EXTENDED MISSION TRAJECTORY OPTIONS FOR THE VAN ALLEN PROBES MISSION

Fazle E. Siddique(1) and Justin A. Atchison(2)

(1)Mission Design and Navigation Lead Engineer, Space Exploration Sector, The Johns Hopkins University Applied Physics Laboratory, (443) 778-9270, Fazle.Siddique@jhuapl.edu
(2)Mission Design and Navigation Engineer, Space Exploration Sector, Justin.Aitchison@jhuapl.edu

Abstract: The Van Allen Probes mission, part of NASA’s Living With a Star Program, successfully launched on August 30th, 2012 from the Cape Canaveral Atlas-V Space Launch Complex 41. The two year primary mission consisted of two spin stabilized spacecraft in highly eccentric Earth orbits. The spacecraft provide insight into the dynamics of Earth’s radiation belts by measuring the relevant in-situ environment (magnetic and electric fields) and key parameters of energetic particles and ions. The two spacecraft have slightly different orbital periods that cause one spacecraft to lap the other approximately four times per year. The difference in orbital elements resulted in an offset in the natural precession rate induced by Earth oblateness, causing the lines of apsides (or petals) of the two orbits to deviate. The project, which is currently in a bridge phase, considered several extended mission trajectory options that alter the rate of petal separation and lapping rate in order to study new Prioritized Science Goals for the first extended mission. This paper details the trajectory adjustment tradespace and subsequent maneuver design for the selected option.

Keywords: Van Allen Probes, Mission Design, Radiation Belts.

1. Introduction

1.1. Background

The Van Allen Probes (formerly the Radiation Belt Storm Probes) Mission is part of NASA’s Living With a Star program and was built and currently operated by The Johns Hopkins University Applied Physics Laboratory (JHUAPL) in Laurel, MD. The primary purpose of the mission is to study the creation and evolution of high energy particles in Earth’s magnetosphere to provide a better understanding of the processes that drive changes within the Earth’s radiation belts. The flight segment of the mission consists of two nearly-identical spin-stabilized spacecraft in similar highly eccentric, low inclination Earth orbits. Each spacecraft hosts an identical suite of eight instruments, which measure the relevant in-situ environment including magnetic and electric fields, particle composition, and wave distributions[1].

The two spacecraft were launched on August 30, 2012 from the Cape Canaveral Atlas-V Space Launch Complex 41 on an Atlas V 401 launch vehicle and began operation on November 1, 2012 following a 60 day commissioning period. They completed their primary two year mission on November 1, 2014 and are currently operating in a bridge phase. During the Senior Review proposal process with NASA headquarters, new Prioritized Science Goals (PSGs) were identified for the first extended mission. The project considered several extended mission trajectory options that alter the orbital parameters in order to satisfy the new PSGs. The limiting factor on the various options was fuel consumption, and the selected option balanced the science goals and mission lifetime.
1.2. Spacecraft Description

The spacecraft have spin stabilized attitude control systems. During the science mission, both spacecraft spin at approximately 5.5 RPM with their +Z spin axes generally pointed towards the Sun. The spin axes are inertially fixed, hence the sun angle drifts about a degree per day. Every 20-22 days, the spin axes are precessed approximately 21° along the sun cone to maintain pointing towards the Sun (Figure 1)[2].

![Figure 1: Expected sun angles for spin axis pointing](image)

Each spacecraft has eight 0.9 N thrusters as shown in Figure 2. The P-thrusters (precession) are used to precess the spin axis by pairing either the P1 & P3 or P2 & P4 thrusters together to provide a nearly pure torque with minimal residual Delta-V ($\Delta v$). The P-thrusters are also used to provide $\Delta v$ capability by pairing either P1 & P2 or P3 & P4 together to provide thrust in the pro-Sun or anti-Sun direction. The S-thrusters are used to vary the spin rate[3].

![Figure 2: Spacecraft thruster configuration](image)
1.3. Nominal Mission

The two spacecraft were launched into nearly identical 10° inclined orbits. After launch, the spacecraft were maneuvered to give a 20 km separation in perigee and a 150 km separation in apogee, with Van Allen Probe A (hence forth referred to as RBSPA) in the inner 605x30550 km orbit and Van Allen Probe B (hence forth referred to as RBSPB) in the outer 625x30700 km orbit. The orbit semi-major axes and inclination were selected to satisfy the science requirement that the line of apsides of the two spacecraft complete a full revolution with respect to the Earth-Sun line during the nominal mission. The solar phase of apsides (δ_{SPA}) angle is defined as the angle between the line of apsides (pointing towards apogee) projected into the ecliptic plane and the Earth-Sun line. This angle is plotted in Figure 3. The quadrants are specified at left and have relevance to the Earth’s magnetosphere. The angle completed a full revolution between the first day of the nominal mission (Day 60) and Day 709.

The slightly different orbital periods also satisfy the science requirement that the two spacecraft “lap” each other at least twice per solar quadrant. This requirement corresponds to a limit on synodic period between the two spacecraft, shown in Figure 4. The period is computed using the long-period Brouwer mean elements[4], which average perturbations faster that the orbit period. After separating from the launch vehicle, each spacecraft executed two maneuvers to target a specific perigee and apogee separation. This increased separation can be observed in Figure 4 as a steep reduction in synodic period. After these maneuvers, the synodic period remained relatively stable with the exception of collision avoidance maneuvers [5].
The difference in semi-major axes and eccentricities resulted in an offset in the natural precession rate induced by Earth oblateness, causing the lines of apsides (or petals) of the two orbits to deviate. Based on the current orbital configuration, the delta in solar phase of apsides is increasing at 0.8°/yr. The extended mission trajectory options considered explored the effects of altering the rate of petal separation and lapping rate.

2. Extended Mission Orbit Adjust Investigation

2.1. Science Rationale

Decreasing the synodic period increases the frequency of close approach “lapping” events. This will yield more opportunities to study PSG1: understanding the role that non-linear mechanisms play in the particle acceleration process. These microscale physics phenomena can be observed by targeting a specific geomagnetic relative position during each close approach event, enabling the spacecraft to spend a period of time near the same Earth magnetic field line and take unique measurements. Increasing the rate of petal separation enables studying PSG3: understanding how mesoscale injections and global transport processes act to transport electrons and ions into the ring current, slot, and inner zones. Sampling different magnetic local times of the radiation belts simultaneously will increase the understanding of what spatial, temporal, and energy distributions are produced by different transport/injection mechanisms.

2.2. Lapping Event On-Orbit Tests

Two such close approach on-orbit test events have already taken place to demonstrate the scientific merit of these unique magnetic field measurements.
2.2.1. February 2015 Test

The first test occurred during the 2-Feb-2015 close approach event. The objective of this test was to minimize the relative X and Y position in the geomagnetic (MAG) coordinate frame between the two spacecraft (see Appendix for frame definition), resulting in the two spacecraft being aligned along the same magnetic meridian, as shown in Figure 5.

Figure 5: Maneuvered state of RBSPB aligned along the same magnetic meridian as RBSPA

This orbit geometry was achieved by executing a small phasing maneuver on RBSPB to slow down the spacecraft (decrease mean motion) in order to ensure time of closest approach (TCA) occurred at the desired true anomaly. A 12.1 cm/s maneuver was executed on 23-Jan-2015 to delay TCA 68.5 s to 2-Feb-2015 19:08:06 UTC. The relative geomagnetic position of the two spacecraft before the maneuver is shown in Figure 6(a). The executed maneuver placed the probes as close as 4.8 km from the same magnetic meridian (Figure 6(b)).
The second test occurred during the 9-Apr-2015 close approach event. The objective of this test was to place both spacecraft in the vicinity of the same magnetic field line. This orbit geometry was achieved by minimizing the inter-satellite range given by

$$\Delta L \phi = \sqrt{(L_A \cos \phi_A - L_B \cos \phi_B)^2 + (L_A \sin \phi_A - L_B \sin \phi_B)^2}$$

Figure 6: Range proximity to common magnetic meridian

2.2.2. April 2015 Test

The second test occurred during the 9-Apr-2015 close approach event. The objective of this test was to place both spacecraft in the vicinity of the same magnetic field line. This orbit geometry was achieved by minimizing the inter-satellite range given by

$$\Delta L \phi = \sqrt{(L_A \cos \phi_A - L_B \cos \phi_B)^2 + (L_A \sin \phi_A - L_B \sin \phi_B)^2}$$

Figure 6: Range proximity to common magnetic meridian
where $L$ is the L-shell value specifying the range of the magnetic field line at the intersection of Earth’s equatorial plane (in Earth radii) and $\phi$ is the magnetic local time (MLT). These values can be determined using the equations

$$ \phi = \arctan\left( \frac{x_{SM}}{y_{SM}} \right) \quad (2) $$

$$ L = \frac{r}{(\cos \lambda)^2} \quad (3) $$

$$ r = \sqrt{x_{SM}^2 + y_{SM}^2 + z_{SM}^2} \quad (4) $$

$$ \lambda = \arctan\left( \frac{z_{SM}}{\sqrt{x_{SM}^2 + y_{SM}^2}} \right) \quad (5) $$

where $r$ is the radial distance of the spacecraft in Earth radii and $\lambda$ is the geomagnetic latitude. The subscripts ($SM$) denotes a coordinate in the solar magnetic coordinate frame (see Appendix for frame definition) and ($A,B$) refer to the two spacecraft. Proximity to the same magnetic field line is guaranteed by minimizing the difference in both L-shell and MLT between both spacecraft. Similar to the previous on-orbit test, this orbit geometry was achieved by executing a small 22.4 cm/s phasing maneuver on RBSPB to slow down the spacecraft. The values of inter-satellite range and $\Delta L \phi$ between the two spacecraft before the maneuver is shown in Figure 7(a). The executed maneuver placed the probes within 0.5 km from the same magnetic field line (Figure 7(b)).
Several trajectory adjust options were investigated in conjunction with the Van Allen Probes Project Scientist. The considered options included various alterations of apogee and perigee altitudes on both spacecraft in order to understand the net effect on petal separation and lapping rate. The limiting factor was fuel consumption and the subsequent reduction in expected mission lifetime.
2.4. Petal Separation

Petal separation rate is predominantly dependent on the difference in nodal precession rate \( \dot{\Omega} \) between each spacecraft caused by Earth’s second order oblateness, given by

\[
\dot{\Omega} = -\left( \frac{3}{2} \right) J_2 R_E^2 \cos(i) \left( \frac{n}{p^2} \right)
\]

where \( J_2 \) is the constant describing the size of Earth’s bulge, \( R_E \) is the equatorial radius of Earth, \( i \) is the orbit inclination, \( n \) is the orbit mean motion, \( p \) is the orbit semi-latus rectum. \( n \) and \( p \) can be found using

\[
n = \sqrt{\left( \frac{\mu}{a^3} \right)}
\]

\[
p = a(1 - e^2)
\]

where \( a \) is the orbit semi-major axis, \( e \) is the orbit eccentricity, and \( \mu \) is Earth’s gravitational constant. Substituting the radius of perigee \( r_p \) and apogee \( r_a \) for the orbit semi-major axis and eccentricity in \( n \) and \( p \), (6) can be written as

\[
\dot{\Omega} = -\left( \frac{3}{2} \right) J_2 R_E^2 \sqrt{\mu/2} \cos(i) \left( \frac{\sqrt{r_a + r_p}}{r_a r_p^2} \right)
\]

Taking the partial derivative of (9) with respect to \( r_p \) and \( r_a \) yields the relationship

\[
\frac{\partial \dot{\Omega}}{\partial r_p} = \frac{r_a(4r_a + 3r_p)}{r_p(3r_a + 4r_p)} = \frac{e^2 + 8e + 7}{e^2 - 8e + 7}
\]

As shown in Figure 8, for the highly elliptic orbits of Van Allen Probes, changes in \( r_p \) are approximately 6 times as effective in changing \( \dot{\Omega} \) versus changes in \( r_a \).

![Figure 8: \( \dot{\Omega} \) sensitivity to changes in \( r_p \) vs. \( r_a \)](image-url)
2.5. Lapping Rate

The lapping rate, or synodic period between the two spacecraft, is given by

\[ T_s = \left( \frac{2\pi}{n_A - n_B} \right) \]  

(11)

Taking the partial derivative of (11) with respect to \( r_a \) and \( r_p \) yields the relationship

\[ \frac{\partial T_s}{\partial r_a} = \frac{\partial T_s}{\partial r_p} \]

(12)

hence, changes in \( r_a \) and \( r_p \) equally impact lapping rate.

2.6. Fuel Usage

The limiting factor on which options could be pursued was fuel consumption. For the highly elliptic class of Van Allen Probe orbits, with eccentricity 0.68, the fuel costs associated with changing perigee altitude is more prohibitive than costs associated with an equivalent change apogee altitude. Per the Vis-viva equation, Figure 9 shows the \( \Delta v \) costs associated for an impulsive maneuver to change the apse altitude for RBSPA.

2.7. Final Options

The down-selected final options that were presented to the Van Allen Probes Science Working Group (SWG) are shown in Table 1. Even though it was demonstrated that it’s much more effective to impact petal separation via changes in perigee altitude, most of the options presented to the SWG were limited to apogee changes due to \( \Delta v \) costs associated with changing perigee on highly elliptic orbits. One amenable option investigated was to reduce the perigee altitude of RBSPA. At the end of the mission, the disposal burns on both spacecraft would result in a 190 km perigee altitude, thus,
any maneuvers for reducing perigee altitude were already allocated in the propellant budget. These options were scrapped due to significant push back from several instrument teams that said their payloads would not work properly at lower altitudes. Similarly, increasing the perigee altitude of RBSPB was considered unfavorable because any selected option had a 2x penalty in propellant usage (propellant used to increase the perigee altitude would require an equivalent increase in propellant to deorbit the spacecraft). If the priority was to impact lapping rate, the $\Delta v$ optimal solution of only changing apogee altitude would be the logical approach.

### Table 1: Orbit Trim Maneuver Options

<table>
<thead>
<tr>
<th>Option</th>
<th>Fuel Used RBSPA (kg)</th>
<th>Cost in Mission Life RBSPA (months)</th>
<th>Fuel Used RBSPB (kg)</th>
<th>Cost in Mission Life RBSPB (months)</th>
<th>Petal Separation in Jan 2019 (days/lap)</th>
<th>Lapping Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do nothing</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>8.7°</td>
<td>67.0</td>
</tr>
<tr>
<td>Dec RBSPA Apogee by 75 km Inc RBSPB Apogee by 75 km</td>
<td>0.70</td>
<td>1.58</td>
<td>0.75</td>
<td>1.70</td>
<td>12.2°</td>
<td>35.0</td>
</tr>
<tr>
<td>Dec RBSPA Apogee by 75 km Inc RBSPB Perigee by 7 km</td>
<td>0.70</td>
<td>1.58</td>
<td>0.38</td>
<td>1.70</td>
<td>11.5°</td>
<td>44.6</td>
</tr>
<tr>
<td>Dec RBSPA Apogee by 70 km Inc RBSPB Apogee by 140 km</td>
<td>0.65</td>
<td>1.47</td>
<td>1.40</td>
<td>3.16</td>
<td>13.5°</td>
<td>29.4</td>
</tr>
<tr>
<td>Dec RBSPA Apogee by 0 km Inc RBSPB Apogee by 210 km</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>4.74</td>
<td>13.5°</td>
<td>29.4</td>
</tr>
<tr>
<td>Dec RBSPA Apogee by 140 km Inc RBSPB Apogee by 140 km</td>
<td>1.30</td>
<td>2.95</td>
<td>1.40</td>
<td>3.16</td>
<td>15.1°</td>
<td>24.8</td>
</tr>
</tbody>
</table>

3. **Extended Mission Maneuver Design and Execution**

3.1. **Selected Option**

In order to balance both science objectives and mission lifetime, the SWG ultimately selected reducing the RBSPA apogee altitude by 75 km and increasing the RBSPB apogee altitude by 75 km. The selected option also gives balanced gains to both petal separation and lapping rate. The requirement to be satisfied by the set of maneuvers was to increase the difference in apogee altitudes $r_{AB}$ by 140 km throughout the remainder of the mission. Figure 10(a) shows the natural evolution of $r_{AB}$ varies between 140 km and 165 km throughout the remainder of the mission if no maneuvers were executed. Figure 10(b) shows the evolution of $r_{AB}$ varies between 284 km and 325 km with the execution of the planned maneuvers, satisfying the science requirement.
A cause of concern for the selected option arose when inspecting the evolution of the difference in perigee altitudes (Figure 11). Due to solar and lunar perturbations, there are a few months in the mission where perigee altitudes cross. However, upon examination of the actual minimum range between the two spacecraft orbit tracks, the maneuver did not have any direct adverse effects on collision risk. Due to $J_2$ precession of argument of perigee $\omega$, it can be seen in Figure 12 that in both cases the orbits cross every 200 days, half the 400 day period for a full $\omega$ cycle. The indirect effect on collision risk is due to the increased synodic period. Risk of collision between the two spacecraft is heightened during periods where the orbits cross and close approach events occur simultaneously. Figure 13 highlights the periods where this geometry is realized for the remainder of the mission.
(a) Post maneuver absolute perigee altitudes for both spacecraft

(b) Post maneuver relative perigee altitudes for both spacecraft

Figure 11: Perigee altitudes for the remainder of the mission

Figure 12: Minimum distance between orbit tracks for nominal and maneuvered case

Figure 13: Predicted dates of orbit crossing and lapping events
3.2. Maneuver Planning

The intended time frame for executing the pair of maneuvers was between March and December 2015. As mentioned previously, the P-thrusters that are used to provide $\Delta v$ capability are along the spacecraft $\pm Z$ axis, with their $+Z$ spin axes pointed in the Sun direction. With thrust limited to only the pro-Sun or anti-Sun direction, the optimal time for executing a $\Delta v$ is dependent on the solar phase of apsides and declination. Figure 14 shows the values of $\delta_{SPA}$ during this time period. The June 2015 epoch would have been ideal to execute the maneuvers due to a $270^\circ$ $\delta_{SPA}$ angle value as the line of apsides crosses from quadrant III to IV. In this orbit geometry, the sun-pointed spacecraft thrust vector would be tangent to the orbit at perigee. Unfortunately, due to the spacecraft pointed $17^\circ$ below the ecliptic plane, orbit inclination, and the Sun being at its highest declination during summer solstice, the minimum value of the angle between the thrust and velocity vector ($\delta_{TV}$) was about $50^\circ$ (Figure 14).

$\delta_{TV}$ became favorable in October 2015. At that time, the $\delta_{SPA}$ angle diverged from the optimal $270^\circ$, but still permitted maneuvers of minimum $\delta_{TV}$ to occur shortly after perigee. To accommodate the 21-day cadence precession maneuvers, the maneuver date for the extended mission orbit trim maneuvers was set for 9-Oct-2015.

![Figure 14: $\delta_{SPA}$ and $\delta_{TV}$ during the maneuver consider time frame](image)

3.3. Maneuver Design

Figure 15 shows the variation in $\delta_{TV}$ on 9-Oct-2015. In the current orbit geometry (based on $\delta_{SPA}$), the minimum values of $\delta_{TV}$ correspond to perigee passes, where maneuvers to adjust apogee altitude must be executed. Out of the permissible maneuver times, the 17:15:00 UTC minimum for RBSPA and the 15:30:00 UTC minimum for RBSPB occurred during standard business hours and were preferred for mission operations. Since the thrust vector is inertially fixed, the larger the orbit arc
required to execute the maneuver, the less efficient the maneuver will be. The propellant savings
from having several maneuvers over multiple orbits was on the order of a few grams and did not
outweigh the streamlined operations benefits of executing a single maneuver per spacecraft.

![Figure 15: Variation in $\delta_{TV}$ during maneuver epoch]

### 3.3.1. Preliminary Maneuver Specification

The relevant details of the designed maneuvers on each spacecraft can be found in Table 2. The
targeted apogee altitude is based on the osculating apogee altitude at the subsequent post maneuver
apogee passage. Using the latest propulsion system parameters (tank pressure and temperature),
estimates of propellant usage were calculated based on maneuver duration. These parameters will
be updated using the latest navigation solution the week of the maneuver.

<table>
<thead>
<tr>
<th></th>
<th>RBSPA</th>
<th></th>
<th>RBSPB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pre-Maneuver Apogee Altitude (km)</td>
<td>30512.6</td>
<td>Pre-Maneuver Apogee Altitude (km)</td>
</tr>
<tr>
<td></td>
<td>Post-Maneuver Apogee Altitude (km)</td>
<td>30437.6</td>
<td>Post-Maneuver Apogee Altitude (km)</td>
</tr>
<tr>
<td>Maneuver Start (UTC)</td>
<td>9 Oct 2015 17:00:00.000</td>
<td></td>
<td>Maneuver Start (UTC) 9 Oct 2015 15:22:00.000</td>
</tr>
<tr>
<td>Duration (sec)</td>
<td>1450</td>
<td></td>
<td>Duration (sec) 1493</td>
</tr>
<tr>
<td>$\Delta v$ (m/s)</td>
<td>2.253</td>
<td></td>
<td>$\Delta v$ (m/s) 2.369</td>
</tr>
<tr>
<td>Fuel (mg)</td>
<td>686</td>
<td></td>
<td>Fuel (mg) 740</td>
</tr>
</tbody>
</table>
3.3.2. Maneuver Start Time Sensitivity

The Van Allen Probes project exclusively uses the APL18 station in Laurel, MD and the Universal Space Network (USN) stations in Hawaii and Australia for ground-based communication. For the 10° inclined Van Allen Probe spacecraft, communication from these ground stations isn’t possible at perigee because of the station latitudes. In order to have timely execution of the maneuvers near perigee, the maneuvers must be commanded through the Tracking and Data Relay Satellite System (TDRSS). The data link between TDRSS and Van Allen Probes is 1.0 bps. Because of the link latency and due to common issues resulting in delayed acquisition of signal, delays in the maneuver execution start time will result in suboptimal maneuver efficiency and falling short of the desired apogee altitude change. The maneuver baseline objective was to change the altitude by 75 km for both spacecraft, with a threshold objective to change the altitude by 70 km. Figure 16 shows the penalty in achieved altitude change caused by delays in maneuver start time. Based on these findings, maneuvers that cannot be initiated within 4 minutes of the prescribed maneuver start time will be canceled and re-planned for another opportunity.

![Figure 16: Effect of delaying the designed maneuver start time](image)

4. Conclusion

The Mission Design and Navigation team has explored several extended mission trajectory options. In order to balance both extended mission Prioritized Science Goals and mission lifetime, the SWG ultimately selected reducing the RBSPA apogee altitude by 75 km and increasing the RBSPB apogee altitude by 75 km. Preliminary maneuver planning and design has been completed and analysis has shown mission objectives will be achieved. The preliminary maneuver parameters will be iterated using the latest navigation solution the week of the maneuver.
5. References


Appendix

The geomagnetic coordinate system (MAG) is defined so that its Z-axis is parallel to the magnetic dipole axis. The geographic coordinates of the dipole axis from the International Geomagnetic Reference Field. The Y-axis of this system is perpendicular to the geographic poles such that if D is the dipole position and S is the south pole Y=DxS. Finally, the X-axis completes a right-handed orthogonal set.

In solar magnetic coordinates (SM) the Z-axis is chosen parallel to the north magnetic pole and the Y-axis perpendicular to the Earth-Sun line towards dusk. The difference between this system and the GSM system is a rotation about the Y-axis. The amount of rotation is simply the dipole tilt angle as defined in the previous section. We note that in this system the X-axis does not point directly at the Sun. As with the GSM system, the SM system rotates with both a yearly and daily period with respect to inertial coordinates.

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The authors wish to acknowledge Gene A. Heyler, who was the Mission Design and Navigation Lead for the Van Allen Probes until his passing in March of 2013.