

# TITAN ATMOSPHERIC DENSITY RESULTS FROM CASSINI'S T107 FLYBY

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**Abstract:** *The Cassini mission's T107 flyby is the second encounter of Titan dedicated to obtaining Doppler tracking through close approach to estimate Titan's atmospheric density profile. A scale factor on a nominal atmosphere is used to estimate the drag experienced during a typical low altitude flyby of Titan but continuous tracking through close approach allows direct estimation of atmospheric density through the drag on the spacecraft. A short arc orbit determination solution is used to estimate corrections to a layered exponential atmospheric density profile. This work describes the navigation filter setup and the atmosphere model used in the estimation process. Telemetry information from the Attitude Articulation and Control Subsystem is used to model the spacecraft thrusting to counteract atmospheric drag. The altitude layering setup is a modification of that applied to the T87 flyby, which had a lower periapsis. The inbound and outbound density profiles are within  $1\sigma$  of each other at close approach. The density estimates are statistically significant since the postfit uncertainty in the base densities are an order of magnitude lower than the values themselves. Compared to the T87 density estimate at an altitude near T107 close approach, the density estimates from the two flybys are statistically consistent.*

**Keywords:** *Cassini, Titan, atmospheric density, navigation, orbit determination.*

## 1. Introduction

This work explores the variation of Titan's atmospheric density using orbit determination results from Cassini's T107 flyby of Titan in December 2014. The Cassini-Huygens mission launched in 1997 and the Cassini spacecraft has been in orbit about Saturn since 2004. Exploration of the Saturn system is driven by gravitational flybys of Titan which alter the spacecraft trajectory. Flybys of Titan are generally "in-the-blind" for navigation, meaning no tracking data is collected during close approach. The spacecraft does experience acceleration from Titan's atmosphere during low flybys of less than 1400 km altitude and a scale factor is estimated in the orbit determination process to adjust the nominal atmosphere model. The T107 flyby is the second opportunity of the mission after T87 to receive two-way Doppler tracking between NASA's Deep Space Network and the spacecraft through close approach of Titan. These flybys were chosen for this experiment due to the orientation of the High-Gain Antenna being near Earth-point. This data allows the Navigation team to estimate an updated density profile for Titan through the orbit determination process.

The methodology of orbit determination is used to determine Titan's atmospheric density, in which a navigation filter minimizes the differences between observed and computed Doppler observations in a least-squares sense. A specialized version of the standard orbit determination setup for Cassini is implemented in the Monte software developed by NASA's Jet Propulsion Laboratory (JPL). A data arc with Doppler tracks before and after the flyby at sixty second count-time and data through Titan close approach at one second count-time is used to adequately resolve the acceleration due to the atmosphere. Telemetry from the Reaction Control Subsystem (RCS) is incorporated during the flyby

to model the thruster firings that maintain the spacecraft attitude in the presence of atmospheric drag. The a priori density model is taken from the results of the first Titan density estimation experiment produced by the T87 flyby. The density model is broken up into layers such that the accumulated acceleration between layers is equal to at least ten times the noise in the Doppler residuals. Separate layer profiles are implemented for the inbound and outbound portions of the flyby allowing different density estimates on either side of close approach. The base densities at these layers are updated in the iterative least-squares process and the density between layers is computed from an exponential profile which enforces continuity at layer boundaries.

This paper details the filtering strategy applied to the tracking data and the exponential atmosphere profile used to model drag on the spacecraft. Plots of Doppler residuals and density variation with altitude are shown and discussed. Estimates of the layered base densities and their formal uncertainties are documented. This Navigation estimate of atmospheric density is used to provide a new best estimate of Titan's atmosphere for future orbit determination solutions.

## **2. Orbit Determination Process**

The Cassini Navigation team uses an epoch-state Kalman filter implemented in JPL's Monte software to estimate the spacecraft's trajectory in orbit about Saturn. Radio tracking data from the Deep Space Network is the observable that allows correction of the spacecraft state along with other quantities related to the forces acting on the spacecraft. This section details the data used, forces modeled, and parameters estimated in the orbit determination filter.

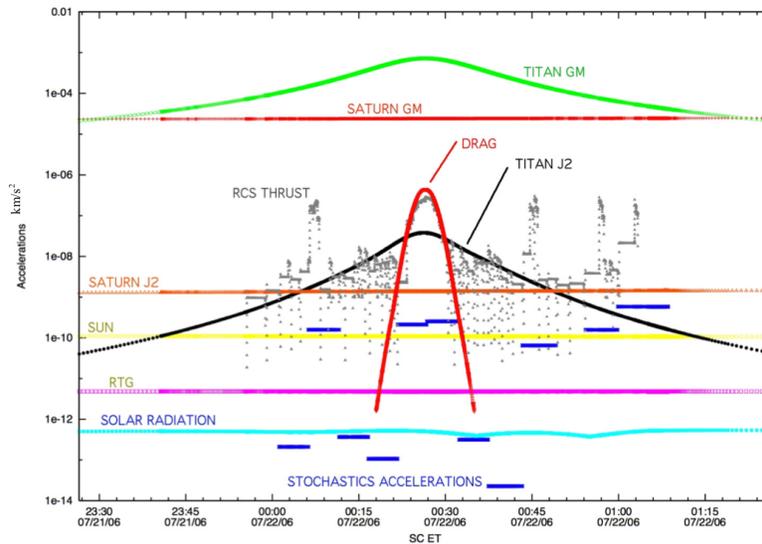
### **2.1. Data Arc**

A traditional data arc for Cassini orbit determination spans from one Titan encounter to the next. Since this study is primarily focused on obtaining the best estimate of atmospheric drag around the T107 flyby of Titan, a shorter data arc is used. The arc starts on 09-DEC-2014 02:00 ET and ends on 12-DEC-2014 06:00 ET, with the close approach of Titan taking place at 10-DEC-2014 22:27:42 ET. The time system used is spacecraft ephemeris time (ET), which is Universal Coordinated Time (UTC) kept at the spacecraft without leap seconds. The filter is anchored on either side of the flyby by a typical sixty second count-time X-band up/X-band down Doppler track from a 70 meter station. One second count time Doppler is taken through the flyby at both X/X and X-band up/Ka-band down. The spacecraft only supports Ka-band down tracking since the Ka-band transponder failed during cruise. The data is weighted at 1xRMS of the Doppler residuals for both X/X and X/Ka data. Tightening or loosening the data weights has negligible effect on the density estimates as the Doppler data through the flyby is very powerful. Range data is not used for this study, information concerning atmospheric drag is in the Doppler data.

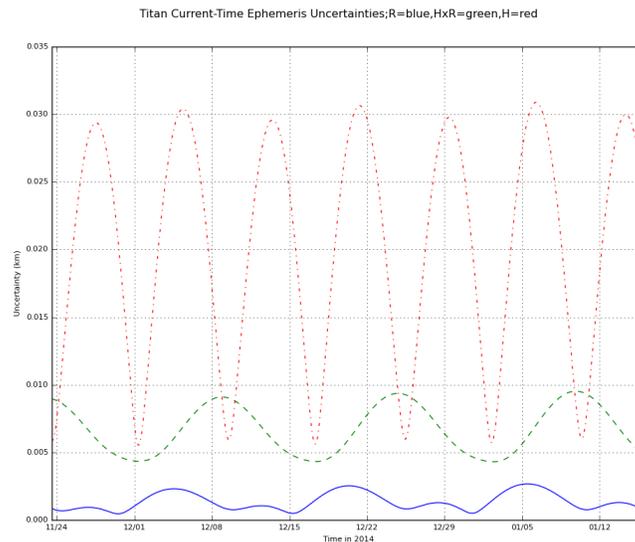
### **2.2. Force Modeling**

In order to separate the drag signature from the other dynamics acting on the spacecraft, it is important to use the most accurate force models possible. Figure 1 shows the accelerations acting on Cassini during a typical low altitude flyby of Titan. At close approach the gravitational acceleration due to Titan eclipses that due to Saturn. The SAT375 Saturn system ephemerides by Jacobson [1] is

used to model the positions and masses of Titan and the other satellites of Saturn. After over one hundred flybys of Titan, the formal uncertainty in Titan's position, shown in Fig. 2 peaks at only thirty meters. SAT375 includes the reconstruction of Titan's position from the T107 flyby, so the Titan ephemeris is not estimated in this study. The acceleration due to Titan oblateness is on par with the acceleration due to drag on either side of close approach. Corrections to the Titan spherical harmonic gravity field are estimated to prevent that signature from aliasing into the atmospheric density estimate.



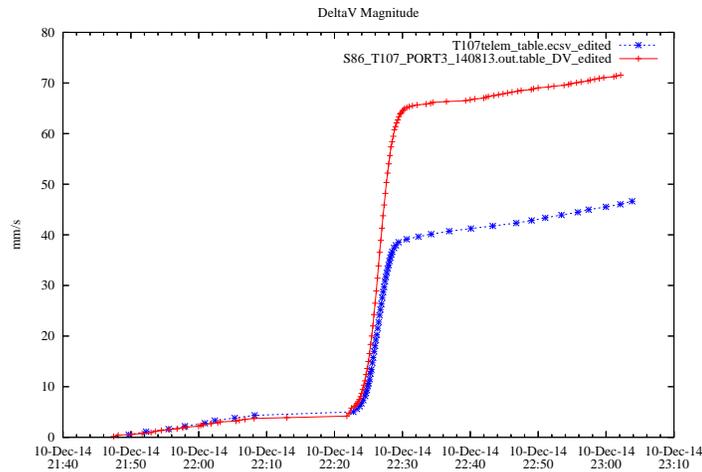
**Figure 1. Accelerations acting on Cassini during typical Titan flyby**



**Figure 2. Uncertainties in Titan ephemeris**

RCS thrusting is used to maintain spacecraft attitude during close approach as well as to provide slewing for other observations. Prior to entering Titan's atmosphere, Cassini's reaction wheels are spun down and then spun back up once out of the atmosphere. These events are modeled as

impulsive burns in the filter. The profile of thrusting during close approach is modeled as a series of accelerations from telemetry data [2] provided by the Attitude Articulation and Control Subsystem (AACS). The predicted and telemetry profiles of RCS thrusting are shown in Fig. 3. The prediction shown in red is nearly double the telemetered profile shown in blue. This shows that the nominal atmosphere model used densities that are too large. The RCS thrusting is modeled as accelerations batched such that the accumulated  $\Delta V$  in each batch is 0.07 mm/s.



**Figure 3. RCS thrusting during T107 encounter**

### 2.3. Filter Setup

Including too many parameters in the filter will dilute the data strength and may corrupt the desired estimate. A simplified version of the Cassini Navigation team’s orbit determination filter configuration is described in Table 1. Corrections to the spacecraft epoch state are estimated using a mapped covariance from the T107 operations arc as an a priori constraint. Spin down and spin up of the reaction wheels are modeled using telemetry and estimated with the majority of the error assigned to the Earth-pointed Z-direction. Four base layers of atmospheric density are estimated in both an inbound and outbound density profile. More discussion of these layers is given in the next section. Corrections to the SAT375 spherical harmonic gravity field of Titan up to order 3 and degree 3 are estimated in the filter using a correlated covariance.

The  $1\sigma$  a priori uncertainty along with whether the quantity is considered or estimated in the filter is given in Table 1. Considered parameters are not estimated but their uncertainty is included in the estimation error covariance to give more realistic uncertainties [3]. Considered parameters include errors in the station locations of the Deep Space Network, a bias in the time system, drift of Earth’s pole, Earth atmospheric delays to the Doppler tracking, and a drag scale factor. The drag scale factor is representative of errors in the spacecraft mass, cross-sectional area, and drag coefficient.

**Table 1. Filter parameter setup**

Parameter	Unit	Estimated/Considered	a priori $\sigma$
Epoch state S/C position - X/Y/Z	km	Estimated	2.35/0.28/3.7
Epoch state S/C velocity - X/Y/Z	mm/s	Estimated	23.9/1.87/8.8
Impulsive spin down burn - X/Y/Z	mm/s	Estimated	0.012/0.12/1.2
Impulsive spin up burn - X/Y/Z	mm/s	Estimated	0.012/0.12/1.2
Inbound Density Layer [0]	kg/km <sup>3</sup>	Estimated	4.73E-10
Outbound Density Layer [0]	kg/km <sup>3</sup>	Estimated	4.73E-10
Inbound Density Layer [1]	kg/km <sup>3</sup>	Estimated	2.79E-10
Outbound Density Layer [1]	kg/km <sup>3</sup>	Estimated	2.79E-10
Inbound Density Layer [2]	kg/km <sup>3</sup>	Estimated	1.48E-10
Outbound Density Layer [2]	kg/km <sup>3</sup>	Estimated	1.48E-10
Inbound Density Layer [3]	kg/km <sup>3</sup>	Estimated	7.17E-11
Outbound Density Layer [3]	kg/km <sup>3</sup>	Estimated	7.17E-11
Titan gravity 3x3 spherical harmonics	unitless	Estimated	SAT375 full covariance
Earth pole motion - X/Y	deg	Considered	8.594E-07
UT1 bias	sec	Considered	2.5E-04
DSN station locations	km/deg	Considered	2003 covariance [4]
Troposphere path delay - wet/dry	km	Considered	1.0E-05/1.0E-05
Ionosphere path delay - day/night	km	Considered	5.5E-04/1.5E-04
Drag scale factor	unitless	Considered	0.105

### 3. Atmospheric Density Model

The acceleration due to atmospheric drag is given by Equation 1:

$$\mathbf{a}_D = -\frac{\rho C_d A V^2}{2m} \hat{\mathbf{V}} \quad (1)$$

where  $\rho$  is the atmospheric density at the current spacecraft location,  $C_d$  is the coefficient of drag for the spacecraft,  $A$  is the cross-sectional area of the spacecraft perpendicular to the velocity vector,  $m$  is the spacecraft mass, and  $\hat{\mathbf{V}}$  is the unit velocity vector. The drag coefficient has a value of 2.1 for Cassini and the most up to date spacecraft mass based on propellant usage is used. Cassini's High Gain Antenna (HGA) is aligned with the spacecraft Z-axis and is pointed at Earth to enable radio tracking during the flyby. The area seen by the atmosphere is approximately constant through the flyby and is set to the AACS value of 21.5 m<sup>2</sup>.

This study implements a density-driven atmospheric drag model in JPL's Monte software. An inbound and outbound profile with multiple base density layers is used, allowing separate density estimates before and after close approach. The density at a given point  $\rho$  is given by Equation 2 where the subscript "i" stands for the characteristics of the base of the layer in altitude  $h$  and scale height  $H$ . The scale height in Equation 3 is computed such that continuity is enforced at the layer boundaries. The only point in the atmospheric density profile where a discontinuity is allowed by

the software is close approach, i.e. the lowest altitude layer.

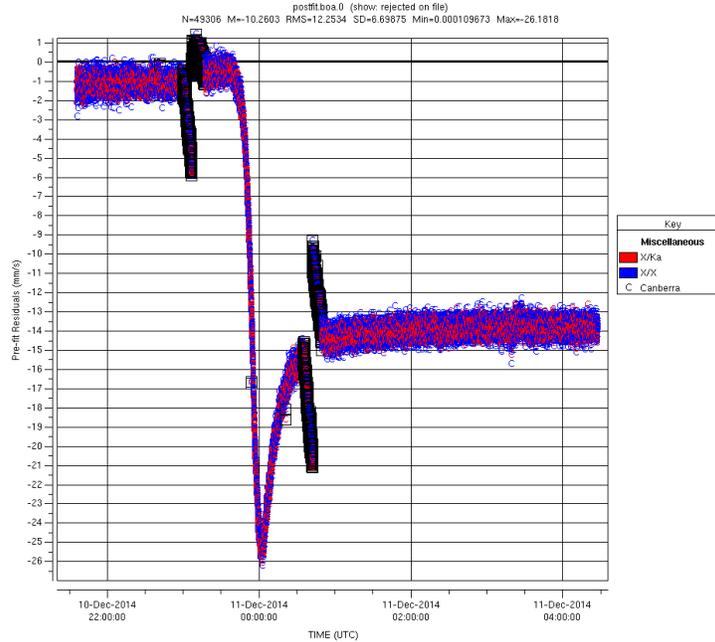
$$\rho = \rho_i \exp\left(\frac{h_i - h}{H_i}\right) \quad (2)$$

$$H_i = \frac{h_i - h_{i+1}}{\log(\rho_{i+1}/\rho_i)} \quad (3)$$

The approach for the T87 atmospheric drag study of Titan placed a density layer at Titan periapsis and then integrated the acceleration due to drag until the change in velocity was equal to ten times the noise in the Doppler residuals. This results in six base density layers for the inbound and outbound profiles at T87. At altitudes above 1150 kilometers [5] the acceleration due to Titan oblateness becomes stronger than that due to atmospheric drag so no additional layers are estimated. Attempting this approach for T107 yielded some difficulty in separating the drag into distinct layers. The filter placed a large discontinuity at close approach, sometimes doubling the density in the lowest base layer and the postfit uncertainty in the layers remained high at 30-40%. Four base density layers gave more physically realistic results and allowed the filter to estimate the base densities more precisely. The choice of a priori base densities is an iterative process. Initially, density values were interpolated from the T87 density profile and 100% uncertainty assigned to each layer. After running several estimation cases, the a priori values equal to the uncertainties given in Table 1 were chosen. The following section details those results of the estimation process.

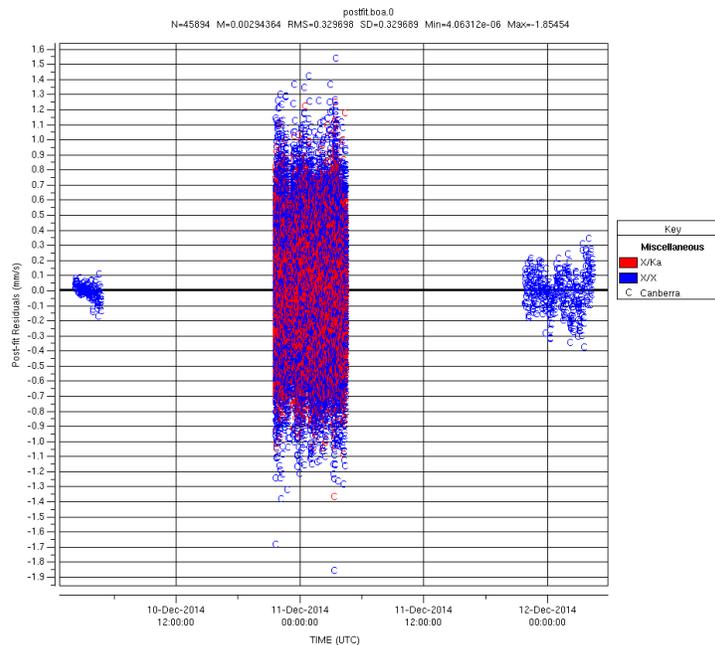
#### 4. Estimation Results

In each filter run, the equations of motion are integrated producing a spacecraft trajectory, the Doppler data is processed and the difference between observed and computed frequencies is calculated and plotted as residual error. Figure 4 shows the prefit Doppler residuals for the T107 data arc, where the blue points represent X/X Doppler and red points represent the X/Ka Doppler tracking, which has a higher down transmit frequency and is thus more sensitive to mismodeled spacecraft motion. The data at close approach has a one second compression time at a 34-meter DSN station and the two tracks bracketing close approach have the standard sixty second compression time at a 70-meter DSN station. The data points boxed in black occur during the reaction wheel spin down and spin up events which are modeled impulsively. These points are not processed in the filter but the data on either side of the spins allows estimation of the resulting  $\Delta V$  on the spacecraft. The dip in Doppler residuals between spin events is due to the difference in the a priori atmospheric drag model and the drag acceleration experienced by the spacecraft. The converged filter solution where the atmospheric drag profile is fit to the Doppler data is shown in the postfit residuals of Fig. 5.



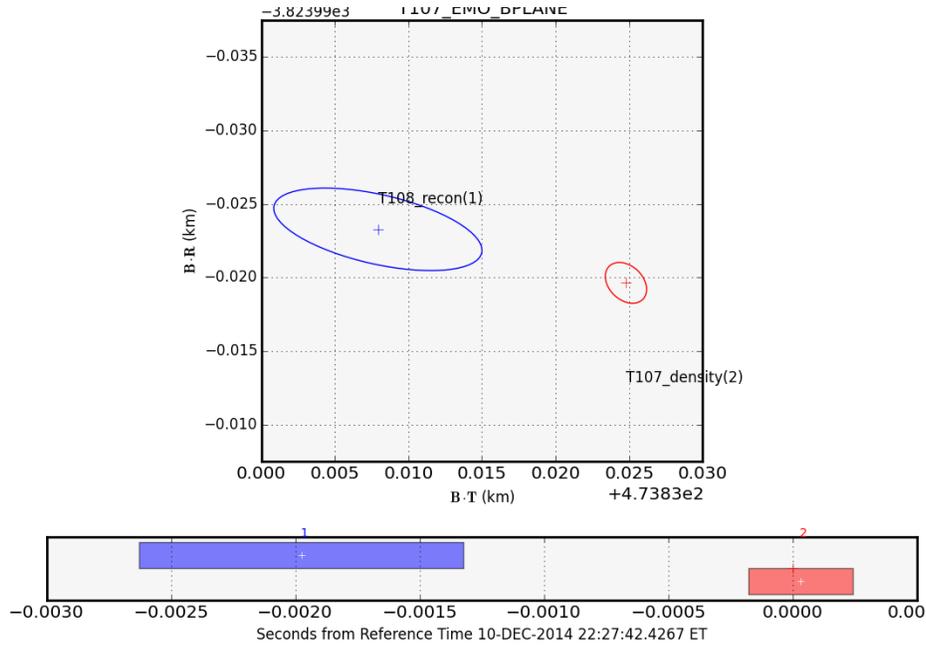
**Figure 4. Prefit Doppler residuals at close approach (mm/s), 1-sec compression**

The two tracks with sixty second compression on either side of close approach are shown along with the drag track. The Ka-down track in red is slightly more accurate and has less noise than the X/X track at close approach due to the higher frequency. There is a factor of  $\sqrt{60}$  difference in the noise between the flyby track and the bracketing tracks due to the different Doppler compression count times. The spacecraft state, base densities and Titan spherical harmonics are corrected based on the tracking so no signature is present in the residuals.



**Figure 5. Postfit Doppler residuals (mm/s), 60-sec compression for tracks bracketing C/A**

The B-plane is a method of showing encounter geometry and timing and comparing between different navigation solutions. Figure 6 shows the B-plane solutions for the T107 Titan encounter from the full operations arc trajectory reconstruction and the T107 density study. The ellipses represent the  $1\sigma$  spacecraft state covariance mapped into B-plane coordinates at the time of close approach. The difference between the full arc and the density arc is on the order of 20 meters. This shows that trajectory differences between the full and short arc solutions are negligible near close approach where drag effects are greatest.



**Figure 6. T107 B-plane for full and short arc reconstructions**

The iterated base density estimates and  $1\sigma$  postfit uncertainties are shown in Tab. 2 for the inbound profile and in Tab. 3 for the outbound profile. The scale heights are computed from the base densities, excepting that for the highest layer which is a filter input set to 80 kilometers. The densities at the close approach altitude of 980.1 kilometers are within  $1\sigma$  of each other and only differ because of the model definition in the software. The 100% a priori uncertainties on the base densities estimate down to an order of magnitude below the estimated values, making these results statistically significant. Changing scale heights or adding additional layers are not necessary, since the computed scale height values are larger than the difference in altitude between the layers. Corrections to the Titan spherical harmonics are very small, for example the corrections to Titan  $J_2$  and  $J_3$  are an order of magnitude smaller than the values themselves.

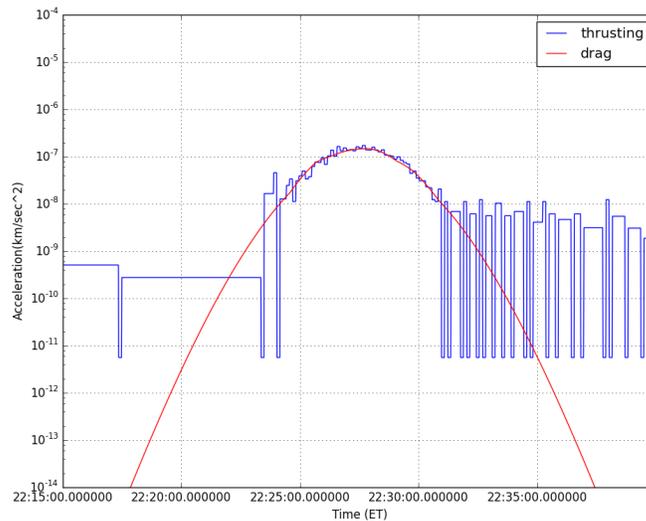
**Table 2. Inbound base density layer estimation results**

Base altitude (km)	Base density (kg/km <sup>3</sup> )	$1\sigma$ Uncertainty (kg/km <sup>3</sup> )	Scale height (km)
980.1	4.28E-10	2.58E-11	199.5
1008.4	3.09E-10	2.79E-11	254.5
1045.2	2.21E-10	2.60E-11	136.5
1133.1	5.03E-11	9.74E-12	80.0

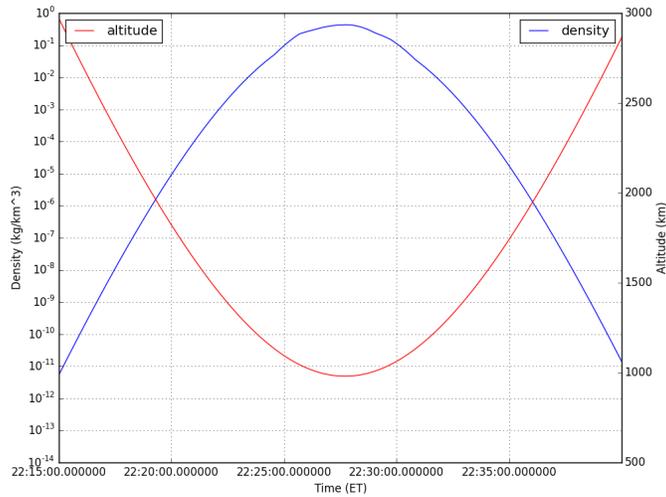
**Table 3. Outbound base density layer estimation results**

Base altitude (km)	Base density (kg/km <sup>3</sup> )	1 $\sigma$ Uncertainty (kg/km <sup>3</sup> )	Scale height (km)
980.1	4.37E-10	2.73E-11	108.4
1008.4	2.40E-10	2.62E-11	181.5
1045.2	1.50E-10	2.40E-11	138.3
1133.1	3.48E-11	9.06E-12	80.0

The accelerations due to RCS thrusting and due to the estimated atmospheric drag profile are shown in Fig. 7. The thruster firing closely follows the drag signature until the drag acceleration drops an order of magnitude from its close approach peak of  $1.5\text{E-}07 \text{ km/s}^3$ . Additional thrusting after the encounter is used for spacecraft turns and other observations. The floor value of thrusting shown in the plot is due to acceleration from the re-radiated heat of Cassini's radioisotope thermoelectric generators.

**Figure 7. Drag and thrusting accelerations during T107**

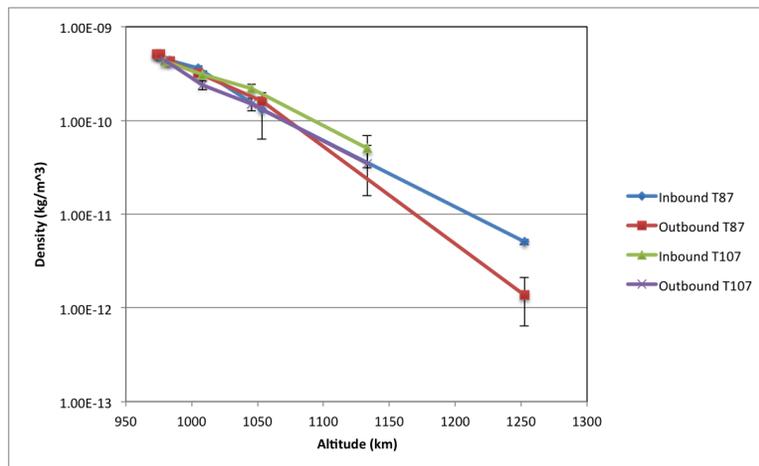
The final estimated atmospheric density profile is plotted in time along with the altitude variation in Fig. 8. The spacecraft quickly climbs past the altitude of the final base density layer within about five minutes of close approach. Zooming in at the drag signature peak would show the small discontinuity in density allowed by the model at close approach. In other atmosphere models with additional density layers this discontinuity would be visible, giving an inbound/outbound split up as great as  $3.0\text{E-}10/6.0\text{E-}10 \text{ kg/km}^3$ . This profile is more physically realistic in that there is no large  $\Delta V$  produced by the drag at close approach and it is more smoothly varying. The layer densities outside of close approach also are not equal, slightly more density is encountered overall on the inbound leg than the outbound.



**Figure 8. T107 estimated density profile**

## 5. Comparison with T87 Flyby

Comparing the density results from T107 to those from T87 is difficult since T87 had a lower close approach altitude of 973.3 kilometers (compared to 980.1 kilometers for T107) as well as different Titan ephemeris models, a different DSN tracking station, etc. The T87 periapsis was also at a low latitude of  $11.4^\circ$  compared to  $53.7^\circ$  latitude for T107. This changes the structure and distribution of the base density layers and results. Figure 9 shows the base density values and uncertainties estimated at T87 and T107 against the altitude layer for each profile. Near close approach the profiles have some overlap, but again, the base altitudes are different for both data sets.



**Figure 9. T87 and T107 estimated density profiles**

The comparison of greatest interest is the density at close approach of T107 against a similar altitude from the T87 profile. At the 983.7 kilometer altitude layer, the base density reported for T87

inbound/outbound is  $4.36\text{E-}10/4.28\text{E-}10$  kg/km<sup>3</sup> [6]. This is not exactly the close approach altitude from T107 but the density values are well within the  $1\sigma$  uncertainties estimated for these densities, showing that the two flyby results are statistically consistent.

## 6. Conclusions and Future Work

Doppler tracking was collected through close approach for only two Titan flybys: T87 and T107. The method of orbit determination is used here to estimate an atmospheric density profile for T107 using an altitude layering setup similar to that applied to the T87 flyby. Telemetry information on spacecraft thrusting is used to estimate corrections to a nominal density profile based on T87 results in an epoch-state Kalman filter. This model allows a discontinuity in density at Titan periapsis to provide separate estimates of the inbound and outbound density profile. The converged density profile estimate yields a discontinuity within  $1\sigma$  of the respective inbound and outbound values. The estimated uncertainty in the base density estimates are an order of magnitude less than the estimated density values, ensuring that the results are statistically significant. The density estimates of T87 at an altitude comparable to T107 close approach are within  $1\sigma$  of each other showing that the two density profiles are statistically consistent. Further validation of these density estimates will include applying the same altitude layering technique to the T87 and T107 data sets and combining both data sets in a multi-arc orbit determination solution to produce a single atmospheric density profile.

## 7. Acknowledgements

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