LUNI-SOLAR PERTURBATIONS FOR MISSION DESIGN IN HIGHLY ELLIPTICAL ORBTS

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Abstract: *This paper presents a method for designing perturbation-enhanced trajectory for re-entry or graveyard disposal. The analysis of the manoeuvre in the phase space allows characterising the long-term evolution of the orbit under the effects of natural perturbations. Then, maps are produced to characterise the stability of several initial conditions in the phase space considering a more detailed model of the dynamics. In this case, single and averaged semi-analytical techniques are used for reducing the computational time, still guarantying a good accuracy. The method applicability is shown for the design of the end-of-life of INTEGRAL and XMM-Newton missions. Previous results in the framework of the study of these missions are here used to explain the general framework of the proposed method.*

Keywords: *Luni-solar perturbation, third-body perturbation, semi-analytical techniques, end-of-life disposal, graveyard orbit, highly-elliptical orbits, long-term propagation.*

1. Introduction

Highly Elliptical Orbits (HEOs) about the Earth are often selected for astronomy missions, such as INTEGRAL [1] and XMM-Newton [2], as well as for Earth missions, such as Molniya or Tundra orbits, as they offer vantage point for the observation of the Earth. In 2018 the Proba-3 satellites will be injected into a HEO to demonstrate formation flying in the context of a large-scale science mission. HEOs guarantees spending most of the time at an altitude outside the Earth's radiation belt; therefore, long periods of uninterrupted scientific observation are possible. Geo-synchronicity is often opted to meet coverage requirements, enhanced at the apogee, and optimise the ground station down-link. If the inclination is properly selected, HEO can minimise the duration of the motion inside the eclipses.

This paper analyses the long-term evolution of spacecraft in HEOs. The dynamics of HEOs with high apogee altitude is mainly influenced by the effect of third body perturbations due to the Moon and the Sun, which induces long-term variations in the eccentricity and the inclination, corresponding to large fluctuations of the orbit perigee. The single and double averaged disturbing potential due to luni-solar perturbations and zonal harmonics of the Earth gravity field are used to study their interaction [3]. The long-term evolution of HEOs is characterised in the phase space of eccentricity, inclination and anomaly of the perigee with respect to the Earth-Moon plane. A method is presented to compute the optimal manoeuvre to perform end-of-life re-entry or transfer into a stable elliptical orbit. The Δv manoeuvre is computed in the eccentricity-inclination-anomaly-of-perigee map, first introduced by Kozai [4]. Given the available delta-v on-board, the reachable space of orbital elements can also be identified. Through these maps, conditions for quasi-frozen, or long-lived libration orbits are identified.

In addition, to allow meeting specific mission constraints, stable conditions for quasi-frozen orbits can be selected as graveyard orbits for the end-of-life of HEO missions, such as the XMM-

Newton mission. On the opposite side, unstable conditions can be exploited to target an Earth re-entry at the end-of-mission, such as the INTEGRAL mission.
The paper is organised as follows, Section 2 summarises the mathematical model used for the study of the orbit evolution: the single and the double averaged dynamics to study the effect of the third body in orbit for with the ration between the semi-major axis and the distance to the small body is small enough. By using the devices model, the analysis of the dynamical system can be performed in the phase space, first using Kozai map, then producing long-term stability maps via numerical integration; the proposed method for perturbation enhanced mission design is shown in Section 3. Finally numerical global optimisation is employed for designing the manoeuvre in the phase space. As example the results of the design of the INTEGRAL [5] and XMM-Newton [3] mission are presented as example of the proposed method.

2. Third body perturbation model in single and double averaged dynamics

In this section the model of third body perturbation used in this work is summarised. The disturbing potential due to the third body perturbation is [6]:

$$R_{3B}(r,r') = \frac{\mu'}{r'}\left(\left(1-2\frac{r}{r'}\cos\psi+\left(\frac{r}{r'}\right)^2\right)^{-1/2} - \frac{r}{r'}\cos\psi\right) \tag{1}$$

where μ' is the gravitational coefficient of the third body, r and r' are the magnitude of the position vector $\mathbf{r}$ and $\mathbf{r}'$ of the satellite and the third body with respect to the central planet, respectively, while their orientation is expressed by the angle ψ between $\mathbf{r}$ and $\mathbf{r}'$. Following the approach by Kauffman and Dasenbrock [7], the disturbing potential can be expressed as function of the spacecraft's orbital elements, choosing as angular variable the eccentric anomaly *E*, the ratio between the orbit semi-major axis and the distance to the third body r' on its circular orbit $\delta = a/r'$ and the orientation of the orbit eccentricity vector with respect to the third body:

$$A = \hat{\mathbf{P}} \cdot \hat{\mathbf{r}}'$$
$$B = \hat{\mathbf{Q}} \cdot \hat{\mathbf{r}}'$$

where the eccentricity unit vector $\hat{\mathbf{P}}$, the semilatus rectum unit vector $\hat{\mathbf{Q}}$ and the unit vector to the third body $\hat{\mathbf{r}}'$ are expressed with respect to the equatorial inertial system, through the composition of rotations [3]. $A(\Omega,i,\omega,\Omega',i',\omega'+f')$ and $B(\Omega,i,\omega,\Omega',i',\omega'+f')$ can be expressed as function of the orbital elements [8] and the variables Ω', ω', i' and f', which are respectively the anomaly of the ascending node, the anomaly of the perigee, the inclination and the true anomaly of the perturbing body on its orbit (described with respect to the Earth-centred equatorial inertial system). Under the assumption that the parameter δ is small (i.e., the spacecraft is far enough from the perturbing body), Eq. (1) can be rewritten as a Taylor series in δ. Then, the average operation in mean anomaly can be performed, assuming that the orbital elements of the spacecraft *a*, *e*, *i*, Ω and ω are constant over one orbit revolution, to obtain the *single* averaged potential of the third body perturbation:

$$\bar{R}_{3B}(r,r') = \frac{\mu'}{r'}\sum_{k=2}^{\infty}\delta^k \bar{F}_k(A,B,e) \tag{2}$$

Under the further assumption that the orbital elements do not change significantly during a full revolution of the perturbing body (e.g., Moon or Sun) around the central body (i.e., Earth), the variation of the orbit over time can be approximately described through the disturbing potential *double* averaged over one orbit evolution of the s/c and over one orbital revolution of the perturbing body around the Earth:

$$\bar{\bar{R}}_{3B}\left(r,r'\right)=\frac{\mu'}{r'}\sum_{k=2}^{\infty}\delta^{k}\bar{\bar{F}}_{k}\left(e,i,\Delta\Omega,\omega,i'\right) \tag{3}$$

where $\Delta\Omega=\Omega-\Omega'$. In [3] we decided to express the double-averaged potential with respect to the Keplerian elements described in the Earth's centred equatorial reference system as this allows embedding the ephemerides of the Moon and the Sun, avoiding the simplification that Moon and Sun's orbit are on the same plane.

Now that the single averaged and the double averaged disturbing potential have been defined in Eq. (2) and Eq. (3), they can be now used for predicting the orbit evolution by computing their particle derivatives with respect to the orbital elements and inserting them in the Lagrange equations [9] to compute the single (using Eq. (2)) and double (using Eq. (3)) variation of the orbital elements. It is important to remind that the modelling of the long-term orbit evolution through Eq. (2) and Eq. (3) represents a good approximation as long as the parameter of the Taylor series $\delta=a/r'$ remain small enough. This is the case of Highly Elliptical orbits such as the one of INTEGRAL and XMM-Newton.

3. Luni-solar perturbation enhanced mission design

The analysis of the long-term orbit evolution under the effects of the third body perturbation due to the Moon and the Sun coupled with the effects of the Earth's oblateness were studied in [3, 4, 10-12].

3.1. Phase space analysis

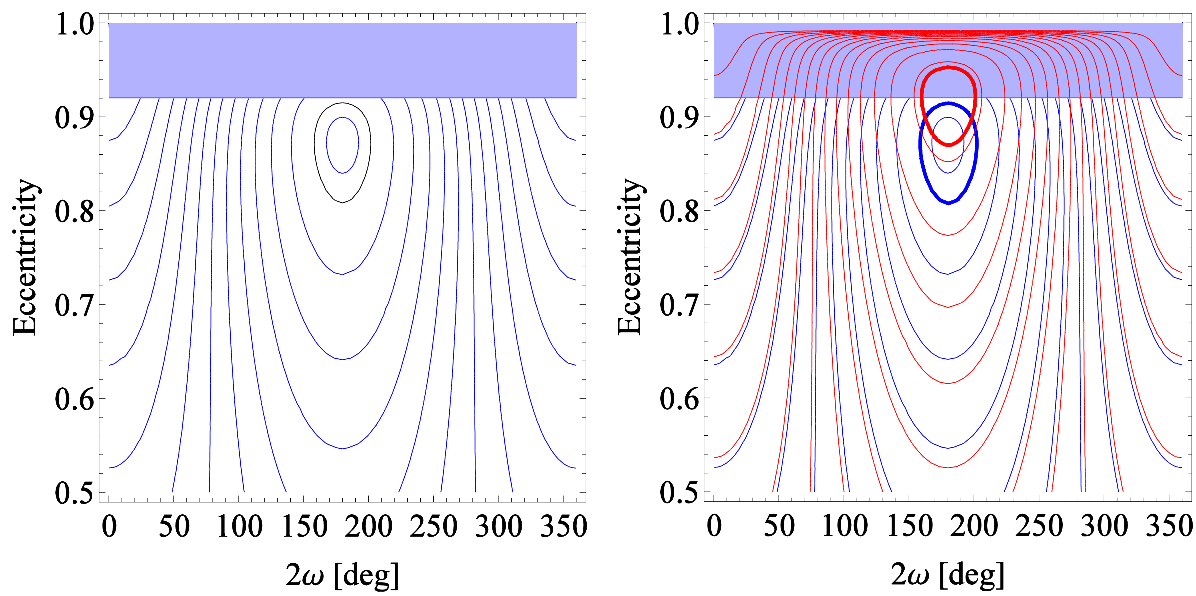
A simplified phase-space analysis can be performed, considering only the third body effect of the Moon [3]. If we change the reference system to Earth-centred equatorial (the convention chosen here) to Earth-centred but with the *x-y* plane corresponding to the Earth-Moon plane, we fall into the convention used by El'yasberg [13] and Costa and Prado [14]. The *x*-axis of this reference system is in the direction of the perturbing body on its orbit and the *z*-axis in the direction of the perturbing body angular momentum while the *y*-axis that completes the reference system. Note that this exactly the same convention used in the modelling of the Circular Restricted Three-Body Problem for the synodic system [15]. By doing this change of coordinates the double averaged potential in Eq. (3) loses the dependence on the anomaly of the ascending node; so, for example, the terms 2 of the summation Eq. (3) becomes:

$$\bar{\bar{F}}_{3\mathrm{Bsys},2}=\frac{1}{32}\left(\left(2+3e_{3\mathrm{Bsys}}^{2}\right)\left(1+3\cos\left(2i_{3\mathrm{Bsys}}\right)\right)+30e_{3\mathrm{Bsys}}^{2}\cos\left(2\omega_{3\mathrm{Bsys}}\right)\sin^{2}i_{3\mathrm{Bsys}}\right) \tag{4}$$

that is equivalent to the expression in [13, 14]. Kozai [4] performed an analytical analysis of the dynamics under the effects of potential in Eq. (4). Indeed, the change of reference system allows obtaining a time-independent Hamiltonian, similarly to the CRTBP and, thus, identifying the constant quantity:

$$\Theta=\left(1-e_{3\mathrm{Bsys}}^{2}\right)\cos i_{3\mathrm{Bsys}} \tag{5}$$

Given a certain value of the semi-major axis, the long-term evolution of under the effect of the Moon can be described by the characteristic lines in the phase space of $e_{3Bsys} - \omega_{3Bsys}$: given the initial condition in e_{3Bsys} and ω_{3Bsys} the following evolution is identified by a line in the $e_{3Bsys} - \omega_{3Bsys}$ phase space (see Figure 1), while the evolution of the inclination can be derived from Eq. (5). As δ increases or Θ decreases, the phase space representation translates at higher eccentricity as shown in red in Figure 1b [16].



a) *b)*

Figure 1. Eccentricity- anomaly of the perigee phase-space evolution under third-body perturbation. (a) Semi-major axis of 87736.34 km and b) change of phase space line for different values of Θ.

The phase space analysis by Kozai highlights the existence of frozen orbits in e_{3Bsys}, ω_{3Bsys} and i_{3Bsys} for which the long-term double averaged variation of the orbital elements under the effects of the third body perturbation is zero. With the semi-major axis considered, these orbits are characterised by $2\omega_{3Bsys} = 180$ degrees, which means that the orbit perigee on top the Earth-Moon plane. For initial conditions close the frozen condition the long-term evolution is libration in ω_{3Bsys} and therefore also libration and limited in e_{3Bsys} and i_{3Bsys}. The solutions departing from $2\omega_{3Bsys} = 90$ or $2\omega_{3Bsys} = 270$ degrees are the one that achieve the highest change in eccentricity. The solutions in the domain $2\omega_{3Bsys} < 90$ degrees or $2\omega_{3Bsys} > 270$ are rotational in ω_{3Bsys}.

3.2. Stability maps

The analysis in Section 3.2 is useful to appreciate the long-term evolution of HEOs, however is limited by several approximations:

- It considers only the effect of one perturbing body; if we would like to describe the effects of both the Sun and the Moon, the reference system chosen by [4, 13, 14] would not be optimal as it is rotating with one of the two bodies.

- The effects of the Earth's oblateness is neglected
- Only the term 2 of the Taylor expansion of the potential is used.

To relax these assumption, the disturbing potential in the Earth-centred equatorial reference system is a better choice. In this system the ephemerides of the Sun and the Moon in the third body potential can be accurately included and it also facilitates the inclusion of the effect of the potential due to the Earth gravity anomaly [3, 7].

With this more detailed model it is not possible to find close form solutions for the orbit evolution, therefore an alternative numerical approach was proposed by Colombo [3]. The aim of this analysis is to characterise the orbit evolution in the phase space and to identify the stability conditions of a wide set of initial conditions. Maps of maximum eccentricity variations were built as a measure of the orbit stability. Indeed, we can expect from the analysis in Section 3.2 that, the lower is the net change of eccentricity over a long time, the more the actual orbit is close to the theoretical frozen orbit identified by Kozai in the simplified case. Conditions with high net change in eccentricity represents solutions close to the theoretical $2\omega_{3Bsys} = 90$ or $2\omega_{3Bsys} = 270$ degrees separatrix line.

To this purpose, a grid was built in the domain of inclination, eccentricity and anomaly of the perigee. Note that, inclination and anomaly of the perigee are described with respect to the Moon plane reference system. Each initial condition of the grid was propagated backward and forward in time over $\Delta t = 30$ years with PlanODyn [3]. For each initial condition the change between the minimum and the maximum eccentricity attained during the period analysed is stored:

$$\begin{aligned}
&\Delta e = e_{\max} - e_{\min} \\
&\text{with} \quad e_{\max} = \max_t e(t) \qquad t \in [-\Delta t \quad +\Delta t] \\
&\qquad\quad\ e_{\min} = \min_t e(t) \qquad t \in [-\Delta t \quad +\Delta t]
\end{aligned}$$

Figure 2 shows the Δe map for a set semi-major axis of 67045.39 km. Note that all initial conditions are integrated with the same stating date, which corresponds to a given initial condition of the Sun and the Moon with respect to the Earth. A different starting date would give a different net change of eccentricity for each initial condition in the $e_{3Bsys} - \omega_{3Bsys}$ plane but would not change the characteristics of the solutions. Comparing Figure 2 with Figure 1 we can recognise the existence of a quasi-frozen solution at $2\omega_{3Bsys} = 180$ degrees and high eccentricity. Also the separatrix lines at $2\omega_{3Bsys} = 90$ or $2\omega_{3Bsys} = 270$ degrees are still present (yellow strips in Figure 2). The maps in Figure 2, however, shows a richer dynamics than the simplified approach in Figure 1) due to the interaction between the third-bod effects of the Moon with the third-body effect of the Sun and the Earth's oblateness.

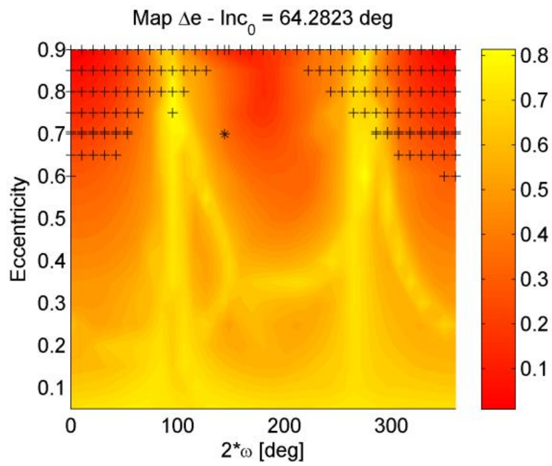


Figure 2. Luni-solar + zonal Δe maps for semi-major axis of 67045.39 km and initial inclination with respect to the orbiting plane of the Moon of 64.2 degrees.

4. Perturbation enhanced mission design

The phase-space analysis in Section 3.2 and [16] and the stability maps proposed in [3] and summarised in Section 3.2 suggest a novel method for the design of perturbation-enhanced manoeuvres which is based on the design of the manoeuvre in the phase space.

In particular, perturbation-enhanced transfer to achieve re-entry can be achieved targeting trajectories in the phase space with a high variation of Δe such the spacecraft eccentricity is increased (at constant semi-major axis) up to the critical eccentricity for Earth e-entry:

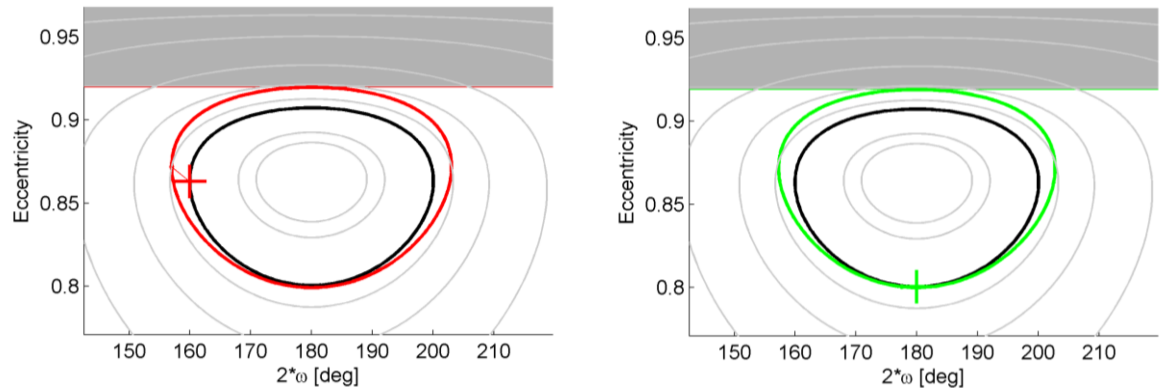
$$e_{\text{crit}} = 1 - \left(R_{\text{Earth}} + h_{p,\,\text{drag}}\right) / a$$

where R_{Earth} is the radius of the Earth and $h_{p,\,\text{drag}}$ the target orbit perigee, which needs to be selected well inside the Earth atmosphere to ensure a safe re-entry [5, 17].

On the opposite side, perturbation-enhanced transfer into a graveyard orbit can be obtained by targeting low variation of Δe in the $e_{3\text{Bsys}} - \omega_{3\text{Bsys}}$ phase space as this represents quasi-stable orbits.

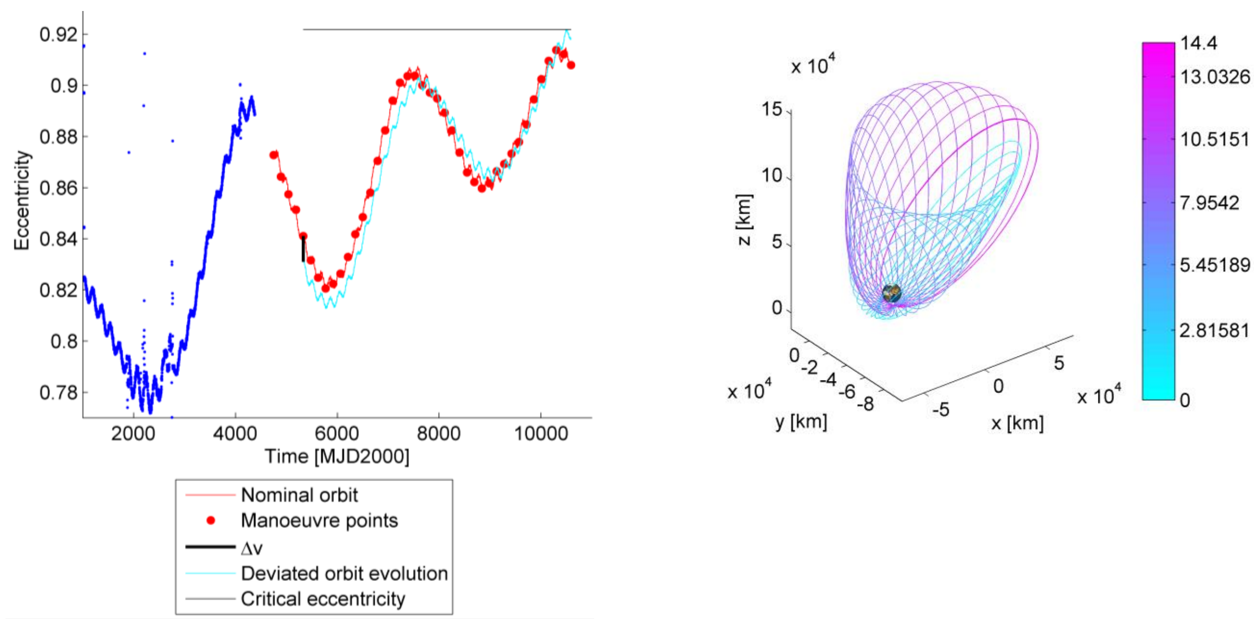
4.1. Design of the disposal of INTEGRAL mission

The design approach proposed in Section 4 was applied to the design of the end-of-life of the INTEGRAL mission in [16] and [5]. In the simplified phase space in Section 3.1 targeting a re-entry with the minimum Δv means transferring to a phase space line which is tangent to the critical eccentricity. This is shown in Figure 3. Depending on the time in which the manoeuvre is given (refer to Figure 3a or b), the spacecraft is on a different position along the nominal trajectory evolution (black line). This means that a different amount of Δv is required depending on the time it is given. It was demonstrated via numerical optimisation in [5] that the minimum required manoeuvre is needed if the injection into the disposal trajectory is given in correspondence of the point along the natural evolution when the eccentricity is at its minimum. The optimised results in the case of INTEGRAL mission [5] is shown in Figure 4.



a) *b)*

Figure 3. Phase-space trajectory to decrease the perigee to 600 km. The cross indicates where the Δv manoeuvre is applied.

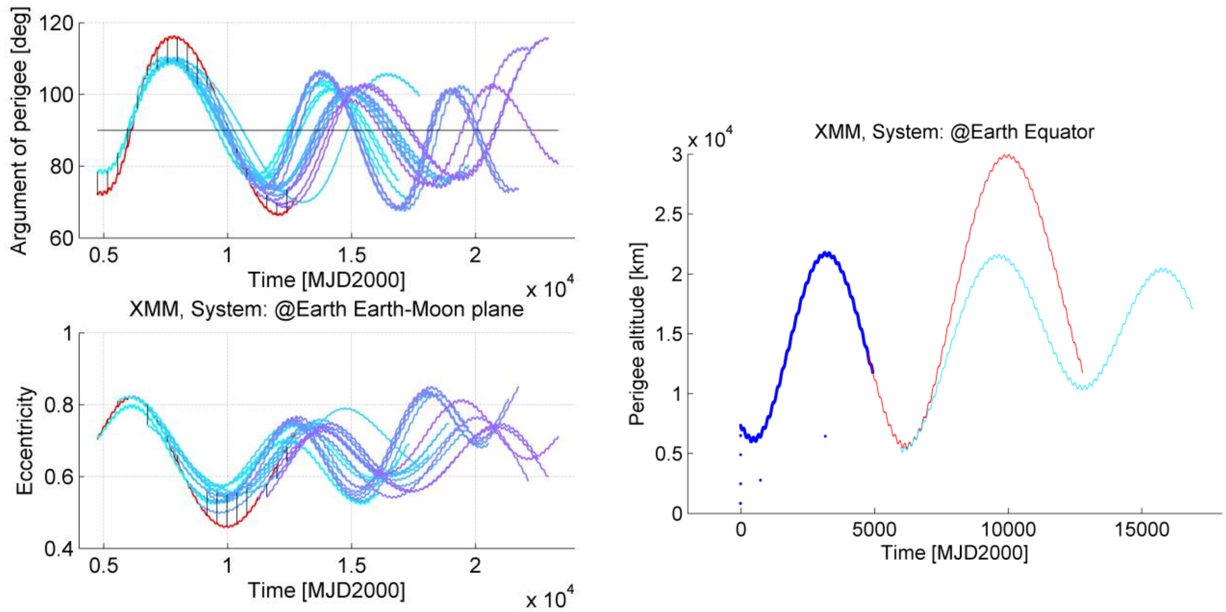


a) *b)*

Figure 4. INTEGRAL re-entry: disposal manoeuvre in 2014, re-entry in 2028.

4.2. Design of the disposal of Newton-XMM mission

The design approach proposed in Section 4 was applied to the design of the end-of-life of the XMM mission in [3]. A graveyard orbit can be identified as a long-term stable orbit, where the evolution of the orbital elements due to natural perturbation is limited. Also in this case a single manoeuvre is given, performed during the natural orbit evolution of the spacecraft under the effect of perturbations. A graveyard orbit is designed imposing that after the manoeuvre, the variation of the eccentricity in time stays limited. This strategy aims at reaching the centre of libration in the phase space (see example phase space in Figure 1). However, due to the limitation in the maximum available Δv on-board the spacecraft, the quasi-stable orbit cannot be reached, but only an orbit that is more stable than the nominal one. The stability map in the case of the XMM-Newton mission is shown in Figure 2a. The black star represent the orbit of XMM spacecraft and the strategy aims at targeting the red area that represents the frozen solutions. A numerical optimisation was performed to determine for each possible available date the required Δv for transfer into a stable orbit, the results are presented in Figure 5a, which shows for each starting date the following evolution of eccentricity and anomaly of the perigee in the timeframe analysed $\Delta t = 30$ years. The magnitude of the manoeuvre is always close to the upper bound of available Δv as is clear that a higher Δv would allow reaching a more stable orbit. However, the new graveyard orbit reduces at least the oscillations in eccentricity. As an example, a disposal trajectory, whose manoeuvre is performed on 20/04/2016, is shown in Figure 5b.



a) *b)*

Figure 5. XMM graveyard disposal manoeuvre in 2016: Evolution of the orbit perigee. Red: nominal predicted orbit. The light blue line represents the evolution after the disposal manoeuvre is performed for the following 30 years

5. Conclusions

The paper present a method for the design of perturbation-enhanced trajectories. The analysis of the manoeuvre in the phase space allows characterising the long-term evolution of the orbit under the effects of natural perturbations. Then, maps are produced to characterise the stability of several initial conditions in the phase space considering a more detailed model of the dynamics. In this case, single and averaged semi-analytical techniques are used for reducing the computational time, still guarantying a good accuracy. The method applicability is shown for the design of the end-of-life of INTEGRAL and XMM-Newton. Previous results in the framework of the study of these missions are here used to explain the general framework of the proposed method. Future work will be devoted to automatise the proposed approach exploiting hybrid optimisation techniques.

6. References

[1] Eismont, N. A., Ditrikh, A. V., Janin, G., Karrask, V. K., Clausen, K., Medvedchikov, A. I., Kulik, S. V., Vtorushin, N. A. and Yakushin, N. I., "Orbit Design for Launching

INTEGRAL on the Proton/Block-Dm Launcher," *Astronomy and Astrophysics*, Vol. 411, No. 1, 2003, pp. L37-L41.
[2] ESA, "Xmm-Newton Users Handbook Issue 2.11," 2013
[3] Colombo, C., "Long-Term Evolution of Highly-Elliptical Orbits: Luni-Solar Perturbation Effects for Stability and Re-Entry," *Proceedings of the 25th AAS/AIAA Space Flight Mechanics Meeting*, Williamsburg, Virginia, 2015.
[4] Kozai, Y., "Secular Perturbations of Asteroids with High Inclination and Eccentricity," *Astronomical Journal*, Vol. 67, 1962, pp. 591-598. doi: 10.1086/108790
[5] Colombo, C., Letizia, F., Alessi, E. M. and Landgraf, M., "End-of-Life Earth Re-Entry for Highly Elliptical Orbits: The INTEGRAL Mission," *Proceedings of the 24th AAS/AIAA Space Flight Mechanics Meeting*, Santa Fe, New Mexico, 2014, AAS 14-325.
[6] Murray, C. D. and Dermott, S. F., *Solar System Dynamics*, Cambridge University Press, New York, 1999.
[7] Kaufman, B. and Dasenbrock, R., "Higher Order Theory for Long-Term Behaviour of Earth and Lunar Orbiters," NRL Repot 7527, 1972.
[8] Chao-Chun, G. C., *Applied Orbit Perturbation and Maintenance*, El Segundo, California, 2005, p.^pp. 23-29.
[9] Battin, R. H., *An Introduction to the Mathematics and Methods of Astrodynamics, Revised Edition*, Aiaa Educational Series, Reston, 1999.
[10] Lidov, M. L., "The Evolution of Orbits of Artificial Satellites of Planets under the Action of Gravitational Perturbations of External Bodies," *Planetary and Space Science*, Vol. 9, No. 10, 1962, pp. 719-759. doi: 10.1016/0032-0633(62)90129-0
[11] Katz, B., Dong, S. and Malhotra, R., "Long-Term Cycling of Kozai-Lidov Cycles: Extreme Eccentricities and Inclinations Excited by a Distant Eccentric Perturber," *Physical Review Letters*, Vol. 107, No. 18, 2011, p. 181101.
[12] Naoz, S., Farr, W. M., Lithwick, Y., Rasio, F. A. and Teyssandier, J., "Secular Dynamics in Hierarchical Three-Body Systems," *Monthly Notices of the Royal Astronomical Society*, Vol. 431, 2013, pp. 2155-2171. doi: 10.1093mnrasstt302
[13] El'yasberg, P. E., " Introduction to the Theory of Flight of Artificial Earth Satellites, (Translated from Russian)," 1967.
[14] Costa, I. V. d. and Prado, A. F. B. d. A., "Orbital Evolution of a Satellite Perturbed by a Third Body," *Advances in Space Dynamics*, 2000.
[15] Sezebehely, V., *Theory of Orbits in the Restricted Problem of the Three Bodies*, New York, 1967.
[16] Colombo, C., Alessi, E. M. and Landgraf, M., "End-of-Life Disposal of Spacecraft in Highly Elliptical Orbits by Means of Luni-Solar Perturbations and Moon Resonances," *Proceedings of the Sixth European Conference on Space Debris*, ESOC, Darmstadt, Germany, 2013.
[17] Armellin, R., San-Juan, J. F. and Lara, M., "End-of-Life Disposal of High Elliptical Orbit Missions: The Case of INTEGRAL," *Advances in Space Research*, Vol. 56, No. 3, 2015, pp. 479-493. doi: 10.1016/j.asr.2015.03.020