ELECTRIC PROPUSION TRANSFER OPTIMIZATION

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Abstract: *GMV has been awarded by ESA with the EPTOS project whose objective is to develop a flight dynamics prototype able to optimize the transfer trajectory of commercial telecomm GEO satellites using electric propulsion, from launch to injection. The final technical solution would be integrated in GMV's focusleop product, part of GMV's focussuite flight dynamics system for ground control of commercial satellites. The new development will be applicable for mission analysis and operations (as required by GEO telecom operators and satellite manufacturers). This paper describes the GMV's solution to satisfy all the mission and system requirements for geostationary telecommunication satellites employing electric propulsion platforms for LEOP that is based on a hybrid optimal control solution technique. This hybrid technique means to provide the robustness and reliability of a direct method while maintaining the swift computation of an indirect method. To achieve this, the secular trajectory is defined by a number of nodes optimized via non-linear programming, and the rapidly changing arcs between nodes are optimized solving a boundary value problem. Every optimization step produces a feasible trajectory, and the convergence radius allows a straight line from the initial to the final orbit to be an adequate initial guess. Furthermore, the number of optimization variables is reduced an order of magnitude with respect to a direct method, yielding a fast and precise trajectory optimization process.*

Keywords: *Electric Propulsion, Low Thrust Trajectory, Orbit Optimization.*

1. Introduction

In the last decades, electric propulsion (EP) has reached enough technological maturity to become the main propulsion system of spacecraft. Several demonstration missions using electric propulsion have been conducted so far (NASA's Deep Space 1, ESA's SMART-1, JAXA's Hayabusa). In addition, EP thrusters are routinely used in multiple GEO satellites for diverse station keeping manoeuvres. From the past missions, the low thrust engines have accumulated thousands of operating hours and hundreds of ignitions. These successes have given the necessary confidence to select electric propulsion for several incoming scientific missions (NASA's Dawn, ESA's BepiColombo), and it was also considered in the past for potential commercial missions such as ConeXpress, SMART-OLEV, and is coming back with the future VEGA EPSM, Vivisat or ESS concepts. This reliability has allowed the production of full electric GEO telecomm satellites that will use only EP for the LEOP transfer phase from the injection orbit to the final GEO slot.

Electric propulsion permits a large reduction on the propellant mass due to its higher efficiency compared to the chemical propulsion, if specific impulse is at least one order of magnitude larger. However, due to its lower thrust (several orders of magnitude) the resulting trajectories are more complex than in the chemical propulsion case, where the instantaneous impulse approximation is valid. Typically low thrust trajectories are composed of an alternating series of thrust and coast arcs, and require longer time of flight. Innovative techniques to optimize such

trajectories are required, in order to cope with all the mission and operations requirements of future electric propulsion transfers.
The geostationary telecommunications satellite market, the domain where EPTOS activity is aimed at, shows a total number of operators of about 80 and a total of some 454 satellites flying, increasing at a yearly rate about 20-30 satellites.

2. Algorithms specifications for the trajectory optimization approach

The present activity deals with a realistic problem, the transfer of a spacecraft (SC) from its initial orbit (in which it is injected by the launcher) to a final orbit using electric propulsion. A currently very interesting problem is to find the minimum-time transfer from GTO to a desired GEO slot with EP thrusters.

2.1. Trajectory optimization formulation

Through the use of EP thrusters it is desired to satisfy the following constraints in terms of: feasibility, satisfying the equations of dynamics and applicable physical and operation constraints; transfer trajectory, make use of electric propulsion to transport the spacecraft from the launch orbit to the target orbit; performance, is usually the final mass (to be maximised), but the time elapsed during the transfer can be an alternate performance index.
For the use of electrical propulsion, a few possibilities of thrust are to be considered:

- Transfer under continuous thrust at constant thrust magnitude.
- Transfer with pulsed thrust, in which the spacecraft flies ballistically over some parts of the trajectory to make more efficient use of the propellant, at the expense of longer duration of the transfer.
- It could be possible to introduce a limited number of high thrust, chemical maneuvers before starting the electric-propulsion phase.

2.2. Optimization solution

The problem complexity and the lack of information make it difficult to reach a direct attempt to a solution. For the EPTOS activity, the following step approach has been followed, each step providing the input trajectory for the next one:

- Step 1: Generation of the initial guess of the transfer by using geoexpress, an in-house tool with issues a “basic trajectory” that:
    - Implements near-optimal thrust steering for trajectory control;
    - Approximately satisfies the equations of dynamics;
    - Satisfies perigee/apogee constraints;
    - Consumes the right amount of propellant.
- Step 2: Refinement of the “basic trajectory” to produce a “feasible trajectory” that:
    - Exactly satisfies the equations of dynamics;
    - Implements optimal thrust steering;

- Satisfies all applicable constraints, with the exception of attitude constraints.

- Step 3: Optimization of the “feasible trajectory” to get a “steering-free optimal trajectory” that:
  - Improves the transfer performance;
  - Still satisfies the equations of dynamics and the applicable constraints, but those on attitude.
- Step 4: Constraint of the “steering-free optimal trajectory” to obtain the “optimal trajectory”, which do satisfy any attitude constraint.

Following the parallel shooting method, the full trajectory is decomposed in a large number of arcs (see Fig. 1). At least fifteen parameters are required to define an arc, and there should be several arcs per orbital revolution. Typically, the number of parameters needed to define the transfer ranges in the tens of thousands. All of them need adjustment to satisfy the applicable constraints while optimizing the trajectory, i.e., the set of these parameters is the core of the set of optimization variables. A small number of other optimization variables may be needed, depending on the mission.

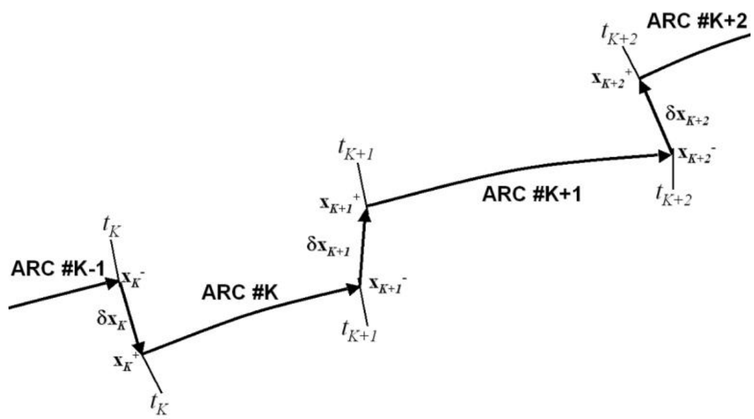

Figure 1. Trajectory decomposition

Each arc is characterized by a reduced set of parameters:

- Initial conditions of the state
  - Arc initial time (1 datum)
  - Arc initial position (3 data)
  - Arc initial velocity (3 data)
  - Arc initial mass (1 datum)
- Parameters of the thrust steering
  - Initial costates of position and velocity (3+3 data)
- Arc duration (1 datum)

2.3 Mathematical Specification of Dynamics and Constraints

2.3.1. Cost Function

Final mass m_F is the objective function in all the transfer types. The corresponding cost function J, to be minimised, reduces to:

$$J = -m_F . \tag{1}$$

In terms of optimization variables, the last formula can be translated into:

$$J = \sum_{k=1}^{k=N_{ARCS}} (\Delta t)_{ON,k} , \tag{2}$$

where the summation extends to all the thrust-on durations.

2.3.2. Arc dynamics

A spacecraft trajectory reduces customarily to the position and velocity of its centre of mass, although spacecraft mass can be incorporated sometimes. Nevertheless, this is not sufficient to describe the spacecraft state in the EP missions, where additional states (the costate) are added to consider trajectory guidance.

The spacecraft evolution over a single arc considers not only position, velocity and mass, but also the time evolution of other variables that have an indirect effect upon the trajectory. The equations of spacecraft motion admit the representation as a set of first-order, explicit differential equations when formulated in the geocentric, equatorial, inertial frame of choice:

$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{v} \\ \dot{\mathbf{v}} &= \mathbf{a} = \mathbf{a}_{CP} + \mathbf{a}_{NS} + \mathbf{a}_{3B} + \mathbf{a}_{SRP} + \mathbf{a}_{PS} \end{aligned} \tag{3}$$

where $\mathbf{r}$ is the position vector, $\mathbf{v}$ the velocity vector, $\mathbf{a}$ the total acceleration, $\mathbf{a}_{CP}$ the gravitational acceleration due to Earth central potential, $\mathbf{a}_{NS}$ the gravitational acceleration due to Earth asphericity, $\mathbf{a}_{3B}$ the Luni-solar third-body differential gravitational acceleration, $\mathbf{a}_{SRP}$ the solar radiation pressure acceleration, and $\mathbf{a}_{PS}$ the thrust acceleration.

In the case of unconstrained thrust steering, the dynamics of the costate is described by the following first-order, explicit differential equations

$$\begin{aligned} \dot{\mathbf{q}} &= +\mathbf{G}\,\mathbf{p} \\ \dot{\mathbf{p}} &= -\mathbf{q} \end{aligned} \tag{4}$$

where $\mathbf{q}$ and $\mathbf{p}$ are the adjoints to position and velocity (also called costates) and $\mathbf{G}$ is the gravity gradient tensor, the Hessian of the gravitational potential U with respect to the position coordinates,

$$\mathbf{G} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \in \Re^{3\times 3} \tag{5}$$

$$G_{ij} \doteq \left[\frac{\partial U}{\partial \xi_i \partial \xi_j} \right], \ \xi_1 \doteq x , \ \xi_2 \doteq y , \ \xi_3 \doteq z \tag{6}$$

A necessary condition for unconstrained optimal thrust steering with prescribed thrust magnitude $T(t)$ is the thrust direction $\hat{\mathbf{u}}_T$ being parallel to the primer vector $\mathbf{p}$, i.e.,

$$\hat{\mathbf{u}}_T = \frac{1}{\|\cdot\|}(\mathbf{p}) \tag{7}$$

where $\frac{1}{\|\cdot\|}$ is the normalization operator.

The initial costate (of every single arc) is part of the set optimization variables. It has to be optimized to attain the optimum of the objective function following that will be taken into consideration at attitude and trajectory guidance stages to follow the optimized trajectory and satisfy attitude and angular rate constraints.

2.3.3. State Evolution in Continuous Thrust Arcs

Assuming constant thrust over the whole arc #1, and in linear approximation, the change in the final original state (position, velocity and mass) depends upon the changes in the initial augmented state and duration of that arc as follows,

$$\begin{Bmatrix} \delta\mathbf{r} \\ \delta\mathbf{v} \\ \delta\mathbf{m} \end{Bmatrix}_{F,i} = \begin{bmatrix} \mathbf{\Phi}_{\mathbf{rr}} & \mathbf{\Phi}_{\mathbf{rv}} & \boldsymbol{\varphi}_{\mathbf{r}m} & \mathbf{\Phi}_{\mathbf{rq}} & \mathbf{\Phi}_{\mathbf{rp}} \\ \mathbf{\Phi}_{\mathbf{vr}} & \mathbf{\Phi}_{\mathbf{vv}} & \boldsymbol{\varphi}_{\mathbf{v}m} & \mathbf{\Phi}_{\mathbf{vq}} & \mathbf{\Phi}_{\mathbf{vp}} \\ \boldsymbol{\varphi}_{m\mathbf{r}} & \boldsymbol{\varphi}_{m\mathbf{r}} & \phi_{mm} & \boldsymbol{\varphi}_{m\mathbf{q}} & \boldsymbol{\varphi}_{m\mathbf{p}} \end{bmatrix} \begin{Bmatrix} \delta\mathbf{r} \\ \delta\mathbf{v} \\ \delta m \\ \delta\mathbf{q} \\ \delta\mathbf{p} \end{Bmatrix}_{0,i} + \delta(\Delta t)_i \begin{Bmatrix} \mathbf{v}_{F,i} \\ \mathbf{a}_{F,i} \\ -\dfrac{T}{g_0 I_{SP}} \end{Bmatrix} \tag{8}$$

where the $\mathbf{\Phi}$'s are blocks of the transition matrix from the initial to the final time of the arc, $\mathbf{v}_{F,i}$ and $\mathbf{a}_{F,i}$ are the velocity and acceleration at arc end, and $\frac{T}{g_0 I_{SP}}$ is the mass rate of the propulsion system. Numerical integration is the only means that allows us to propagate augmented state and state transition matrix.

In order to facilitate the optimization it is necessary to impose several constraints on the trajectory. In the following paragraph, the constraints that are considered in the EPTOS activity are detailed.

The trajectory must satisfy a number of conditions that can be mathematically translated into constraints. The following table presents the list of constraints retained in the project.

Table 1. Arc constraints

At Arc Start: all of the following constraints
The vector of velocity adjoints has unit module (1 equality constraint per arc)
Perigee/apogee altitude constraints (up to 2 inequality constraints per arc)
On Arc Duration
Arc duration is a fixed proportion of total transfer duration (1 equality constraints per arc)
Defect Constraints (At Arc Junctions): all of the following constraints
Position defects (3 equality constraints per junction)
Velocity defects (3 equality constraints per junction)

Mass defect (1 equality constraint per junction)
Time defect (1 equality constraint per junction)
At Transfer Departure
Launch state (position, velocity and mass) and time are prescribed (8 equality constraints)
At Transfer Arrival
Prescribed target orbit (six equality constraints) (Mandatory)

The optimal thrust direction is insensitive to scale changes. Unit norm is imposed on the primer vector $\mathbf{p}_{0,i}$ at the beginning of every trajectory arc to prevent ambiguities in the optimisation as well as to avoid numerical problems if these optimisation variables drift along the optimisation.

- Constraint at departure from the delivery orbit of a launcher.

The vector of velocity adjoints has unit module, to prevent ambiguities in the optimization in order to avoid numerical problems if these optimisation variables drift along the optimization. Also constraint on the perigee/apogee are imposed, where $h_{\pi,T}$ and $h_{\alpha,T}$ are the bounds of perigee and apogee altitudes, and h_{π} and h_{α} are the osculating perigee and apogee altitudes, such as:

$$g_{h,\pi} = h_{\pi} - h_{\pi,T} \geq 0 \qquad (9)$$
$$g_{h,\alpha} = h_{\alpha} - h_{\alpha,T} \leq 0$$

- Constraint at arrival in the target orbit.

The target orbit can be specified by a desired point in the state space (of position and velocity), but there is a manifold of Cartesian states that correspond to the same orbit. It is therefore convenient to relax one degree of freedom, the spacecraft anomaly. The approach here is to add one ballistic (no thrust) arc after transfer end, its duration being adjusted to get the specified state, and such a duration becomes one additional optimization variable.

For the case of a fully described final orbit, the constraint is evaluated as

$$\mathbf{g}_{FO} = \begin{Bmatrix} \mathbf{r}_{F,FF} \\ \mathbf{v}_{F,FF} \end{Bmatrix} - \begin{Bmatrix} \mathbf{r}_{TO} \\ \mathbf{v}_{TO} \end{Bmatrix} = \mathbf{0} \in \Re^{6} \qquad (10)$$

where $\mathbf{r}_{TO}$ and $\mathbf{v}_{TO}$ are the position and velocity of a point in the target orbit, and $\mathbf{r}_{F,FF}$ and $\mathbf{v}_{F,FF}$ are the final position and velocity in a ballistic arc of duration $(\Delta t)_{FF}$ and whose initial condition is given by the final time, position, velocity and mass of the transfer (i.e., end condition of the last transfer arc).
When the final orbit is considered with free ascending node and argument of perigee, the constraint definition is described by eq. 2 with the note that $\mathbf{r}_{TO}$ and $\mathbf{v}_{TO}$ are not fixed but calculated as:

$$\mathbf{r}_{TO} = \mathbf{T}\mathbf{r}_{TO,REF}$$
$$\mathbf{v}_{TO} = \mathbf{T}\mathbf{v}_{TO,REF} \qquad (11)$$

$\mathbf{T}$ being a rotation matrix that introduces the effect of the changes in the angles.

- Constraints on Final Geographical Longitude

The constraint on final geographic longitude makes sense only in transfers to the geostationary orbit. The constraint can be formulated as

$$g_{\Lambda} = \mathrm{mod}(\, RA_F - GHA(t_F) - \Lambda_T + \pi \,,\, 2\pi\,) - \pi \in [-\pi, +\pi] \tag{12}$$

where RA_F is the final right ascension

$$RA_F = atan2\,(y_F, x_F) \tag{13}$$

being x_F and y_F the x and y coordinates at transfer end; $GHA(t_F)$ is Greenwich hour angle, which can be computed with a Newcomb's type formula, such as

$$GHA(t) = a + b(t - t_{REF}) + c(t - t_{REF})^2 \tag{14}$$

and Λ_T is the target geographic longitude.

- Constraints on arc duration

There are two constraints that apply to the duration of every trajectory arc in transfers with continuous thrust on. First, an arc duration cannot be negative, and this is an inequality constraint

$$g_{\Delta t,i,1} = \Delta t_{ON,i} \geq 0,\; i \in \{1, 2, \ldots, N_{ARCS}\} \tag{15}$$

where $\Delta t_{ON,i}$ is the duration of the i-th arc.
Second, every arc duration must be a fixed proportion of the total transfer duration to prevent ambiguities and the optimization variables to wander without an actual change of the transfer. This is an equality constraint

$$g_{\Delta t,i,2} = \Delta t_{ON,i} - k_{\Delta t,i}\,\Delta t_{TRANSFER} = 0,\; i \in \{1, 2, \ldots, N_{ARCS}\} \tag{16}$$

where $k_{\Delta t,i}$ is a positive constant –it does not change along the optimisation-, and $\Delta t_{TRANSFER}$ is the accumulated transfer duration

$$\Delta t_{TRANSFER} = \sum_{k=1}^{k=N_{ARCS}} \Delta t_{ON,k} \tag{17}$$

- Constraints at the beginning of the arcs, the bounds of perigee and apogee altitudes are checked.
- Constraints at the junctions between adjacent arcs, where the continuity in mass, position and velocity is imposed by means of equality constraints.

3. Results

Proving the versatility of the process, the low thrust optimizer has been tested for a transfer from a GTO with an inclination of 7 degrees, 11 hours osculating period to an inclined GSO with 56 degrees inclination.

Figure 2 shows the optimal trajectory obtained after the optimization**.** The gradual increase in inclination depicted here **Fehler! Verweisquelle konnte nicht gefunden werden.**can be confirmed below in Figure 3. The variation of the main orbital parameters during the transfer can be seen in Figure 3**Fehler! Verweisquelle konnte nicht gefunden werden.**: semi-major axis, inclination and RAAN.
The optimal thrust law obtained for this transfer is shown in Figure 4, divided by its components. The resulting mass profile can be seen in Figure 4**Fehler! Verweisquelle konnte nicht gefunden werden.**. As this result aims for a minimum time optimization, no coast arcs appear and consequently mass expenditure is constant throughout the entire transfer.

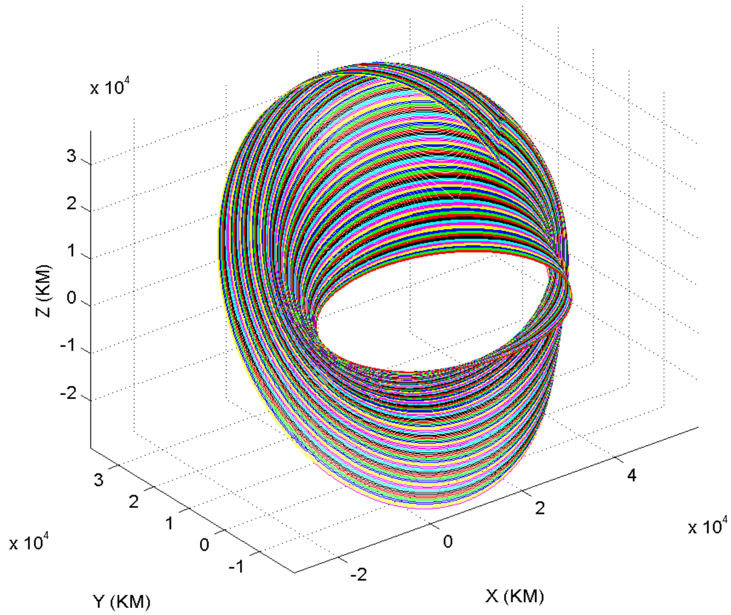

Figure 2: 3D Trajectory

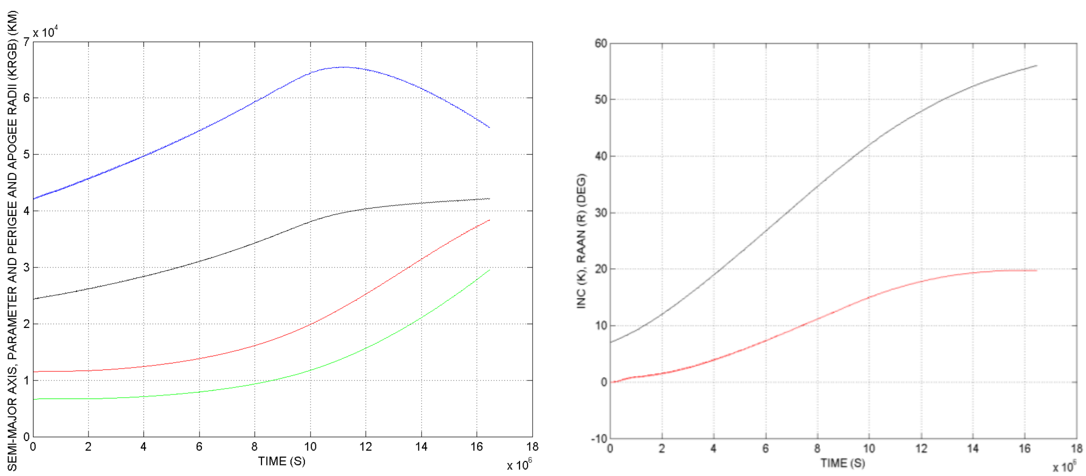

Figure 3: Orbital evolution of SC height (left), inclination and RAAN (right)

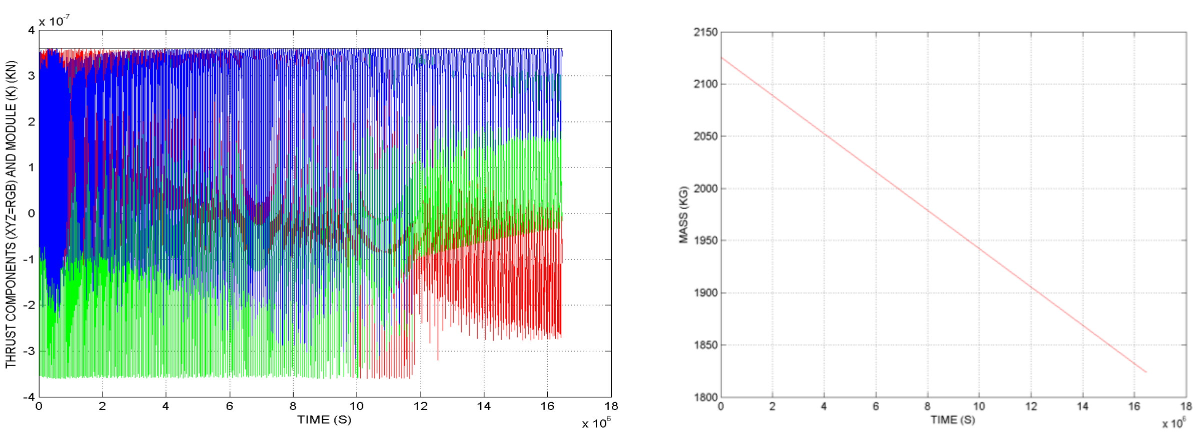

Figure 4: Optimal thrust components (left) and mass profile (right)

From the presented results, it can be seen that the optimization method proposed is apt to generate valid minimum time trajectories from GTO to IGSO orbits. The defect between arcs is reduced to the order of mm, mm/s and milligrams thus accurately linking the arcs to generate a flyable trajectory.

The method developed is a robust optimizer, capable of providing the most efficient transfer available for a given target (not only IGSO but any other type of elliptical orbits) either in terms of time spent or propellant used. This is of paramount importance to unfold the potential of electric propulsion, enabling mission analysts to accurately evaluate the advantages of this type of engine as well as providing a tool for operators to prepare optimal transfers with ease.

4. References

[1] Morley, T. and Budnik, F. "Effects on Spacecraft Radiometric Data at Superior Solar Conjunction." Proceedings 20th International Symposium on Space Flight Dynamics – 20th ISSFD. Annapolis, MD, USA, 2007.

[2] Radice, G. "Deployment Considerations for Spacecraft Formation at Sun-Earth L2 Point." Nonlinear Dynamics and Systems Theory, Vol. 6, No. 4, pp. 401-411, 2006.

[3] J. Gil-Fernandez, L. Tarabini, M. Graziano. "Very low-thrust optimal transfers from GTO to GEO (AAS 06-133)". Advances in the Astronautical Sciences, Vol. 124. Univelt Publishers. Proceedings of the AAS/AIAA Space Flight Mechanics Meeting, Tampa, Florida.

[4] S. Geffroy, R. Epenoy, "Optimal low-thrust transfers with constraints – generalization of averaging techniques", Acta Astronautica, Vol. 41, No 3, 1997

[5] Gil-Fernandez, J., Tarabini, L., Graziano, M., and Gomez-Tierno, M. A. "Optimization and Guidance of Very Low-Thrust Transfers to Geostationary Orbit," Journal of the British Interplanetary Society, Vol. 61, No. 4, April 2008, pp. 139-145.

[6] Lappas, V., Mengali, G., Quarta, A., Gil-Fernandez, J. et al., "Practical Systems Design for an Earth-Magnetotail-Monitoring Solar Sail Mission", Journal of Spacecraft and Rockets, Vol. 46, No. 2, March–April 2009, pp. 381-393

[7] J. T. Betts, "Very low-thrust trajectory optimization using a direct SQP method" Journal of Computational and Applied Mathematics 120, pp. 27-40, 2000

[8] Long, A.C. et al. (Ed.), Goddard Trajectory Determination System (GTDS) – Mathematical Theory – Revision 1, NASA CSC/TR-89/6001, Goddard SFC, 1989.