Relative equilibrium and dynamics characteristic of Spacecraft electromagnetic formation

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Abstract: *Electromagnetic Formation Flying (EMFF) is a novel technology that uses superconducting electromagnetic coils to provide electromagnetic force/torque, which can be used to control the relative position and attitude between different satellites in a formation. This concept could effectively solve the problem of propellant consumption and plume contamination, and offer non-contacting continuous reversible and synchronous controllability. With the development of composite spacecraft formation technology, the EMFF technology turns into research hotspot increasingly. Nevertheless, because the characteristic of electromagnetic internal force/ internal moment and the high nonlinear/ coupling of the dynamic equation, the groundwork research of static electromagnetic formation have important significance.*

This paper mainly concentrates on the dynamics problems of spacecraft electromagnetic formation flight, and mainly investigates the Hamilton modeling, relative equilibrium analysis and dynamics characteristic etc. First, the relative equilibrium for the vector mechanics model is analyzed and the necessary conditions for circularly restricted static electromagnetic formation are derived. Second, the 6-DOF coupled nonlinear dynamic models by Hamiltonian method for the multi-spacecraft electromagnetic formation was obtained under earth imperturbation. Then, the equilibrium solution and the magnetic dipole of the given static formation were gained. Finally, Base on the equilibrium conditions, Hamilton dynamic models and position of equilibrium solution the characteristic of stability and controllability are investigated.

In conclusion, the study of Electromagnetic Formation Flying include many aspect，the dynamical research is fundamental of the control. All of these explorations base on the theories analysis and calculator simulation, and provide the foundation for future research on electromagnetic formation flight technology.

Key words: *Electromagnetic Formation Flying; Hamilton dynamic model; relative equilibrium; dynamics characteristic*

1. Introduction

Spacecraft electromagnetic formation as a non-contacting force innovation formation mode, which eliminate the disadvantages of the traditional thruster formation model and solve the technical problem of the plume flow contamination and propellant consumption[1]. And more importantly, Spacecraft electromagnetic formation offers reversible and synchronous controllability, providing a novel approach of controlling spacecraft formation flight. Inter-satellite non-contacting force is produced by interaction of fields between spacecraft. The internal force and torque can be used to control the relative trajectory and attitude of spacecraft to satisfy specified formation mission requirements. With the development of spacecraft formation technology, it has increasingly become a formation technology research focus and front subject.

Several representative inter-satellite non-contacting forces such as Coulomb force, electromagnetic force, flux-pinning force etc. have already been investigated. Electromagnetic formation [2,3] has been studied by MIT systematically. Miller's team proposed the electromagnetic formation flight (EMFF) concept in which each satellite is equipped with superconducting magnetic coils and the magnetic fields electrically generated by all the satellites are used to control the relative degrees of freedom of the formation.

In the formation dynamics modeling, the most widely applied is traditional Newtonian mechanics theory, either directly establishing a fully nonlinear model [4], or setting up a linearized model [5] based on Hill equation. Umair Ahsun [6] establishes the relative dynamics model considering the J2 and the nonlinear adaptive control is applied to formation maintenance and reconstruction. C. Norman [7] analyzes the nonlinear dynamic model of spacecraft formation system, which uses the Lagrange method.

The dynamic characteristics, especially the equilibrium characteristics for spacecraft formation with inter-satellite non-contacting force are essentially necessary for reference geometry and control law design. Natarajan [8,9] analyzed the relative equilibrium stability of two-craft coulomb formation along radial, along-track and normal direction, respectively. Schaub [10] proposed necessary conditions for static coulomb formation in a circular orbit. Huang [11] analyze the dynamics and equilibrium of spacecraft formation with inter-satellite non-contacting force. A general dynamic model for multi-spacecraft formation with inter-satellite non-contacting force is developed based on the Kane method. Analyzing the dynamics equilibrium for circularly restricted spacecraft formation, the necessary conditions to achieve static formation with inter-satellite non-contacting force are derived.

2. Problem statement

2.1 System description

The physical model of Multi-spacecraft electromagnetic formation shown in Figure 1, this system can be distributed in space by a certain formation configuration and formation control is enabled by non-contacting internal forces and torques among spacecraft. Treat each spacecraft as homogeneous spherical rigid-body; enumerated as

$1,\cdots,N$, each of the satellites is fitted with 3 orthogonal electromagnetic coils. When the coil is energized, the controllable electromagnetic force / torque can be generated between the satellites.

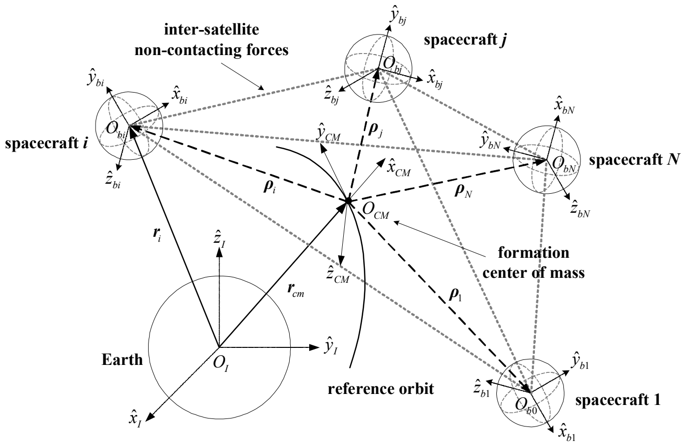

Fig.1 Spacecraft Formation with Non-contacting Internal Force

Assume as following: (i) The Earth's gravitational field is regarded as a central gravitational field; (ii) The formation centre of mass travels on a circular orbit, and orbit perturbation and disturbance are ignored.

2.2 Related reference frame

As shown in Fig. 1, Denote O_{CM} as the formation centre of mass and $\boldsymbol{\rho}_i$ as position vector from O_{CM} to centre of mass of spacecraft-i. For analysis convenience, the following four reference frames are introduced, as shown in Fig. 1.

(i) The Earth-centred inertial frame $\mathcal{N}$. It is a non-rotating frame with its origin at the centre of Earth O_I. The unit vectors of three-axes are defined as $\hat{\boldsymbol{x}}_I, \hat{\boldsymbol{y}}_I, \hat{\boldsymbol{z}}_I$, respectively, among which the x_I axis points to vernal equinox, the z_I axis is perpendicular to the equatorial plane and points to the north pole, and the y_I axis completes the right-hand system.

(ii) The orbital reference frame $\mathcal{H}$. Its origin is attached to the formation centre of mass O_{CM}, and the unit vectors of three-axes are $\hat{\boldsymbol{x}}_{CM}, \hat{\boldsymbol{y}}_{CM}, \hat{\boldsymbol{z}}_{CM}$, with x_{CM}, y_{CM} and z_{CM} axis along the directions of orbit radial, along-track and normal direction respectively.

(iii) The formation fixed reference frame $\mathcal{B}$. The origin is attached to O_{CM}, and the unit vectors of three-axes are defined as $\hat{\boldsymbol{x}}_B, \hat{\boldsymbol{y}}_B, \hat{\boldsymbol{z}}_B$, respectively. The three axes are fixed with the principal inertia axes of the formation.

(iv) The body reference frame $\mathcal{B}_i$. Its origin is attached to the i-th ($i = 1,\cdots,N$) spacecraft centre of mass O_{bi}, and the unit vectors of three-axes are $\hat{\boldsymbol{x}}_{bi}, \hat{\boldsymbol{y}}_{bi}, \hat{\boldsymbol{z}}_{bi}$, respectively, the three axes are fixed with the principal inertia axes of the spacecraft.

2.3 Generalized coordinates

The transformation relationship between each coordinate system is established and the relative position and attitude of the spacecraft are determined. The motion characteristic of the electromagnetic formation system is described by the following.

$$\begin{cases} \boldsymbol{q} = \left[\boldsymbol{q}_0 \ \ \boldsymbol{q}_1 \ \cdots \ \boldsymbol{q}_i \ \cdots \ \boldsymbol{q}_N\right]^{\mathrm{T}} \\ \boldsymbol{q}_0 = \left[q_{01} \ \ q_{02} \ \ q_{03}\right]^{\mathrm{T}} = \left[\varphi \ \ \theta \ \ \psi\right]^{\mathrm{T}} \\ \boldsymbol{q}_i = \left[q_{i1} \ \ q_{i2} \ \cdots \ q_{i6}\right]^{\mathrm{T}} = \left[\rho_{ix} \ \ \rho_{iy} \ \ \rho_{iz} \ \ \alpha_i \ \ \beta_i \ \ \gamma_i\right]^{\mathrm{T}} \end{cases} \tag{1}$$

Among them, $\rho_{ix}, \rho_{iy}, \rho_{iz}$ are expressed as the relative position component in the formation fixed reference frame $\mathcal{B}$; $\alpha_i, \beta_i, \gamma_i$ are expressed as Euler angles between the reference frame $\mathcal{B}_i$ .and $\mathcal{B}$ by the rotation of the 3-2-1sequence, the rotating matrix is noted as ${}^{\mathcal{B}_i}\boldsymbol{M}^{\mathcal{B}}$.; φ, θ, ψ are expressed as Euler angles between $\mathcal{B}$ and $\mathcal{H}$ by the rotation of the 2-3-1 sequence, the rotating matrix is noted as ${}^{\mathcal{B}}\boldsymbol{M}^{\mathcal{H}}$.

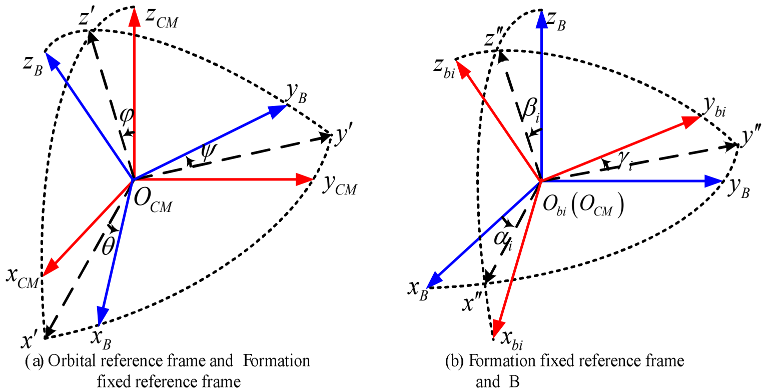

Fig.2 The relationship of Euler angles transformation

3. Vector dynamics model

In this section, the vector mechanics model is analyzed, the orbital and attitude dynamics equation are obtained. First, the orbital dynamics equation of member spacecraft-*i* and the center of mass of formation system in the inertial system are:

$$\begin{cases} \dfrac{{}^{\mathcal{N}}d^2\boldsymbol{r}_{CM}}{dt^2} = -\dfrac{\mu}{r_{CM}^3}\boldsymbol{r}_{CM} + \boldsymbol{f}_{CM}^d \\ \dfrac{{}^{\mathcal{N}}d^2\boldsymbol{r}_i}{dt^2} = -\dfrac{\mu}{r_i^3}\boldsymbol{r}_i + \dfrac{\boldsymbol{F}_i^{EM}}{m_i} + \boldsymbol{f}_i^d \end{cases} \tag{2}$$

μ is the Earth gravitational constant; $\boldsymbol{F}_i^{EM}$ is the electromagnetic force ; $\boldsymbol{f}^d$ is the perturbation acceleration. Nonlinear kinetics equation of electromagnetic formation system in inertial coordinate frame is:

$$\frac{{}^{\mathcal{B}}d^2\boldsymbol{\rho}_i}{dt^2} = \frac{\mu}{r_{CM}^3}\left[\boldsymbol{r}_{CM} - \left(\frac{r_{CM}}{r_i}\right)^3\boldsymbol{r}_i\right] + \frac{\boldsymbol{F}_i^{EM}}{m_i} + \Delta\boldsymbol{f}_i^d - 2\boldsymbol{\omega}^{\mathcal{B}/\mathcal{N}} \times \frac{{}^{\mathcal{B}}d\boldsymbol{\rho}_i}{dt} - \frac{d\boldsymbol{\omega}^{\mathcal{B}/\mathcal{N}}}{dt} \times \boldsymbol{\rho}_i - \boldsymbol{\omega}^{\mathcal{B}/\mathcal{N}} \times (\boldsymbol{\omega}^{\mathcal{B}/\mathcal{N}} \times \boldsymbol{\rho}_i) \tag{3}$$

Among them, $\boldsymbol{\omega}^{\mathcal{B}/\mathcal{N}}$ is the rotational angular velocity of $\mathcal{B}$ respect to $\mathcal{N}$ .Euler dynamics equation of electromagnetic formation system is:

$$\boldsymbol{J}_i\dot{\boldsymbol{\omega}}^{\mathcal{B}_i/\mathcal{N}} + \tilde{\boldsymbol{\omega}}^{\mathcal{B}_i/\mathcal{N}} \times \boldsymbol{J}_i\boldsymbol{\omega}^{\mathcal{B}_i/\mathcal{N}} = \boldsymbol{\tau}_i^{EM} + \boldsymbol{\tau}_i^c + \boldsymbol{\tau}_i^d \tag{4}$$

Among them, $\boldsymbol{\omega}^{\mathcal{B}_i/\mathcal{N}}$ is the rotational angular velocity of $\mathcal{B}_i$ respect to $\mathcal{N}$, $\boldsymbol{\tau}_i^{EM}$ is the electromagnetic torque acting on the spacecraft-*i*; $\boldsymbol{\tau}_i^d$ is the perturbation torque. $\boldsymbol{\tau}_i^c$ is the control torque.

4. Relative equilibrium conditions

4.1 The condition of electromagnetic moment

In this section, The situation and attitude of each satellite in the formation system are obtained and the configuration of entire electromagnetic formation system is analyzed :

$$\boldsymbol{\omega}^{\mathcal{B}/\mathcal{H}} = \boldsymbol{0}, \quad \boldsymbol{\omega}^{\mathcal{B}_i/\mathcal{B}} = \boldsymbol{0} \tag{5}$$

$$\boldsymbol{\omega}^{\mathcal{B}/\mathcal{H}} = \boldsymbol{u}, \quad \boldsymbol{\omega}^{\mathcal{B}_i/\mathcal{B}} = \boldsymbol{0} \tag{6}$$

Formula (5) and (6) describe the static non-spin and spin formation, respectively.

The equilibrium constraint condition is obtained by formula (6). The relative equilibrium satisfies the following conditions.:

$$^{\mathcal{H}}\dot{\boldsymbol{q}}_0 = \boldsymbol{u},\ ^{\mathcal{H}}\ddot{\boldsymbol{q}}_0 = \boldsymbol{0},\ ^{\mathcal{B}}\dot{\boldsymbol{q}}_i = \boldsymbol{0}\ ,^{\mathcal{B}}\ \ddot{\boldsymbol{q}}_i = \boldsymbol{0} \tag{7}$$

Formula (7) Description: The relative first order and second order derivative of the generalized coordinates $\boldsymbol{q}_0$ in the reference frame $\mathcal{H}$ is a constant vector; the relative first order and second order derivative of the generalized coordinates $\boldsymbol{q}_i$ in the reference frame $\mathcal{B}$ is a zero.

For the spacecraft formation with inter-satellite non-contacting force, the forces applied to a spacecraft generally include the gravitational force $\boldsymbol{F}_i^g$ and the internal force $\boldsymbol{F}_i$, and the torque is internal torque $\boldsymbol{\tau}_i$ .The relative perturbation factor and the influence of the earth's magnetic field on the satellite system are neglected. A superscript mark "–" is used to denote the equilibrium state. Substituting constraint condition (7) into the Eq.(4), we obtain the attitude dynamic equations of member spacecraft-*i* under static formation relative equilibrium state:

$$\tilde{\overline{\boldsymbol{\omega}}}^{\mathcal{B}/\mathcal{N}}\ {}^{\mathcal{B}_i}\boldsymbol{J}_i\overline{\boldsymbol{\omega}}^{\mathcal{B}/\mathcal{N}} = \overline{\boldsymbol{\tau}}_i^{EM} \tag{8}$$

Substituting the angular velocity $\boldsymbol{\omega}^{\mathcal{B}/\mathcal{N}}$ into the Eq.(8), the torque under relative equilibrium state of static formation:

$$\overline{\boldsymbol{\tau}}^{EM} = {}^{\mathcal{B}}\begin{bmatrix}\overline{\tau}_x^{EM} & \overline{\tau}_y^{EM} & \overline{\tau}_z^{EM}\end{bmatrix}^{\mathrm{T}} = \boldsymbol{0} \tag{9}$$

From above, the mass distribution of the spacecraft is uniform, that is, the mass of the "rigid body" is distributed uniformly to the axis of the given coordinate axis. From Eq. (8), it is known that the internal torque applied to all spacecraft are zero in dynamics equilibrium.

4.2 A necessary condition of relative equilibrium

Static formation means that the formation appears frozen or stationary with respect to $\mathcal{H}$-frame following the motion of formation centre of mass. At this point, the internal force and torque are used to cancel all relative motion of spacecraft, so the spacecraft perform non-Keplerian orbits and maintain a constant relative trajectory and attitude. Hence, the formation would keep its geometry unvaried and behave equivalently to a single rigid body in orbit.

Substituting the equilibrium constraint condition Eq.(7) into the nonlinear relative motion kinetic equation Eq.(3), vector equation of static formation is:

$$-\overline{\boldsymbol{\omega}}^{\mathcal{B}/\mathcal{N}} \times (\overline{\boldsymbol{\omega}}^{\mathcal{B}/\mathcal{N}} \times \overline{\boldsymbol{\rho}}_i) + \frac{\mu}{r_{CM}^3}\left[\boldsymbol{r}_{CM} - \left(\frac{r_{CM}}{\overline{r}_i}\right)^3 \overline{\boldsymbol{r}}_i\right] + \frac{\overline{\boldsymbol{F}}_i^{EM}}{m_i} = \boldsymbol{0} \tag{10}$$

Multiply m_i both sides of Eq.(10) and make a weighted sum:

$$\sum_{i=1}^{N} -m_i\overline{\boldsymbol{\omega}}^{\mathcal{B}/\mathcal{N}} \times (\overline{\boldsymbol{\omega}}^{\mathcal{B}/\mathcal{N}} \times \overline{\boldsymbol{\rho}}_i) + \sum_{i=1}^{N} m_i\frac{\mu}{r_{CM}^3}\left[\boldsymbol{r}_{CM} - \left(\frac{r_{CM}}{\overline{r}_i}\right)^3 \overline{\boldsymbol{r}}_i\right] + \sum_{i=1}^{N}\overline{\boldsymbol{F}}_i^{EM} = \boldsymbol{0} \tag{11}$$

Substituting simplifying conditions into Eq. (11), we obtain::

$$\sum_{i=1}^{N} m_i\overline{\boldsymbol{\omega}}^{\mathcal{B}/\mathcal{N}} \times (\overline{\boldsymbol{\omega}}^{\mathcal{B}/\mathcal{N}} \times \overline{\boldsymbol{\rho}}_i) = \boldsymbol{0} \tag{12}$$

Expanding Eq.(12) with components formulation as:

$$\sum_{i=1}^{N} m_i \begin{bmatrix} \bar{\omega}_y^{\mathcal{B}/\mathcal{N}2} + \bar{\omega}_z^{\mathcal{B}/\mathcal{N}2} & -\bar{\omega}_x^{\mathcal{B}/\mathcal{N}} \bar{\omega}_y^{\mathcal{B}/\mathcal{N}} & -\bar{\omega}_x^{\mathcal{B}/\mathcal{N}} \bar{\omega}_z^{\mathcal{B}/\mathcal{N}} \\ -\bar{\omega}_x^{\mathcal{B}/\mathcal{N}} \bar{\omega}_y^{\mathcal{B}/\mathcal{N}} & \bar{\omega}_x^{\mathcal{B}/\mathcal{N}2} + \bar{\omega}_z^{\mathcal{B}/\mathcal{N}2} & -\bar{\omega}_y^{\mathcal{B}/\mathcal{N}} \bar{\omega}_z^{\mathcal{B}/\mathcal{N}} \\ -\bar{\omega}_x^{\mathcal{B}/\mathcal{N}} \bar{\omega}_z^{\mathcal{B}/\mathcal{N}} & -\bar{\omega}_y^{\mathcal{B}/\mathcal{N}} \bar{\omega}_z^{\mathcal{B}/\mathcal{N}} & \bar{\omega}_x^{\mathcal{B}/\mathcal{N}2} + \bar{\omega}_y^{\mathcal{B}/\mathcal{N}2} \end{bmatrix} \begin{bmatrix} \bar{q}_{i1} \\ \bar{q}_{i2} \\ \bar{q}_{i3} \end{bmatrix} = \boldsymbol{0} \tag{13}$$

To make Eq.(13) rationally tenable, we may have feasible cases as follows and get the results listed in Table 1. ($\bar{u}_{01}=\bar{\omega}_x^{\mathcal{B}/\mathcal{N}}, \bar{u}_{02}=\bar{\omega}_y^{\mathcal{B}/\mathcal{N}}, \bar{u}_{03}=\bar{\omega}_z^{\mathcal{B}/\mathcal{N}}$)

Table 1 Equilibrium conditions

	Conditions	Feasible solutions for $\bar{\varphi}$, $\bar{\theta}$, $\bar{\psi}$
1	$\bar{u}_{01} = \bar{u}_{02} = \bar{u}_{03} = 0$	No solution
2	$\bar{u}_{01} = \bar{u}_{02} = 0,\ \bar{u}_{03} = \omega_l$	$\bar{\varphi} = 0,\ \bar{\theta} = 0、\pi/2, \bar{\psi} = 0$ or $\bar{\varphi} = \pi/2,\ \bar{\theta} = \pi/2,\ \bar{\psi} = \pi/2$
3	$\bar{u}_{01} = \bar{u}_{03} = 0,\ \bar{u}_{02} = \omega_l$	$\bar{\varphi} = 0,\ \bar{\theta} = 0、\pi/2, \bar{\psi} = \pi/2$ or $\bar{\varphi} = \pi/2,\ \bar{\theta} = \pi/2,\ \bar{\psi} = 0$
4	$\bar{u}_{02} = \bar{u}_{03} = 0,\ \bar{u}_{01} = \omega_l$	$\bar{\varphi} = \pi/2,\ \bar{\theta} = 0, \bar{\psi} = 0、\pi/2$
5	$\bar{u}_{01} = 0,\ \bar{u}_{02} = \bar{u}_{03} = \omega_l / \sqrt{2}$	$\bar{\varphi} = 0,\ \bar{\theta} = 0、\pi/2, \bar{\psi} = \pi/4$
6	$\bar{u}_{02} = 0,\ \bar{u}_{01} = \bar{u}_{03} = \omega_l / \sqrt{2}$	$\bar{\varphi} = \pi/2,\ \bar{\theta} = \pi/4, \bar{\psi} = \pi/2$ or $\bar{\varphi} = \pi/4,\ \bar{\theta} = 0, \bar{\psi} = 0$
7	$\bar{u}_{03} = 0,\ \bar{u}_{01} = \bar{u}_{02} = \omega_l / \sqrt{2}$	$\bar{\varphi} = \pi/2,\ \bar{\theta} = \pi/4, \bar{\psi} = 0$ or $\bar{\varphi} = \pi/4,\ \bar{\theta} = 0, \bar{\psi} = \pi/2$

Since φ, θ, ψ constituting 2-3-1 Euler angle sequence between the frame $\mathcal{H}$ and $\mathcal{B}$, and there are at least two angles being 0 or $\pi/2$ according to the results, which means that at least two axes of the frame $\mathcal{B}$ are parallel with axes of the frame $\mathcal{H}$.

The supplement equilibrium conditions is derived from the momentum perspective, from the Eq.(10) , we can get:

$$\sum_{i=1}^{N} \bar{\boldsymbol{\rho}}_i \times m_i \frac{\mu}{r_{CM}^3} \left[\boldsymbol{r}_{CM} - \left(\frac{r_{CM}}{\bar{r}_i} \right)^3 \bar{\boldsymbol{r}}_i \right] + \sum_{i=1}^{N} \bar{\boldsymbol{\rho}}_i \times \bar{\boldsymbol{F}}_i^{EM} - \sum_{i=1}^{N} \bar{\boldsymbol{\rho}}_i \times m_i \bar{\boldsymbol{\omega}}^{\mathcal{B}/\mathcal{N}} \times (\bar{\boldsymbol{\omega}}^{\mathcal{B}/\mathcal{N}} \times \bar{\boldsymbol{\rho}}_i) = \boldsymbol{0} \tag{14}$$

Substituting the condition into Eq.(14), we obtain:

$$\sum_{i=1}^{N} m_i \bar{\boldsymbol{\rho}}_i \times \bar{\boldsymbol{\omega}}^{\mathcal{B}/\mathcal{N}} \times (\bar{\boldsymbol{\omega}}^{\mathcal{B}/\mathcal{N}} \times \bar{\boldsymbol{\rho}}_i) = \boldsymbol{0} \tag{15}$$

Expanding Eq.(15) with components formulation as:

$$\sum_{i=1}^{N} m_i \begin{bmatrix} \left(\bar{q}_{i2}\bar{\omega}_z^{\mathcal{B}/\mathcal{N}} - \bar{q}_{i3}\bar{\omega}_y^{\mathcal{B}/\mathcal{N}}\right)\left(\bar{q}_{i1}\bar{\omega}_x^{\mathcal{B}/\mathcal{N}} + \bar{q}_{i2}\bar{\omega}_y^{\mathcal{B}/\mathcal{N}} + \bar{q}_{i3}\bar{\omega}_z^{\mathcal{B}/\mathcal{N}}\right) \\ \left(\bar{q}_{i3}\bar{\omega}_x^{\mathcal{B}/\mathcal{N}} - \bar{q}_{i1}\bar{\omega}_z^{\mathcal{B}/\mathcal{N}}\right)\left(\bar{q}_{i1}\bar{\omega}_x^{\mathcal{B}/\mathcal{N}} + \bar{q}_{i2}\bar{\omega}_y^{\mathcal{B}/\mathcal{N}} + \bar{q}_{i3}\bar{\omega}_z^{\mathcal{B}/\mathcal{N}}\right) \\ \left(\bar{q}_{i1}\bar{\omega}_y^{\mathcal{B}/\mathcal{N}} - \bar{q}_{i2}\bar{\omega}_x^{\mathcal{B}/\mathcal{N}}\right)\left(\bar{q}_{i1}\bar{\omega}_x^{\mathcal{B}/\mathcal{N}} + \bar{q}_{i2}\bar{\omega}_y^{\mathcal{B}/\mathcal{N}} + \bar{q}_{i3}\bar{\omega}_z^{\mathcal{B}/\mathcal{N}}\right) \end{bmatrix} = \boldsymbol{0} \tag{16}$$

This conclusion is in accordance with the conclusion with respect to Kane electromagnetic formation dynamics model in Ref. [11], and verify the conclusion of vector mechanics equilibrium state conditions.

5. Hamilton dynamics model

In this section, we study a formation case which contains two simplified spacecraft operating by inter-satellite electromagnetic force and torque, and the formation center of mass travels on a circular orbit. As shown in Figure 3, the related reference frame and generalized coordinate parameter can be obtained from the figure.

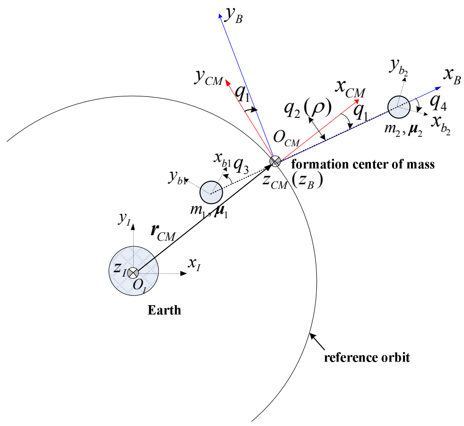

Fig.3 The simplified model of two-vehicle electromagnetic formation

Lagrangian function of the electromagnetic formation system:

$$\begin{aligned} L = & \frac{(m_1 m_2 \dot{q}_1^2 q_2^2 + 2m_1 m_2 \omega_0 \dot{q}_1 q_2^2 + m_1 m_2 \dot{q}_2^2 + m_1^2 \omega_0^2 r_{CM}^2 + m_1 m_2 \omega_0^2 q_2^2 + 2m_1 m_2 \omega_0^2 r_{CM}^2 + m_2^2 \omega_0^2 r_{CM}^2)}{2(m_1+m_2)} \\ & + \frac{J_1(\dot{q}_1+\dot{q}_3+\omega_0)^2}{2} + \frac{J_2(\dot{q}_1+\dot{q}_4+\omega_0)^2}{2} - \frac{\mu_0 \mu_1 \mu_2 \left[\cos(q_3-q_4)+3\cos(q_3+q_4)\right]}{4\pi q_2^3} - \frac{\mu m_2}{r_1} - \frac{\mu m_1}{r_2} \end{aligned} \tag{17}$$

Among them, the r_1、r_2:

$$r_1 = \sqrt{r_{CM}^2 - 2r_{CM}\cos q_1 \frac{m_2}{M} q_2 + \left(\frac{m_2}{M} q_2\right)^2}, \quad r_2 = \sqrt{r_{CM}^2 + 2r_{CM}\cos q_1 \frac{m_1}{M} q_2 + \left(\frac{m_1}{M} q_2\right)^2}$$

Based on the theory of Hamilton mechanics, the generalized momentum of the formation system is computed. The Hamilton dynamic model for the two-vehicle electromagnetic formation can be derived:

$$\begin{aligned} \dot{p}_1 &= \frac{\mu_e m_2 \left(2r_{CM}\, signf_3 \sin q_1 - f_1\right)}{2\left(f_3^2+f_4\right)^{3/2}} + \frac{\mu_e m_1 \left(2r_{CM}\left|f_2\right| signf_2 \sin q_1 - f_1\right)}{2\left(f_2^2+f_4\right)^{3/2}} \\ \dot{p}_2 &= \frac{2m_1 m_2 q_2 \dot{q}_1^2 + 4m_1 m_2 q_2 \dot{q}_1 \omega_0 + 2m_1 m_2 q_2 \omega_0^2}{2\left(m_1+m_2\right)} + \frac{3\mu_0 \mu_1 \mu_2 \left[\cos(q_3-q_4)+3\cos(q_3+q_4)\right]}{4\pi q_2^4} \\ & \quad - \frac{\mu_e m_1 m_2 \left|f_3\right| signf_3}{\left(m_1+m_2\right)\left(f_3^2+f_4\right)^{3/2}} + \frac{\mu_e m_1 m_2 \left|f_2\right| signf_2}{\left(m_1+m_2\right)\left(f_2^2+f_4\right)^{3/2}} \\ \dot{p}_3 &= -\frac{\mu_0 \mu_1 \mu_2 \left[sin(q_3-q_4)+3sin(q_3+q_4)\right]}{4\pi q_2^3} \\ \dot{p}_4 &= \frac{\mu_0 \mu_1 \mu_2 \left[sin(q_3-q_4)-3sin(q_3+q_4)\right]}{4\pi q_2^3} \\ \dot{q}_1 &= -\frac{\left[m_1 m_2 \omega_0 q_2^2 + (m_1+m_2)(J_1+J_2)\omega_0 - (m_1+m_2)p_1\right]}{m_1 m_2 q_2^2} - \frac{(m_1+m_2)(p_3-J_1\omega_0)}{m_1 m_2 q_2^2} - \frac{(m_1+m_2)(p_4-J_2\omega_0)}{m_1 m_2 q_2^2} \\ \dot{q}_2 &= \frac{(m_1+m_2)p_2}{m_1 m_2} \\ \dot{q}_3 &= \frac{\left[m_1 m_2 \omega_0 q_2^2 + (m_1+m_2)(J_1+J_2)\omega_0 - (m_1+m_2)p_1\right]}{m_1 m_2 q_2^2} + \frac{(m_1+m_2)(p_4-J_2\omega_0)}{m_1 m_2 q_2^2} + \frac{(p_3-J_1\omega_0)(J_1 m_1 + J_1 m_2 + m_1 m_2 q_2^2)}{m_1 m_2 J_1 q_2^2} \\ \dot{q}_4 &= \frac{\left[m_1 m_2 \omega_0 q_2^2 + (m_1+m_2)(J_1+J_2)\omega_0 - (m_1+m_2)p_1\right]}{m_1 m_2 q_2^2} + \frac{(m_1+m_2)(p_3-J_1\omega_0)}{m_1 m_2 q_2^2} + \frac{(p_4-J_2\omega_0)(J_2 m_1 + J_2 m_2 + m_1 m_2 q_2^2)}{m_1 m_2 J_2 q_2^2} \end{aligned} \tag{18}$$

Among them, the function f_1、f_2、f_3、f_4 is:

$$\begin{cases} f_1 = 2r_{CM}{}^2\left|\sin q_1\right| sign(\sin q_1)\cos q_1 \quad , \quad f_2 = r_{CM}\cos q_1 - \dfrac{m_2 q_2}{m_1 + m_2} \\ f_3 = r_{CM}\cos q_1 + \dfrac{m_1 q_2}{m_1 + m_2} \quad , \quad f_4 = r_{CM}{}^2\left|\sin q_1\right|^2 \end{cases}$$

6. Calculate relative equilibrium

When the system is equilibrium at a certain position $\boldsymbol{X}(q_1, q_2, q_3, q_4)$, the generalized coordinate's partial derivative of the system's total potential energy $\boldsymbol{U}$ is zero.

$$\left.\frac{\partial U}{\partial q_{ie}}\right|_e = 0, (i=1,2,3,4) \tag{19}$$

q_{ie} is the corresponding generalized coordinates value of formation system equilibrium position.

Solving equations(19), the equilibrium value of the electromagnetic formation system is obtained. The numerical result is shown in Table.2 and the corresponding deployment of equilibrium state are shown in Fig.4.

Table 2 The solution of Equilibrium conditions

	q_1	q_2	q_3	q_4
Case (a)	0	$\bar{q}_2$	0	0
	the deployment of orbit radial: all spacecraft are aligned with the axis of Ox; **magnetic moments of dipoles:** $\boldsymbol{\mu}_i // \boldsymbol{\mu}_j$, $\boldsymbol{\mu}_i = \pm{}^B[\mu_i \ \ 0 \ \ 0]^{\mathrm{T}}$, $\boldsymbol{\mu}_j = \pm{}^B[\mu_j \ \ 0 \ \ 0]^{\mathrm{T}}$			
Case (b)	$\pm\pi/2$	$\bar{q}_2$	$\pm\pi/2$	$\pm\pi/2$
	the deployment of along-track: all spacecraft are aligned with the axis of Oy; **magnetic moments of dipoles:** $\boldsymbol{\mu}_i // \boldsymbol{\mu}_j$, $\boldsymbol{\mu}_i = \pm{}^B[0 \ \ \mu_i \ \ 0]^{\mathrm{T}}$, $\boldsymbol{\mu}_j = \pm{}^B[0 \ \ \mu_j \ \ 0]^{\mathrm{T}}$			

And the corresponding deployments of equilibrium state are shown in Fig.4.

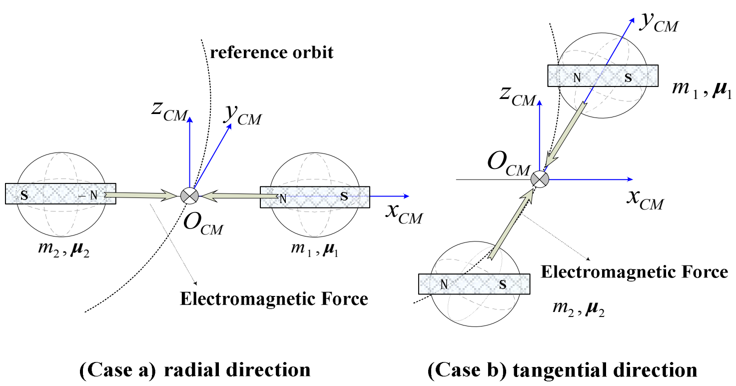

Fig.4 The deployment of two-vehicle electromagnetic formation

From the above results, we find the two equilibrium states of two-vehicle simplified electromagnetic formation system. The radial distribution（case a）and tangential distribution（case b） of the formation system are obtained. The magnetic dipole configuration of the satellite formation system is obtained.

7. Stability analysis

Stability theory: If the potential energy function of the equilibrium position is strict local maximum, then the equilibrium position is stable. To determine balance stability of complete conservative systems, we need to find extreme value of function ***U***, given a sufficient condition for the function extreme value.

For the multi-degree freedom of the system，The specific mark is introduced：

$$U^0_{q_1^{p_1} q_2^{p_2} \dots q_n^{p_n}} = \left(\frac{\partial^k U}{\partial q_1^{p_1} q_2^{p_2} \dots q_n^{p_n}} \right)_0 , k = \sum_{i=1}^{n} p_i \tag{20}$$

Calculation determinant D_i：

$$D_i = \begin{vmatrix} U^0_{q_1^2} & U^0_{q_1 q_2} & \dots & U^0_{q_1 q_i} \\ U^0_{q_2 q_1} & U^0_{q_2^2} & \dots & U^0_{q_2 q_i} \\ \dots & \dots & \dots & \dots \\ U^0_{q_i q_1} & U^0_{q_i q_2} & \dots & U^0_{q_i^2} \end{vmatrix} \tag{21}$$

When $D_i < 0 (i = 1,3,5,\dots), D_i > 0 (i = 2,4,6,\dots)$, ***U*** has maximum value; when $D_i < 0$ $(i = 1,2,3,\dots,n)$, ***U*** is minimum value.

Based on the Stability theory, we can draw the conclusion: The determinant value of the case (a) does not satisfies the stability condition, so the equilibrium position is unstable. By contrast, the determinant value of the case (b) satisfies the stability condition, so the equilibrium position is stable.

In order to verify the validity of the relative equilibrium conditions and Hamilton dynamics model，the numerical simulation of this two-vehicle electromagnetic formation is achieved. In the simulation, spacecraft mass and moment of inertia are set as $m_1 = m_2 = 250\text{kg}$, $I_1 = I_2 = 20\text{kg}\cdot\text{m}^2$, $\mu_1 = \mu_2 = 24115\,\text{A}\cdot\text{m}^2$ the orbital altitude 500km, and the initial states are chosen as $\boldsymbol{q}_0 = \begin{bmatrix} 2^\circ & 15 & 0 & 0 \end{bmatrix}^T$.

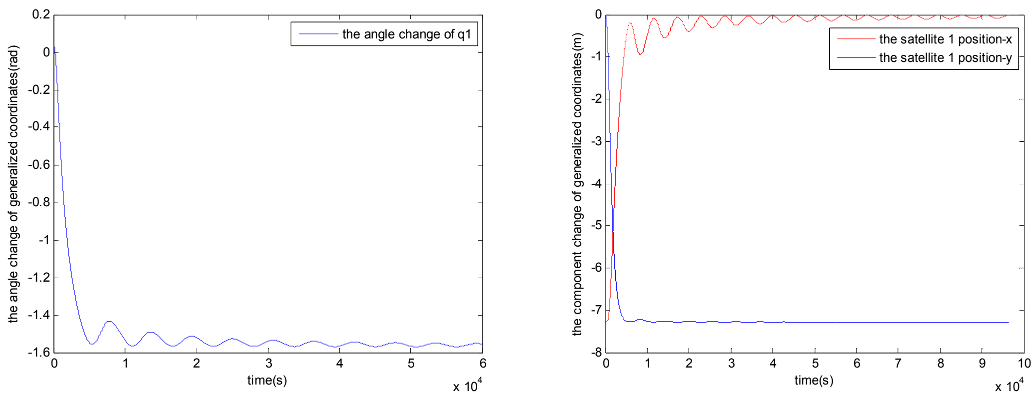

Fig.5 The change curve of the generalized coordinate

The simulation results which based on the Hamilton dynamics model are shown in Fig. 5. From the change curve of the generalized coordinate，the correctness and the validity of the theory and operation is verified reasonably.

8. Conclusions

This paper mainly studies the Hamilton dynamics and relative equilibrium of multi-spacecraft formation with non-contacting internal force. First, the relative equilibrium for the vector mechanics model is analyzed and the necessary conditions for circularly restricted static electromagnetic formation are derived. Second, the 6-DOF coupled nonlinear dynamic models by Hamiltonian method for the multi-spacecraft electromagnetic formation was obtained under earth imperturbation. Then, the equilibrium solution and the magnetic dipole of the given static formation were gained. Finally, Base on the equilibrium conditions, Hamilton dynamic models and position of equilibrium solution the characteristic of stability and controllability are investigated.

9. References

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