Super-twisting Adaptive Sliding Mode Disturbance Observer based Attitude Control for Mars Entry under Uncertainty

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Abstract: *A novel super-twisting adaptive sliding mode observer based attitude controller is designed for Mars entry. A continuous uncertain term is assumed to be bounded with unknown boundary. The proposed super-twisting adaptive sliding mode disturbance observer is designed to approximate the uncertainty and disturbance in attitude control loop. Sliding compensation control is designed to offset the harmful effect by application of Lyapunov method. A numerical example confirms the efficacy of the proposed strategy.*

Keywords: *Mars entry, Uncertainty and disturbance, Super-twisting algorithm, Disturbance observer, Sliding mode control.*

1. Introduction

Mars landing exploration activities have been and will continue to gather scientific data and deepen the current understanding about the life origin and the solar system formation process. All Mars landers to date continue to rely on the entry, descent and landing (EDL) technologies developed for the Viking missions in the mid-seventies of the last century, which lead to larger landing error ellipse. With the advances of technologies, estimated Mars landing accuracy to date has gradually improved from ~150 km of Mars Pathfinder to ~35 km for the Mars Exploration Rovers to ~10 km for 2012 Mars Science Laboratory (MSL) [1]. It is believed that MSL is challenging the capabilities of Viking-heritage EDL technologies, defining an upper bound on the performance of the first generation EDL systems and GNC mode. Future Mars missions, such as Mars sample return, manned Mars landing and Mars base, need to achieve the pin-point Mars landing (safe landing within tens of meters to 100 m of a preselected target site) [2]. Since the current EDL system and GNC methods can not satisfy the requirements for future pinpoint Mars landing missions, the next generation of EDL system and GNC methodologies are required in order to deliver the larger and most capable lander/rover to date to the surface of Mars.

It is believed that the model parameter uncertainty and external disturbance are the main impedient for further improving Mars landing accuracy [3]. For Mars entry, due to the lack of reliable model of Martian atmosphere at the current stage, and the aerodynamic parameters of entry vehicle model are obtained by ground wind tunnel test, as well as complex varied flight environment introduces external disturbance moment on Mars entry vehicle [1,3]. They usually result in large uncertainty between designed model and real flight state, which inevitably degrades the performance of Mars entry guidance and control algorithms [3]. In order to ensure Mars landing missions are executed safely and accurately, the flight control system of Mars entry vehicle should be designed to compensate and suppress those uncertainty and disturbance.

Disturbance observer is one of the effective means to solve the flight control problem with uncertainty and disturbance, which utilize some known data to estimate unknown data [4,5]. For Mars entry with large uncertainty and disturbance, the disturbance observer can online approximate the composite uncertain item, which is conducive to design a compensation controller, so as to achieve high precision and robustness. However, the boundary of disturbance is needed for traditional sliding mode control. High order sliding mode not only can commendable overcome the chattering problem of one order sliding mode, but also retain the merits of the latter [6]. As one of useful second order sliding mode control methods, super-twisting algorithm is the unique that don't need any differential information of sliding mode variable respect to time in advance, and it has less adaptive learning parameters to be propitious to real-time control [7]. Since super-twisting algorithm contains a discontinuous function under the integral, chattering is not eliminated but attenuated [8].

The aim of this paper is to develop new robust tracking control in order to further improve the robustness and accuracy of Mars atmospheric entry in the presence of larger uncertainty/ disturbance. Motivated by the preceding works, we design the novel super-twisting adaptive sliding mode disturbance observer based attitude control low that continuously drives the sliding variable and its derivative to zero in the presence of the bounded disturbance with the unknown boundary. The proof is based on recently proposed Lyapunov function that is used for the derivation of the novel adaptive super-twisting algorithm. Based on the real-time approximate value of the uncertainty/disturbance during Mars entry, the attitude of entry vehicle can be tracked quickly and smoothly by sliding controller.

This paper is organized as follows. Firstly, six degree-of-freedom dynamic model for Mars atmospheric entry with uncertainty/disturbance is established. Secondly, super-twisting adaptive sliding mode disturbance observer is designed to estimate the uncertainty and disturbance in attitude loop. Thirdly, sliding compensation control is designed by application of Lyapunov method. The information of uncertainty and disturbance approximated by super-twisting adaptive sliding mode disturbance observer is feed back to control system, and the harmful effect is offset by compensation control. Finally, the effectiveness of this method is demonstrated through the simulation test.

2. Description of Mars entry attitude dynamics

In this paper, the following nonlinear rigid body dynamics equations with six degree-of-freedom and twelve states are used to describe the Mars entry vehicle dynamic model. The subsequent control law is designed based on these equations as follows [6].

$$\dot{x} = V\cos\gamma\cos\chi \tag{1}$$

$$\dot{y} = V\cos\gamma\sin\chi \tag{2}$$

$$\dot{z} = -V\sin\gamma \tag{3}$$

$$\dot{V} = \frac{1}{M}(-D + Y\sin\beta - Mg\sin\gamma) + \frac{1}{M}(T_{rx}\cos\beta\cos\alpha + T_{ry}\sin\beta + T_{rz}\cos\beta\sin\alpha) \tag{4}$$

$$\begin{aligned}\dot{\chi} = {} & \frac{1}{MV\cos\gamma}(L\sin\mu + Y\cos\mu\cos\beta + T_{rx}\sin\mu\sin\alpha - T_{rx}\cos\mu\sin\beta\cos\alpha + \\ & T_{ry}\cos\mu\cdot\cos\beta - T_{rz}\sin\mu\cos\alpha - T_{rz}\cos\mu\sin\beta\sin\alpha)\end{aligned} \tag{5}$$

$$\begin{aligned}\dot{\gamma} = {} & \frac{1}{MV}[L\cos\mu - Y\sin\mu\cos\beta - Mg\cos\gamma + T_{rx}(\sin\mu\sin\beta\cos\alpha + \cos\mu\sin\alpha) - \\ & T_{ry}\sin\mu\cos\beta + T_{rz}(\sin\mu\sin\beta\sin\alpha - \cos\mu\cos\alpha)]\end{aligned} \tag{6}$$

$$\dot{\alpha} = q - \tan\beta(p\cos\alpha + r\sin\alpha) + \frac{1}{MV\cos\beta}(-L + Mg\cos\gamma\cos\mu - T_{rx}\sin\alpha + T_{rz}\cos\alpha) \tag{7}$$

$$\dot{\beta} = -r\cos\alpha + p\sin\alpha + \frac{1}{MV}(Y + Mg\cos\gamma\sin\mu - T_{rx}\sin\beta\cos\alpha + T_{ry}\cos\beta - T_{rz}\sin\beta\sin\alpha) \tag{8}$$

$$\dot{\mu} = \sec\beta(p\cos\alpha + r\sin\alpha) + \frac{1}{MV}[L\tan\gamma\sin\mu + L\tan\beta - Mg\cos\gamma\cos\mu\tan\beta + (T_{rx}\sin\alpha - T_{rz}\cos\alpha)(\tan\gamma\sin\mu + \tan\beta) - (T_{rx}\cos\alpha + T_{rz}\sin\alpha)\tan\gamma\cos\mu\sin\beta + (Y + T_{ry})\tan\gamma\cos\mu] \tag{9}$$

$$\dot{p} = \frac{I_{yy} - I_{zz}}{I_{xx}}qr + \frac{1}{I_{xx}}(-\dot{I}_{xx}p + l_A + l_{Tr}) \tag{10}$$

$$\dot{q} = \frac{I_{zz} - I_{xx}}{I_{yy}}pr + \frac{1}{I_{yy}}(-\dot{I}_{yy}q + m_A + m_{Tr}) \tag{11}$$

$$\dot{r} = \frac{I_{yy} - I_{xx}}{I_{zz}}pq + \frac{1}{I_{zz}}(-\dot{I}_{zz}r + n_A + n_{Tr}) \tag{12}$$

where, x , y , z are the position of center of mass of entry vehicle, V is the vehicle's velocity; χ , γ , α , β , μ are azimuth angle, flight path angle, angle of attack, angle of sideslip, and roll angle, respectively; p 、 q 、 r are roll rate, pitch rate, and yaw rate, respectively; M is mass of the vehicle; L , Y , D are lift, yawing force, and drag force, respectively; l_A , m_A , n_A are rolling moment, pitching moment, and yawing moment, respectively; T_{rx} , T_{ry} , T_{rz} are tri-axial force by reaction control system (RCS), respectively; l_{Tr} , m_{Tr} , n_{Tr} are rolling moment, pitching moment, and yawing moment by RCS, respectively; I_{xx} , I_{yy} , I_{zz} are rotational inertia for x , y , z axis, respectively; $\dot{I}_{xx}$, $\dot{I}_{yy}$, $\dot{I}_{zz}$ are rate of rotational inertia for x , y , z axis, respectively.

The system state of fast loop is $\boldsymbol{\omega} = [p,q,r]^{\mathrm{T}}$. The controller should output the RCS command according to some required command $\boldsymbol{\omega}_c$. Based on the equations (10) to (12), the attitude dynamics model can be described as an affine nonlinear system:

$$\dot{\boldsymbol{\omega}} = \boldsymbol{f}_f + \boldsymbol{\Delta f}_f + \boldsymbol{g}_f \boldsymbol{M}_C + \boldsymbol{d} \tag{13}$$

where, $\boldsymbol{M}_C$ denotes the control moment, $\boldsymbol{\Delta f}_f$ is the uncertain part in the $\boldsymbol{f}_f$, $\boldsymbol{d}$ is external moment disturbance; let $\boldsymbol{D}_f = \boldsymbol{\Delta f}_f + \boldsymbol{d}$.

3. Super-twisting Adaptive Sliding Mode Disturbance Observer based Attitude Control

Because of the moment disturbance has a more influence on the angular velocity while few influence on the attitude angle, during Mars entry process. Hence, the external disturbance is considered to be in the fast loop. In fact, it is impossible to predict the complex interference, much less the change rate of the external disturbance. In this paper, the Super-twisting Adaptive Sliding Mode Disturbance Observer (SASMDO) is designed to estimate the disturbance of the fast loop and the compensation control law is derived.

Levant first proposed the idea of higher order sliding mode control [9]. The higher order sliding mode control can avoids the chattering problem of a first order sliding mode control, and also inherits the advantages of the latter. As one of the second order sliding mode control algorithms, super-twisting algorithm is the only one that does not require time derivative data of any sliding mode variables. The super-twisting controller u is calculated as follows:

$$\begin{cases} u = -\lambda_1 |\sigma| sign(\sigma) + u_1 \\ \dot{u}_1 = -\lambda_2 sign(\sigma) \end{cases} \tag{14}$$

where, σ is the sliding surface; and $\dot{\sigma}$ is not required. Therefore, the super-twisting controller has robustness.

Theorem 1: Let $|D_i| \le \delta_i |\sigma_i|^{\frac{1}{2}}$, such that $|\boldsymbol{D}| \le \boldsymbol{\delta} \cdot |\boldsymbol{\sigma}|^{\frac{1}{2}}$; where, $i = 1, 2, 3$, unknown $\delta_i > 0$. For fast loop nonlinear system (1), the Adaptive Sliding Mode Disturbance Observer (ASMDO) is constructed as:

$$\begin{cases} \boldsymbol{\sigma} = \boldsymbol{\omega} + \boldsymbol{z} \\ \dot{\boldsymbol{z}} = -\boldsymbol{f}_f - \boldsymbol{g}_f \boldsymbol{u} - \boldsymbol{v} \\ \hat{\boldsymbol{D}} = \boldsymbol{v} \end{cases} \tag{15}$$

where

$$\boldsymbol{v} = \boldsymbol{l}_1 \cdot |\boldsymbol{\sigma}|^{\frac{1}{2}} \cdot sign(\boldsymbol{\sigma}) + \boldsymbol{l}_2 \cdot \int sign(\boldsymbol{\sigma}) d\boldsymbol{\tau} \tag{16}$$

Adaptive law of parameter $\boldsymbol{l}_1$ is

$$\dot{l}_{i1} = \rho_i \|\omega_i\| \|\sigma_i\| \tag{17}$$

Relationship between $\boldsymbol{l}_2$ and $\boldsymbol{l}_1$ is

$$l_{i2} = \frac{\varepsilon}{2} l_{i1} + \frac{1}{2}\varepsilon^2 + \frac{1}{2}\lambda \tag{18}$$

where, $\boldsymbol{\sigma}$ is assist sliding surface, $\boldsymbol{\omega}$ is the state of fast loop, z is the state of observer, v is the input of assist control, λ and ε are any positive number.

Lemma 1: Consider the nonlinear system as follows [8,10]:

$$\begin{cases} \dot{x}_1 = -\iota |x_1|^{\frac{1}{2}} sign(x_1) + x_2 + \xi(t) \\ \dot{x}_2 = -\varpi sign(x_1) \end{cases} \tag{19}$$

This equation can be rewritten as $\dot{x}_1 + \iota |x_1|^{\frac{1}{2}} sign(x_1) + \varpi \int sign(x_1) d\tau = \xi(t)$, parameters ι and ϖ are designed to enable x_1 and $\dot{x}_1$ converge to zero in a limited time. Therefore, parameters $\boldsymbol{l}_1$ and $\boldsymbol{l}_2$ should be designed to enable the sliding surface $\boldsymbol{\sigma}$ and $\dot{\boldsymbol{\sigma}}$ converge to zero in a limited time, and $\boldsymbol{l}_1$ reaches its stable value $\boldsymbol{l}_{l_0}$. Here, $\boldsymbol{\sigma}$ and $\dot{\boldsymbol{\sigma}}$ corresponding to x_1 and $\dot{x}_1$. Then, the compound disturbance observation $\hat{\boldsymbol{D}}$ could uniform converges to the real value.

Proof: Time derivation of $\boldsymbol{\sigma}$ equation is

$$\dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{\omega}} + \dot{\boldsymbol{z}} = \boldsymbol{f}_f + \boldsymbol{g}_f \boldsymbol{u} + \boldsymbol{D} - \boldsymbol{f}_f - \boldsymbol{g}_f \boldsymbol{u} - \boldsymbol{v} = \boldsymbol{D} - \boldsymbol{v} \tag{20}$$

After transposition, that is

$$\dot{\boldsymbol{\sigma}} + \boldsymbol{v} = \boldsymbol{D} \tag{21}$$

Let's plug eq. (16) into eq. (21), and rewritten eq. (21) as the form of eq. (19), that is

$$\begin{cases} \dot{\boldsymbol{\sigma}} = -\boldsymbol{l}_1 . |\boldsymbol{\sigma}|^{\frac{1}{2}} .sign(\boldsymbol{\sigma}) + \boldsymbol{\upsilon} + \boldsymbol{D} \\ \dot{\boldsymbol{\upsilon}} = -\boldsymbol{l}_2 .sign(\boldsymbol{\sigma}) \end{cases} \tag{22}$$

Let $\boldsymbol{\varsigma}_i = \begin{pmatrix} \varsigma_{i1} \\ \varsigma_{i2} \end{pmatrix} = \begin{pmatrix} |\sigma_i|^{\frac{1}{2}} sign(\sigma_i) \\ \upsilon_i \end{pmatrix}$, we have $\boldsymbol{\varsigma}_i^2 = \varsigma_{i1}^2 + \varsigma_{i2}^2 = |\sigma_i| + \upsilon_i^2$, $sign(\sigma_i) = sign(\varsigma_{i1})$, $|\varsigma_{i1}| = |\sigma_i|^{\frac{1}{2}}$. According to the eq. (22), we have:

$$\begin{cases} \dot{\varsigma}_{i1}=\dfrac{1}{2|\sigma_i|^{\frac{1}{2}}}(-l_{i1}\varsigma_{i1}+\varsigma_{i2}+D_i) \\ \dot{\varsigma}_{i2}=-l_{i2}sign(\sigma_i)=-l_{i2}sign(\varsigma_{i1})=-\dfrac{1}{|\sigma_i|^{\frac{1}{2}}}l_{i2}\varsigma_{i1} \end{cases} \tag{23}$$

That is

$$\dot{\boldsymbol{\varsigma}}_{\boldsymbol{i}}=\frac{1}{|\sigma_i|^{\frac{1}{2}}}\begin{bmatrix} -\dfrac{l_{i1}}{2} & \dfrac{1}{2} \\ -l_{i2} & 0 \end{bmatrix}\boldsymbol{\varsigma}_{\boldsymbol{i}}+\frac{1}{|\sigma_i|^{\frac{1}{2}}}\begin{bmatrix} \dfrac{1}{2} \\ 0 \end{bmatrix}D_i \tag{24}$$

It can be rewritten as

$$\dot{\boldsymbol{\varsigma}}_{\boldsymbol{i}}=\boldsymbol{A}_{\boldsymbol{i}}\boldsymbol{\varsigma}_{\boldsymbol{i}}+\boldsymbol{B}_{\boldsymbol{i}}D_i \tag{25}$$

where, $\boldsymbol{A}_{\boldsymbol{i}}=\frac{1}{|\sigma_i|^{\frac{1}{2}}}\begin{bmatrix} -\frac{l_{i1}}{2} & \frac{1}{2} \\ -l_{i2} & 0 \end{bmatrix}$, $\boldsymbol{B}_{\boldsymbol{i}}=\frac{1}{|\sigma_i|^{\frac{1}{2}}}\begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}$.

In fact, if ς_{i1} and ς_{i2} can converge to zero in a limited time, then σ_i and $\dot{\sigma}_i$ can also converge to zero in a limited time.

We select the Lyapunov function as:

$$V=\boldsymbol{\varsigma}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma}_{\boldsymbol{i}}+\frac{1}{2\kappa}(l_{i1}-\hat{l}_{i1})^2 \tag{26}$$

where, κ denotes any positive number. Let $V_0=\boldsymbol{\varsigma}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma}_{\boldsymbol{i}}$, then eq. (26) could be rewritten as:

$$V=V_0+\frac{1}{2\kappa}(l_{i1}-\hat{l}_{i1})^2 \tag{27}$$

where, $\boldsymbol{P}=\begin{bmatrix} \lambda+\varepsilon^2 & -\varepsilon \\ -\varepsilon & 1 \end{bmatrix}$, λ and ε denote any positive number. Therefore, $\boldsymbol{P}$ is a positive defined matrix.

Time derivation of V_0 is

$$\dot{V}_0=\dot{\boldsymbol{\varsigma}}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma}_{\boldsymbol{i}}+\boldsymbol{\varsigma}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{P}\dot{\boldsymbol{\varsigma}}_{\boldsymbol{i}}=\boldsymbol{\varsigma}_{\boldsymbol{i}}^{\mathrm{T}}(\boldsymbol{P}\boldsymbol{A}_{\boldsymbol{i}}+\boldsymbol{A}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{P})\boldsymbol{\varsigma}_{\boldsymbol{i}}+2D_i\boldsymbol{B}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma}_{\boldsymbol{i}}=-\frac{1}{|\sigma_i|^{\frac{1}{2}}}\boldsymbol{\varsigma}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{\varsigma}_{\boldsymbol{i}}+2D_i\boldsymbol{B}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\varsigma}_{\boldsymbol{i}} \tag{28}$$

According to eqs. (24) and (25), we get

$$\boldsymbol{Q}=\begin{bmatrix} l_{i1}(\lambda+\varepsilon^2)-2l_{i2}\varepsilon & l_{i2}-\dfrac{l_{i1}}{2}\varepsilon-\dfrac{1}{2}(\lambda+\varepsilon^2) \\ * & \varepsilon \end{bmatrix} \tag{29}$$

$$2\boldsymbol{B}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{P}=\frac{1}{|\sigma_i|^{\frac{1}{2}}}\begin{bmatrix} \lambda+\varepsilon^2 & -\varepsilon \end{bmatrix} \tag{30}$$

Here, let $\boldsymbol{G}=-\begin{bmatrix} \lambda+\varepsilon^2 & -\varepsilon \end{bmatrix}$, we have

$$\dot{V}_0=-\frac{1}{|\sigma_i|^{\frac{1}{2}}}\boldsymbol{\varsigma}_{\boldsymbol{i}}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{\varsigma}_{\boldsymbol{i}}-\frac{D_i}{|\sigma_i|^{\frac{1}{2}}}\boldsymbol{G}\boldsymbol{\varsigma}_{\boldsymbol{i}} \tag{31}$$

Let $\dot{V}_0\le-\frac{1}{|\sigma_i|^{\frac{1}{2}}}\varsigma_i^{\mathrm{T}}\tilde{\boldsymbol{Q}}\varsigma_i=-\frac{1}{|\sigma_i|^{\frac{1}{2}}}\varsigma_i^{\mathrm{T}}(\boldsymbol{Q}+\hat{\boldsymbol{Q}})\varsigma_i$, because if $|D_i|\le\delta_i|\sigma_i|^{\frac{1}{2}}$, then

$$\hat{\boldsymbol{Q}} = -\frac{1}{|\sigma_i|^{\frac{1}{2}}}\begin{bmatrix} -\delta_i(\lambda+\varepsilon^2) & \frac{1}{2}\delta_i\varepsilon \\ * & 0 \end{bmatrix} \tag{32}$$

Hence,

$$\tilde{\boldsymbol{Q}} = \boldsymbol{Q} + \hat{\boldsymbol{Q}} = \begin{bmatrix} (l_{i1}-\delta_i)(\lambda+\varepsilon^2) - 2l_{i2}\varepsilon & l_{i2} - \frac{1}{2}\varepsilon l_{i1} - \frac{1}{2}(\lambda+\varepsilon^2) + \frac{1}{2}\delta_i\varepsilon \\ * & \varepsilon \end{bmatrix} \tag{33}$$

Let

$$l_{i2} = \frac{1}{2}\varepsilon l_{i1} + \frac{1}{2}\varepsilon^2 + \frac{1}{2}\lambda \tag{34}$$

Let's plug eq. (32) into eq. (31), then we get

$$\tilde{\boldsymbol{Q}} = \begin{bmatrix} l_{i1}\lambda - (\varepsilon+\delta_i)(\lambda+\varepsilon^2) & \frac{1}{2}\delta_i\varepsilon \\ * & \varepsilon \end{bmatrix} \tag{35}$$

Hence, if $l_{i1} \geq \dfrac{(\varepsilon+\delta_i)(\lambda+\varepsilon^2)+\frac{\varepsilon}{2}(\delta_i^2+1)}{\lambda}$, $\lambda_{\min}(\tilde{Q}) \geq \dfrac{\varepsilon}{2}$, we have

$$\dot{V}_0 = -\frac{\lambda_{\min}(\tilde{Q})\lambda_{\min}^{\frac{1}{2}}(P)}{\lambda_{\max}(P)}V_0^{\frac{1}{2}} \tag{36}$$

Let $\vartheta = \dfrac{\lambda_{\min}(\tilde{Q})\lambda_{\min}^{\frac{1}{2}}(P)}{\lambda_{\max}(P)}$, then we have

$$\dot{V} = \dot{V}_0 + \frac{1}{\kappa}(l_{i1} - l_{i1_0})\dot{l}_{i1} = -\vartheta V_0^{\frac{1}{2}} + \frac{1}{\kappa}(l_{i1} - \hat{l}_{i1})\dot{l}_{i1} < 0 \tag{37}$$

This completes the proof.

Therefore, for adaptive sliding mode disturbance observer (15), under the assist control of $\boldsymbol{v}$, the assist sliding surface $\boldsymbol{\sigma}$ and $\dot{\boldsymbol{\sigma}}$ can converged to zero in a limited time; in this case, $\hat{\boldsymbol{D}} = \boldsymbol{v}$, which means $\boldsymbol{v}$ is a precise estimation of uncertain item $\boldsymbol{D}$. Then, we get the compensation control law $\boldsymbol{M}_1 = -\boldsymbol{g}_f^{-1}\hat{\boldsymbol{D}}$ for the uncertainty. Hence, the control law of fast loop is

$$\boldsymbol{M}_c = -\boldsymbol{g}_f^{-1}(\boldsymbol{f}_f - \dot{\boldsymbol{\omega}}_c - \boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) - \boldsymbol{g}_f \cdot \boldsymbol{h}(\boldsymbol{\omega}_{aw}) + a_2\boldsymbol{\omega}_e + b_2\boldsymbol{\omega}_e^{q_2/p_2} + K \cdot \boldsymbol{s} + \hat{\boldsymbol{D}}) \tag{38}$$

where, $\boldsymbol{M}_c = [l_{ctrl}, m_{ctrl}, n_{ctrl}]^{\mathrm{T}}$ are the control moment on rolling, pitching, and yawing; their expression are

$$\boldsymbol{M}_c = \boldsymbol{g}_{f,\delta}\boldsymbol{\delta}_c + \boldsymbol{M}_{RCS} \tag{39}$$

$$\boldsymbol{M}_{aero} = \boldsymbol{g}_{f,\delta}\boldsymbol{\delta}_c = (1-k_c)\boldsymbol{M}_c \tag{40}$$

$$\boldsymbol{M}_{RCS} = k_c\boldsymbol{M}_c \tag{41}$$

where, $\boldsymbol{\delta}_c$ is the deflection angle of aerodynamic surface, $\boldsymbol{M}_{aero}$ is the moment of force generated by aerodynamic surface, $\boldsymbol{M}_{RCS}$ is the moment of force generated by RCS, k_c is the control authority coefficient which is determined by the aerodynamic press $\hat{q}$.

System close-loop stability analysis

According to the description above, the adaptive sliding mode disturbance observer (ASMDO) is able to approximate the uncertainty, and the compensation control law is utilized to offset the influence of uncertainty.

Theorem 2: Adopting the adaptive sliding mode disturbance observer (15), the system (13) could be asymptotically stable by the control law (38).

Proof: We plug the control law (38) into the state equation (13), then we get

$$\dot{\boldsymbol{e}}_f = -a_2\boldsymbol{e}_f - b_2{\boldsymbol{e}_f}^{q_2/p_2} + \boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) + \boldsymbol{g}_f\boldsymbol{h}(\boldsymbol{\omega}_{aw}) - \boldsymbol{K}\cdot\boldsymbol{s} + \boldsymbol{D} - \hat{\boldsymbol{D}} \tag{42}$$

It can be rewritten as

$$\dot{\boldsymbol{e}}_f = -\boldsymbol{A}_f\boldsymbol{e}_f + \boldsymbol{f}_f(\boldsymbol{\omega}_{aw}) + \boldsymbol{g}_f\boldsymbol{h}(\boldsymbol{\omega}_{aw}) - \boldsymbol{K}\cdot\boldsymbol{s} + \boldsymbol{D} - \hat{\boldsymbol{D}} \tag{43}$$

where, $-\boldsymbol{A}_f$ is a diagonal Hurwitz matrix. Symmetric matrix $\boldsymbol{P}_f$ exists and meets

$$\boldsymbol{P}_f\boldsymbol{A}_f + {\boldsymbol{A}_f}^{\mathrm{T}}\boldsymbol{P}_f = -\boldsymbol{Q}_f \tag{44}$$

where, $\boldsymbol{Q}_f = \boldsymbol{Q}_f^{\mathrm{T}} > 0$. We expand the close-loop error vector as

$$\boldsymbol{\Psi}_f = [\boldsymbol{e}_f^{\mathrm{T}} \quad (\boldsymbol{L}\varsigma_i)^{\mathrm{T}}]^{\mathrm{T}} \tag{45}$$

Here, $\boldsymbol{P} = \boldsymbol{L}^{\mathrm{T}}\boldsymbol{L}$.

We select the Lyapunov fuction of the entire close-loop system as

$$V_{\Psi_f} = \boldsymbol{\Psi}_f^{\mathrm{T}}\overline{\boldsymbol{P}}_f\boldsymbol{\Psi}_f \tag{46}$$

where, $\overline{\boldsymbol{P}}_f = \dfrac{1}{2}\begin{bmatrix} \boldsymbol{P}_f & 0 \\ 0 & \boldsymbol{I}_2 \end{bmatrix}$.

Time derivation of V_{Ψ_f} is

$$\begin{aligned}
\dot{V}_{\Psi_f} &= \frac{1}{2}(\dot{\boldsymbol{e}}_f^{\mathrm{T}}\boldsymbol{P}_f\boldsymbol{e}_f + \boldsymbol{e}_f^{\mathrm{T}}\boldsymbol{P}_f\dot{\boldsymbol{e}}_f) + \frac{1}{2}(\dot{\varsigma}_i^{\mathrm{T}}\boldsymbol{P}\varsigma_i + \varsigma_i^{\mathrm{T}}\boldsymbol{P}\dot{\varsigma}_i) \\
&\le -\frac{1}{2}\chi_{\min}(\boldsymbol{Q}_f)\|\boldsymbol{e}_f\|^2 + \frac{1}{2}(\dot{\varsigma}_i^{\mathrm{T}}\boldsymbol{P}\varsigma_i + \varsigma_i^{\mathrm{T}}\boldsymbol{P}\dot{\varsigma}_i) \\
&= -\frac{1}{2}\chi_{\min}(\boldsymbol{Q}_f)\|\boldsymbol{e}_f\|^2 + \frac{1}{2}\dot{V}_0 < 0
\end{aligned} \tag{47}$$

where, $\chi_{\min}(\boldsymbol{Q}_f) > 0$ is the least eigenvalue of $\boldsymbol{Q}_f$.

Hence, the compound system consisting of eq. (13) and eq. (15) is asymptotically stable by the control law (38). This completes the proof.

4. Simulation Case

For preliminary verify the contribution described above, a simulation case is given. We adopt the Mars entry vehicle simulation parameters mentioned in literature [6]. Some initial value is set as $\chi = \gamma = 0$, $\alpha = 1^\circ$, $\beta = 2.5^\circ$, $\mu = 3^\circ$, and $p = q = r = 0$. Tracking command: $1 \sim 4s$: $\alpha_c = 6^\circ$, $\beta_c = 0^\circ$, $\mu_c = 0^\circ$; $4 \sim 7s$: $\alpha_c = 12^\circ$, $\beta_c = 0^\circ$, $\mu_c = 0^\circ$. Fast-loop uncertain disturbance moments of forces: $d_1 = 1\times10^5(\sin(4t)+0.2)$, $d_2 = 2\times10^6(\sin(11t)-0.6)$, $d_3 = 2\times10^6(\sin(5t)+0.2)$. Slow-loop control parameters: $a_1 = 2$, $b_1 = 1$, $q_1 = 7$, $p_1 = 9$, $k = 7/9$. Fast-loop control parameters: $a_1 = 1$, $b_1 = 3$, $q_1 = 7$, $p_1 = 9$, $K = diag(15,15,15)$, initial value of l_1 is $l_1^* = 10$. Authority coefficient of moment of force is $k_c = (285 - \hat{q}/1000)/1000$. To simulate the large uncertainty, here, aerodynamic coefficients multiply $1+0.5\sin(t)$, aerodynamic moments of forces multiply $1+0.4\sin(t)$. The simulation results are shown as follows.

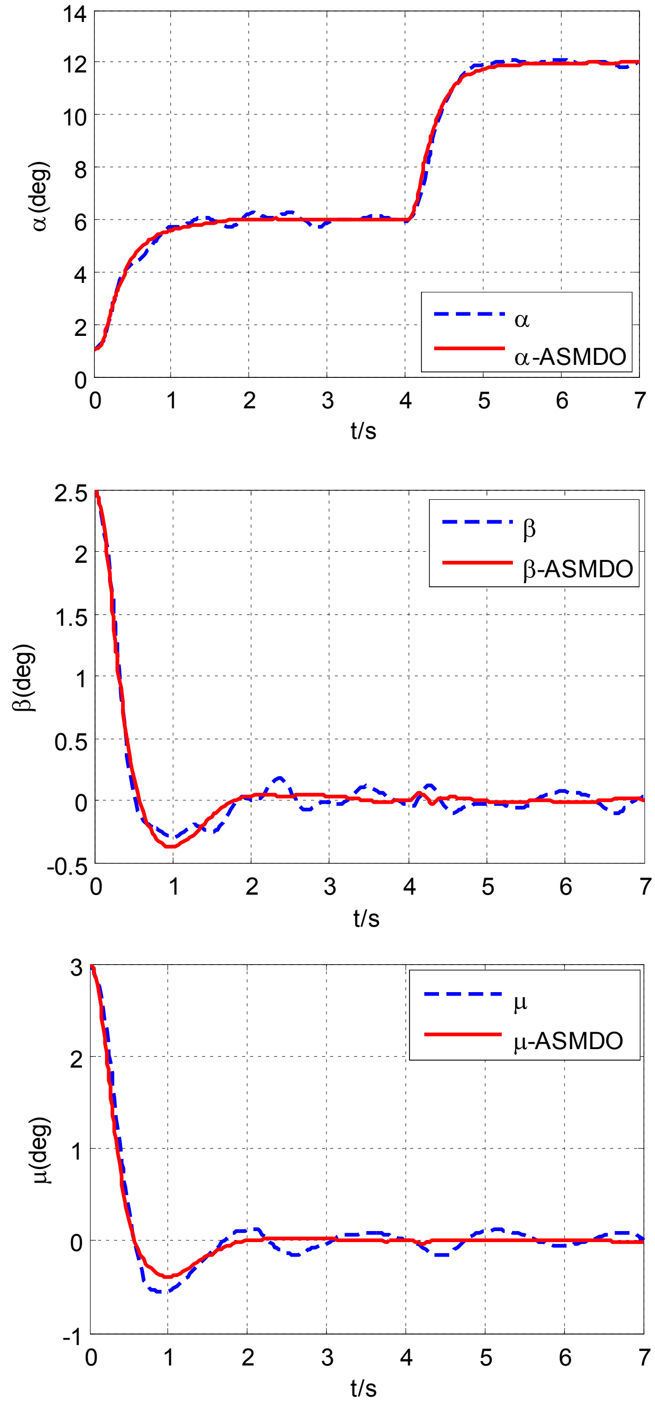

Figure 1. Attitude angles tracking profiles

In Figure 1, we can found that in the case of large uncertainty exists and ASMDO absents, the commands of attitude angles are oscillating tracked. While after the ASMDO presents, the observer can timely observe the external uncertain disturbances and feedback them to the control system, the designed compensation control law is able to counteract the adverse effects caused by uncertainty. They enable the commands of attitude angles are tracked rapidly and smoothly.

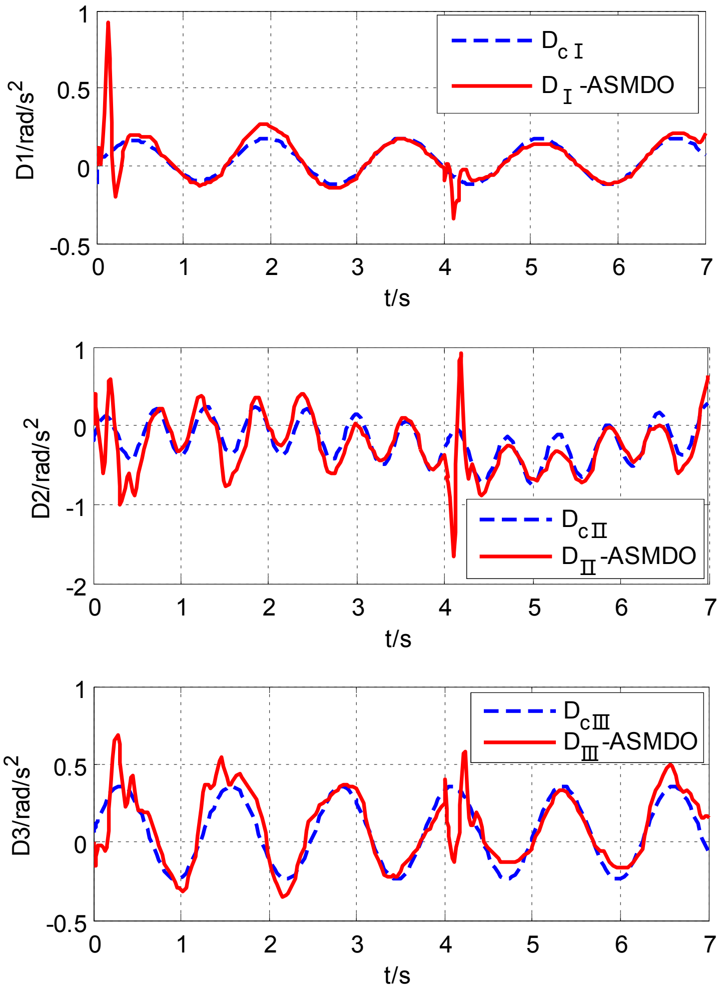

Figure 2. Observation of uncertainty

In figure 2, we can found that the ASMDO can timely approximate the disturbance and uncertainty. The derivation of disturbance is not required, so it has good robustness.

5. Conclusions

The external uncertain disturbances of moments of forces exist during the Mars atmospheric entry process, and the derivative boundary value of the uncertainty is unpredictable. In this paper, an adaptive sliding mode disturbance observer based on super-twisting algorithm is designed, which does not require a priori value of external uncertainty. The learning law of adaptive parameters is derived, and the adaptive learning parameters are little. The adaptive sliding mode disturbance observer based attitude control law is suitable for real-time operation. The preliminary simulation results show that the observer can approximate the uncertain disturbance while the commands of attitude angles can be tracked rapidly and smoothly.

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