and velocity but also body parameters, such as shape, surface properties and attitude are relevant. The state can be extended however, then observability is thwarted.

A new method is proposed, to determine observability under inclusion of measurement noise. This allows to evaluated observability in dependence of sensor and detector parameters as well as the observation scenario. Furthermore, measurement noise is also an additional source of information, when it is realistically modeled. Further research is needed in order to fully explore the next concept.

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