ON SPIN MANOEUVRES WITH A SYMMETRIC SATELLITE

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ABSTRACT

This paper contains some theoretical results and practical formulae to calculate non-ideal spin-
manoeuvres with a symmetric satellite. In a non-
ideal spin manoeuvre the spacecraft is not rotat-
ing uniformly at the start of the manoeuvre and
the thruster used is not a pure tangential or spin
thrustur. Such manoeuvres generate a nutational
motion and an attitude change. These aspects can
be studied as a special case of the theoretical
problem known as "the self-excited rigid body".
The notation is described by Fresnel integrals
from which practical formulae giving the maximum
rotation and its rate of decrease are extracted.
A numerical illustration is given by the spin-up
of Meteosat-2. A closed form result for a small
attitude change caused by a pure spin up thruster
is in the presence of initial nutation and by a
general thruster on a Kovalevski top (Inertia
A, A, A) is mentioned.

Keywords: Spin-up, nutation, self-excited rigid
body, Meteosat, Kovalevski.

NOTATIONS

\( A \): lateral moment of inertia
\( C \): inertia around the nominal spin axis
\( \mathbf{m}^e \): torque components in the principal axis
system
\( n^e \): \( n^e = n^e_1 + n^e_2 \) lateral component of
the torque as a complex number
\( m^c \): spin component of the torque. Throughout
this paper \( m^c \geq 0 \).
\( z \): \( \exp(j \omega T) \) normalised time. Replaces \( t \)
as independent variable
\( \alpha \): \( \alpha = \alpha^e_0, T \) initial value of \( z \) at the start
of the manoeuvres
\( \gamma \): intermediate variable for the
attitude change
\( \omega_0, 2\omega \): Fresnel-integrals
\( \beta \): intermediate variable that locates the
instant of maximal nutation when starting
from a uniform spin
\( \gamma \): nutation angle. Instantaneous angle
between the spin-axis and the angular
momentum. In terms of the components of
the angular velocity \( \omega_0 \), \( \omega_0 = \omega_0^x + j \omega_0^y \)
\( \beta \): nutation angle at \( \omega_0^x \) when starting from
rest
\( \eta \): initial nutation \( \eta \) when present
\( A \): slenderness \( A = A_0 / A \) (oblate body
(spin axis stable), \( A < 1 \) prolate
body (spin axis unstable in the presence

of energy dissipation)

\( \omega_0, \omega, \lambda \): components of the angular velocity in
the principal axis system
\( \alpha^e \): \( \alpha^e = \alpha^e_0 + \alpha^e_0 \) lateral component of the \( \omega -
vector as a complex number
\( \gamma \): \( \gamma = \gamma^e_0 + \gamma^e_0 \) time constant
\( \nu, \nu, \psi \): Euler angles

1. INTRODUCTION

Spin adjustment manoeuvres are quite common activities in spacecraft operations because different
phases in the lifetime of a spin stabilized space-
craft require a different nominal spin rate.
Typical examples are: launcher constraints, deploy-
ment activities and firing of an apogee boost motor.

In an ideal spin manoeuvre one starts from a uni-
formly rotating spacecraft and the spin thruster
gives only a constant spin up or spin down torque.
Under these assumptions the spin manoeuvre causes
no nutation and no attitude change. The calcula-
tion of the burn duration from the initial and
desired spin rate is trivial. In an operational
environment one has often to consider spin
manoeuvres where one or both of these assumptions
are not met. The influence of an initial nutation
must be assessed as well as thruster misalignments.
Some spacecraft have no pure spin thruster. This is
the case with Meteosat-2 for which this study
was undertaken. The spin changes are executed
with thrusters having a more important lateral as
spin-up component.

A reasonable model for non-ideal spin manoeuvres
is provided by the theoretical problem known as
"the self-excited rigid body" (Ref. 1, 2). Self-
excitn means that the torque vector has a constant
direction in a body-fixed reference frame. We
assume that the torque has also a constant magni-
itude which is equivalent with a constant force,
assumption for the thruster. The moments of
inertia of the spacecraft are also treated as
constants, their variation due to out-flowing mass
is neglected. Furthermore we neglect internal
energy dissipation and consider only symmetric
spacecraft.

The first step of the complete solution of the
formulated problem is to obtain expressions for the
instantaneous rotation vector \( \omega(t) \) in a body fixed
frame. Integration of the Euler equations shows
that \( \omega(t) \) can be expressed in terms of the Fresnel-
integrals (Ref. 1, 2). This result is rederived.
in a simple way and permits a detailed discussion of the evolution of the nutation angle. It is shown that for a spin-up from rest the maximum nutational angle equals the angle between the torque direction and the spin-axis. A practical formula for the nutation induced with an arbitrary thruster and starting from a uniform spin is also given.

To complete the solution of the formulated problem one needs the time history of a suitable set of orientation parameters between the body-fixed and inertial reference frame to calculate the attitude change. The corresponding differential equations contain the previous result for pure and their structure depends highly on the choice of the orientation parameters. Ref. 3 reduces the case of a pure spin thruster to a Weber equation in orientation parameters related to the Cayley-Klein parameters. In terms of the rotation matrix one obtains a linear first-order matrix differential equation (Ref. 1) for which a converging recurrence formula is given in Ref. 4. The assumption of a small attitude change does not introduce simplifications in each of these mentioned approaches.

Using the Euler angles the remaining differential equations reduce to one linear differential equation under the assumption that the attitude change is small. Analytical results can then be obtained for a pure spin torque in the presence of initial nutation and for a general torque applied to a Kovalevsky top (inertia’s A, A, A/2) which was spinning uniformly.

2. BASIC EQUATIONS FOR THE NUTATION ANGLE

Consider a symmetric rigid body with inertia A about any axis in the principal plane (x,y) through the center of mass and inertia C around the z-axis. The Z-axis is the third principal axis and the nominal spin-axis. From t = 0 onwards the body is subjected to a constant external torque with components (d,d,d) in the principal axis system (x,y,z).

The time-history of the nutation angle $\theta(t)$ can be calculated from the components of the angular velocity $\omega(t)$. These components are obtained by integrating the classical Euler equations:

$$\begin{align*}
A \omega_1 \times \dot{A} c_1 \omega_1 + \omega_2 & = \omega_1,
A \omega_2 \times \dot{A} c_2 \omega_2 + \omega_3 & = \omega_2,
C \omega_3 \times \dot{C} c_3 \omega_3 & = \omega_3
\end{align*}$$

(1) (2) (3)

with $\omega_i(t) = \omega_i(0)$

Due to the assumed symmetry Eq. 3 is decoupled from Eqs. 1 - 2 and easily integrated:

$$\omega_i(t) = \omega_i(0) + \frac{\tau_i}{A} C_i$$

(4)

So a spin-up always occurs when $\tau_i > 0$. This result is not true for an asymmetrical rigid body. Eq. 4 is now substituted into Eqs. 1 - 2 which are combined into a single linear differential equation in the complex variable $\omega^2 \omega_1 + \omega_2 + \omega_3$

$$\frac{\tau_1}{A} C_{1 x} \omega_1 + \frac{\tau_2}{A} C_{1 y} \omega_2 + \frac{\tau_3}{A} C_{1 z} \omega_3$$

(5)

where $C_{1 x}$ is the slenderness ratio $m_{1 x} m_{2 y} m_{3}$ the equatorial or lateral part of the torque

and $\omega^2 \omega_1 + \omega_1 + \omega_3 = \omega^2$

The general solution of Eq. 5 is:

$$\omega^2(t) = \left\{ \int \frac{e^{ x (x-3)} \frac{\tau_1}{A} C_{1 x} \omega_1 + \frac{\tau_2}{A} C_{1 y} \omega_2 + \frac{\tau_3}{A} C_{1 z} \omega_3}{x} \omega^2 \omega_1 + \omega_2 + \omega_3 \right\}$$

(6)

Note that Eq. 6 has been obtained without any transformation of variables in the original Euler equations. To evaluate the integrals remaining in Eq. 6 we first rewrite it as:

$$\omega^2(t) = \left\{ \frac{e^{ x (x-3)} \frac{\tau_1}{A} C_{1 x} \omega_1 + \frac{\tau_2}{A} C_{1 y} \omega_2 + \frac{\tau_3}{A} C_{1 z} \omega_3}{x} \omega^2 \omega_1 + \omega_2 + \omega_3 \right\}$$

(7)

Eq. 7 is only valid when a spin-up component is present ($\tau_3 \neq 0$). The remaining integral cannot be expressed in terms of elementary functions. It is easily put in the following normalised form

$$\int_{-\infty}^{+\infty} \frac{e^{ x (x-3)} \frac{\tau_1}{A} C_{1 x} \omega_1 + \frac{\tau_2}{A} C_{1 y} \omega_2 + \frac{\tau_3}{A} C_{1 z} \omega_3}{x} \omega^2 \omega_1 + \omega_2 + \omega_3 \right\}$$

(8)

where

$$\int_{-\infty}^{+\infty} \frac{e^{ x (x-3)} \frac{\tau_1}{A} C_{1 x} \omega_1 + \frac{\tau_2}{A} C_{1 y} \omega_2 + \frac{\tau_3}{A} C_{1 z} \omega_3}{x} \omega^2 \omega_1 + \omega_2 + \omega_3 \right\}$$

(9)

are the Fresnel-integrals (Ref. 5). The notations are further simplified by using the time constant $\tau$:

$$\tau = \frac{\pi C}{\tau_3 (x)}$$

(10)

and the non-dimensionless variable $x$

$$x = \frac{\omega^2(t)}{\tau^2} = \frac{\omega^2(t)}{\tau^2}$$

(11)

$$\omega^2(t) = e^{-x (x-3)} \left\{ \omega^2 - e^{x (x-3)} \frac{\tau_1}{A} C_{1 x} \omega_1 + \frac{\tau_2}{A} C_{1 y} \omega_2 + \frac{\tau_3}{A} C_{1 z} \omega_3 \right\}$$

(12)

Now we can use Eqs. 12, 13 to calculate the nutation angle $\theta(t)$. By definition we have:

$$\theta(t) = \frac{1}{\omega^2(t)}$$

(13)

where $1/\omega^2(t)$ stands for the module of the complex number

Using Eqs. 12, 13 we obtain for Eq. 14

$$\theta(t) = \frac{1}{\omega^2(t)}$$

(14)

In the next points the meaning of Eq. 15 will be discussed for some special cases of the initial conditions and direction of the torque vector.

Table 1 summarises the cases to which Eq. 15 can be specialized. The analysis of the evolution of the nutation angle from Eq. 15 will be more complex when the 2 additive terms are present as there is
no simple rule for the module of the sum of two complex numbers.

2.1 Spin-up From Rest with a General Thruster

Case III

Starting from rest the initial conditions are $\dot{\theta}_0 = 0$, $\omega_0 = 0$ or $\omega_0$ and Eq. 15 becomes

$$\dot{\theta}_0 = \frac{m}{2\mu} |P_1(\cos \phi)|$$

The starting value of the nutation angle $\theta_n(0)$ is not given immediately by Eq. 16 as $\theta_n(0) = 0$ makes Eq. 16 undetermined. The series expansion of $C_n(\phi)$ (Ref. 5, 6) gives

$$\lim_{x \to \infty} |\frac{C_n(x)}{x}| = \frac{\omega_0}{x}$$

Therefore

$$\dot{\theta}_0 = \frac{4m}{\mu} C_n(x) |\frac{C_n(x)}{x}|$$

The nutation angle which is not defined before the thruster is on takes at $t = \theta_0$ the value $\theta_0$, $\theta_0$ is the angle between the spin-axis and the torque direction (Fig. 1). This result can also be obtained from Eqs. 1 - 3 when the product terms, which are of second order for small $\theta$, are neglected.

Now Eq. 16 can be rewritten as

$$\dot{\theta}_0 = \frac{\omega_0}{x} |\frac{C_n(x)}{x}|$$

In Fig. 2, the locus of complex numbers $\frac{\theta_0}{x}[C_n(x)]/x$ is represented. Looking how the module $|f(x)|$ changes with time (or $x$) we see that $|f(x)|$ is maximum at $\frac{\theta_0}{x}$ or $\frac{\theta_0}{x}$.

So $\omega_0$ is also the maximal value of the nutation angle during a spin-up from zero. This result gives an upper bound for the nutation angle when the nominal spin-up is with a tangential thruster ($m \times \omega_0$, $m \times \omega_0$) and the influence of misalignments $m \times \omega_0$ is evaluated. So, the maximal nutation due to misalignments equals the maximal nutation angle. Initially the decrease of the nutation angle is monotonic: For $x < 0$ or $x > 0$ $\theta_0(\theta_0) > \theta_0$ and for $x = 0$ $\theta_0(\theta_0) > \theta_0$. For larger values of $x$ we see that $\theta_0$ spirals to zero with curls of decreasing "amplitude" and centered on the first bisector. The evolution of the nutation angle can be described by an average value $\theta_0(\theta_0)$ taken on the first bisector and superimposed deviations $\delta \theta_0(\theta_0)$, both $\theta_0(\theta_0)$ and $\delta \theta_0(\theta_0)$ tend to zero as time goes on.

Using the property that

$$\lim_{x \to \infty} \frac{\omega_0}{x} |\frac{C_n(x)}{x}| = \frac{\omega_0}{x}$$

one obtains easily the following approximate expressions:

$$\dot{\theta}_0 = \frac{\omega_0}{x}$$

The decrease of $\theta_0(\theta_0)$ is faster than the decrease of $\theta_0(\theta_0)$.

2.2 Spin-up From a Uniform Spin with a General Thruster

Case IV

At $t = \infty$, the satellite is spinning uniformly $m \times \omega_0$ and a thruster with equatorial torque $m \times \omega_0$ and spin-up torque $\omega_0$ is activated. Eq. 15 with $\omega_0 = 0$ becomes

$$\dot{\theta}_0 = \frac{\omega_0}{x} |\frac{C_n(x)}{x}|$$

where $\omega_0 = \frac{C_n(x)}{x}$

Eq. 21 shows that the maximal value of the nutation caused by this maneuver is less than the residual nutation level after a spin-up from zero to $\omega_0$. An approximate expression for the maximal nutation is derived in Ref. 6 by replacing $\theta_0(\theta_0)$ by its osculating circle at $x = \omega_0$, one obtains

$$\frac{\omega_0}{x} |\frac{C_n(x)}{x}| = \frac{\omega_0}{x} - \frac{\omega_0}{x} = \frac{\omega_0}{x}$$

where $\theta_0$ is the solution of

$$\frac{\omega_0}{x} - \frac{\omega_0}{x} = \frac{\omega_0}{x}$$

For $x > 2$, this result can be replaced by

$$\frac{\omega_0}{x} = \frac{\omega_0}{x}$$

Eq. 24 gives directly $\theta_0(\theta_0)$ as a function of $\omega_0$ and $\omega_0$, without solving the transcendental equation 23. Table II compares the results of Eqs. 22 - 23 versus Eq. 24.

<table>
<thead>
<tr>
<th>$\omega_0$</th>
<th>$\omega_0$</th>
<th>$\omega_0$</th>
<th>$\omega_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
</tr>
<tr>
<td>2</td>
<td>0.63</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td>3</td>
<td>0.64</td>
<td>0.64</td>
<td>0.64</td>
</tr>
<tr>
<td>4</td>
<td>0.67</td>
<td>0.67</td>
<td>0.67</td>
</tr>
<tr>
<td>5</td>
<td>0.68</td>
<td>0.68</td>
<td>0.68</td>
</tr>
</tbody>
</table>

**TABLE II**

From Eq. 21 we can also derive a decrease of the average nutation level $\omega_0(\theta_0)$ from the limiting value

$$\dot{\theta}_0 = \frac{\omega_0}{x} |\frac{C_n(x)}{x}|$$

This averaged decrease is also inversely proportional to the time.

2.3 A Comment on the General Formula

CASE V

Equation 15 gives the evolution of the nutation angle, when before the activation of the general thrust ($m \times \omega_0$, $m \times \omega_0$) the satellite is spinning with an initial nutation angle $\theta_0(\theta_0)$. Numerical applications of Eq. 15 are straightforward when a table of the Fresnel-integrals is available.
Using Eq. 19, $\mathbf{Q}(1) = x$ becomes constant as time goes on:

$$
\mathbf{Q}(1) = \mathbf{x} \quad \text{as} \quad t \to \infty
$$

where the $x$-axis is chosen coinciding with $\mathbf{m}_x$. When Eq. 26 is maximal the initial phasing, $\mathbf{u}_0$, was the worst possible. Eq. 26 can be written

$$
\left( \frac{\partial}{\partial t} \right) \mathbf{Q}(1) = \mathbf{u}_0 \times \mathbf{Q}(1)
$$

and as $\mathbf{Q}(1)$ tends to circle around $(1, 1, 1)$, $\mathbf{Q}(1)$ becomes perpendicular to $\mathbf{Q}(1)$ as $t$ increases. To make the two terms of Eq. 27 co-linear $\mathbf{u}_0$ becomes perpendicular to $\mathbf{Q}(1)$ which means that the thrust direction is perpendicular to $\mathbf{m}_x^*$. Eq. 27 becomes

$$
\left( \frac{\partial}{\partial t} \right) \mathbf{Q}(1) = \mathbf{u}_0 \times \mathbf{Q}(1)
$$

from which the maximum nutation level can be calculated if it is always smaller than the nutation calculated in 2.2. augmented by $\mathbf{Q}_c$.

3. ATTITUDE CHANGE

The determination of the principal axes frame with respect to an inertial frame completes the solution of the self-excited rigid body problem. This part is much more difficult than the calculations of the nutation angle. In fact a complete analytical solution has not yet been found. We will use the Euler angles (Fig. 3) to define the orientation of the body-fixed frame in inertial space. At the connection between the two frames is given by the initial values $\psi, \theta, \phi, \mathbf{p}_e$. During the manoeuvre one expects eventually a stabilisation of the spin-axis in inertial space. It was indeed shown that the nutation angle goes to zero which implies in turn that the influence of the equatorial torque ($\mathbf{m}_x^*$) on the angular momentum averages out over a nutation period. The limiting motion $\mathbf{m}_x(t)$ is a uniform spin about some unknown direction: $\mathbf{Q}_x$.

The differential equation that define the evolution of the Euler angles are (Ref. 3, 6):

$$
\dot{\psi} = \omega_x \sin \theta \sin \phi - \omega_y \cos \theta
$$

$$
\dot{\theta} = \omega_x \cos \theta \cos \phi - \omega_y \sin \theta
$$

$$
\dot{\phi} = \omega_y - \omega_x \cos \theta \sin \phi
$$

Introducing the small angle approximation on $\theta$ we can use Eq. 30 to eliminate $\theta$ in Eqs. 28, 29:

$$
\dot{\psi} = \dot{\theta} \sin \theta \sin \phi + \omega_x \sin \phi
$$

$$
\dot{\phi} = \omega_y - \omega_x \cos \theta \sin \phi
$$

With the complex variable

$$
\mathbf{v} = \mathbf{y} + i \mathbf{z} = \mathbf{y} + i \mathbf{w}(t)
$$

Eqs. 31 and 32 are combined into:

$$
\dot{\mathbf{v}}(t) = \mathbf{w}(t)
$$

$$
\mathbf{w}(t) = \dot{\mathbf{v}}(t) = \dot{\mathbf{y}}(t) + i \dot{\mathbf{z}}(t)
$$

which is also a linear first order differential equation with as general solution

$$
\mathbf{v}(t) = \mathbf{v}(0) e^{\int_{t_0}^{t} \mathbf{w}(\tau) d\tau}
$$

where

$$
\mathbf{w}(t) = \dot{\mathbf{y}}(t) + i \dot{\mathbf{z}}(t) = \frac{\mathbf{m}_x^*}{\psi_x} (\dot{\mathbf{y}}(t) - i \dot{\mathbf{z}}(t))
$$

For the attitude change $\theta = |\mathbf{y}(t)|$, Eq. 36 becomes:

$$
\theta(t) = \int_{t_0}^{t} \mathbf{w}(\tau) d\tau + \theta_0
$$

To eliminate the remaining integral the expression $\mathbf{w}(t)$ corresponding to the particular case considered has to be used.

3.1 A Pure Spin-up Torque on a Spacecraft with Initial Notation

When $\mathbf{m}_x = 0$, $\phi = \frac{\omega_x}{\sqrt{\tau}}$ we have (Ref. 6):

$$
\mathbf{w}(t) = \omega_x e^{-i \frac{\omega_x t}{\sqrt{\tau}}}
$$

and Eq. 38 becomes:

$$
\theta(t) = \frac{\omega_x \sqrt{2\tau}}{\sqrt{3}} e^{-i \frac{\omega_x t}{\sqrt{\tau}}} \left[ \mathbf{v}(0) \right]_x
$$

$$
\theta(t) = \frac{\omega_x \sqrt{2\tau}}{\sqrt{3}} \left[ \mathbf{v}(0) \right]_x
$$

Defining $\psi, \theta$ such that all constant terms in Eq. 40 disappear:

$$
\theta = \tau \left[ \omega_x \sqrt{\tau} \right] \left[ \mathbf{v}(0) \right]_x
$$

$$
\theta(t) = \frac{\omega_x \sqrt{2\tau}}{\sqrt{3}} \left[ \mathbf{v}(0) \right]_x
$$

and

$$
\theta = \frac{\omega_x \sqrt{2\tau}}{\sqrt{3}} \left[ \mathbf{v}(0) \right]_x
$$

with $\tau = \sqrt{\frac{2\tau}{\sqrt{3}}}$ as the relevant time constant.

Notice that this initial orientation of the inertial frame is not on the angular momentum. The limiting value of the attitude change is after some calculations found to be:

$$
\theta = \frac{\omega_x \sqrt{2\tau}}{\sqrt{3}} \left[ \mathbf{v}(0) \right]_x
$$
3.2 Spin-up from a Uniform Rotation with a General Thruster

For this case \( \omega \) is given by:

\[
\omega_n = \frac{m_0}{A} e^{-\lambda x} \left[ \frac{F_n(t)}{r_n(t)} - \frac{F_n(t)}{x(t)} \right] \tag{45}
\]

and the remaining integral in Eq. 38 denoted \( J_n \) becomes:

\[
J_n = \int_0^t e^{\lambda x} \frac{m_0}{A} \frac{d}{dt} \left( e^{-\lambda x} \frac{F_n(t)}{r_n(t)} \right) \frac{dx}{dt}
\]

When \( \lambda x = 0 \) (top of Kovaleski) Eq. 46 reduces to:

\[
J_n = \frac{m_0}{A} e^{\lambda x} \left( \frac{F_n(t)}{r_n(t)} - \frac{F_n(t)}{x(t)} \right) ^2 \tag{47}
\]

No closed form of Eq. 46 for other values of \( \lambda x \) was found. The equations 43, 46 constitute a good starting point for further study of the attitude changes taking place during spin manoeuvres.

4. NUMERICAL APPLICATION TO METEOSAT-2

The formulae derived in point 2 have been used to calculate the nutation during the spin-up from 10 to 100 RPM of Meteosat-2 which will be launched in June 81. Meteosat-2 has no pure spin thrusters. Spin-up manoeuvres can be executed with either of the 2 thrusters called \( R \) and \( V \). Both of these thrusters generate a large equatorial torque (Table III) and the inertia figures of the Meteosat-2 are: \( \lambda = 29 \) E and \( \mu = 4 \) kg m\(^2\).

<table>
<thead>
<tr>
<th>( \lambda = 29 ) E</th>
<th>( \mu = 4 ) kg m(^2)</th>
<th>( R )</th>
<th>( V )</th>
<th>( \omega_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.59</td>
<td>1.92</td>
<td>10.23</td>
<td>84.40</td>
<td>( \omega_n ) = 23.79</td>
</tr>
<tr>
<td>2.67</td>
<td>0.74</td>
<td>3.588</td>
<td>74.43</td>
<td>( \omega_n ) = 38.22</td>
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TABLE III - Thruster data

<table>
<thead>
<tr>
<th>( \omega_n ) (RPM)</th>
<th>( x_0 )</th>
<th>Man.duration</th>
<th>Max.nut.</th>
<th>End Nut.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4.837</td>
<td>1035&quot;17'16&quot;</td>
<td>14.91°</td>
<td>0.8°</td>
</tr>
<tr>
<td>5</td>
<td>2.419</td>
<td>1092&quot;18'13&quot;</td>
<td>43.47°</td>
<td>1.58°</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1151&quot;19'11&quot;</td>
<td>84.40°</td>
<td>8.54°</td>
</tr>
</tbody>
</table>

\( R \) (\( x_0 = 29.77 \))

<table>
<thead>
<tr>
<th>( \omega_n ) (RPM)</th>
<th>( x_0 )</th>
<th>Man.duration</th>
<th>Max.nut.</th>
<th>End Nut.</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
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<td>2.13°</td>
<td>0.10°</td>
</tr>
<tr>
<td>5</td>
<td>3.885</td>
<td>2821&quot;17'1&quot;</td>
<td>6.68°</td>
<td>0.22°</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>2970&quot;19'30&quot;</td>
<td>74.83°</td>
<td>1.87°</td>
</tr>
</tbody>
</table>

\( V \) (\( x_0 = 29.77 \))

Table IV shows that for initial spin-rates between 5 to 10 RPM the final level of the nutation is quite acceptable. The major difference is the maximum nutation level which occurs at the beginning of the manoeuvre. Ref. 6 contains more numerical results and a comparison of these results against integration on an analogue computer. The agreement is good. A major difficulty during the simulation is the large range of the variables involved, a scaling of the variables which covers the range 0 to 100 RPM allows not a sufficient precision on the output variables. Moreover the simulations are time consuming, they had to be done in the real time option to avoid the influence of internal filtering which showed up in the past option. If the same problem is treated with a digital computer a variable step integration method must be chosen to avoid excessive run times.

Finally one may not forget the two most important assumptions under which these results are derived:

1) Internal energy dissipation is neglected. As Meteosat-2 is unstable the dissipation of energy tends to increase the nutation. At a certain spin-rate and nutation level, this effect can cancel the spin-up component of a given thruster. If this happens one has first to reduce the nutation by pulsed thrusting before the same thruster can spin the spacecraft further on. With the dissipation data of Meteosat-1 this problem does not arise with either of the 2 thrusters (\( R \) or \( V \)).

2) The spacecraft is symmetric. Although Meteosat is an almost symmetric spacecraft there is a fundamental difference in the behaviour of symmetric and asymmetric bodies. For symmetric bodies \( \lambda > 0 \) is sufficient to establish any desired value of the spin-rate, independent of \( \omega_n \). For asymmetric bodies, the so-called separation plane defines two regions where the average spin-rate is or about the nominal axis or about the transverse principal axis. When the initial rotation \( \omega_n \) is about the transverse axis (\( x_1 = x_2 \)) it is not trivial if a thruster with \( m_0 \) such that \( |x_1| \) decreases and \( \lambda > 0 \) will succeed in establishing a spin-rate around the desired spin-axis. This problem of flat-spin recovery (Refs. 7, 8) does not exist for a symmetric rigid body and for a nearly symmetric rigid body it is always present for high-enough initial transverse rates. It was checked that irrespective of the sign of the flat spin and initial transverse rates of 8 RPM (corresponding to 10 RPM about the nominal axis) and a nominal orientation of the inertia ellipsoid the recovery took always place just by firing the \( R \) or \( V \) thruster. The formulae presented in this paper can be used for a nearly-symmetric body as long as the initial spin rate is about the nominal spin-axis (which excludes Case V of Table II).

REFERENCES

1. Leimanas E 1965, The general problem of the motion of coupled rigid bodies about a fixed point, Springer-Verlag.


3. Luré L 1968, Mécanique analytique, Masson et Cie.


7. EWP 807 (ESTEC) 1974, Flat spin dynamics of Meteosat, by M. Kluiters.


| I    | O   | O   | O   |
| I    | O   | O   | O   |
| II   | O   | O   | O   |
| III  | O   | O   | O   |
| IV   | O   | O   | O   |
| V    | O   | O   | O   |
| VI   | O   | O   | O   |

**TABLE 1**

<table>
<thead>
<tr>
<th>Case</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( \theta_0(t) = 0 )</td>
</tr>
<tr>
<td>II</td>
<td>( \lambda \theta_0(t) + \frac{15}{8} )</td>
</tr>
<tr>
<td>III</td>
<td>( \lambda \theta_0(t) + \frac{15}{8} )</td>
</tr>
<tr>
<td>IV</td>
<td>( \lambda \theta_0(t) + \frac{15}{8} )</td>
</tr>
<tr>
<td>V</td>
<td>( \lambda \theta_0(t) + \frac{15}{8} )</td>
</tr>
<tr>
<td>VI</td>
<td>( \lambda \theta_0(t) + \frac{15}{8} )</td>
</tr>
</tbody>
</table>

**FIGURE 2** - Plot of \( \frac{F(t)}{F_0} \) vs. \( t \)