

## SOME ASPECTS IN MEASURING AND CORRECTING OF SATELLITE ORBIT

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## ABSTRACT

This article deals with the method of orbit determination and orbit correction. For determination of the orbit, one of the method which we use is based on the initial ephemeris of the orbit and the Doppler-frequency data obtained from four ground stations. Using differential correction method, the more accurate orbit parameters can be obtained. To correct the initial elliptical orbit, a method using small thrust and short operation duration is discussed. The model of orbit transfer and the control equations are derived, and the numerical examples are presented.

## 1. INTRODUCTION

The SSTC space programme now comprises a scientific satellite project and an earth resources satellite project. During life time of the scientific satellite, the orbit will be determined, using the differential correction mentioned here. The orbit of the earth resources satellite planned is circular and sun-synchronous. To eliminate the injection errors and compensate the effects of perturbing forces, orbit correction is necessary and will be performed by a co-planar transfer which corrects the size and shape, and a non-coplanar transfer which corrects the inclination of the initial orbit.

## 2. Orbital differential correction

The purpose of differential correction is to estimate the orbital parameters more accurately. Using the least square principle, the sum of the square of the weighted differences between measured and computed values is minimized. By solving the normal equations, we can obtain the fine estimation of orbital elements. The block diagram is shown in Fig. 1.

The orbit of the scientific satellite is nearly circular. In order to eliminate the possibility of divergence, instead of  $e, \omega$ , we take  $\xi = e \cos \omega$  and  $\eta = -e \sin \omega$  as variables. In calculations of the

perturbation, we take only the first order short period parts and second order secular of perturbing functions.

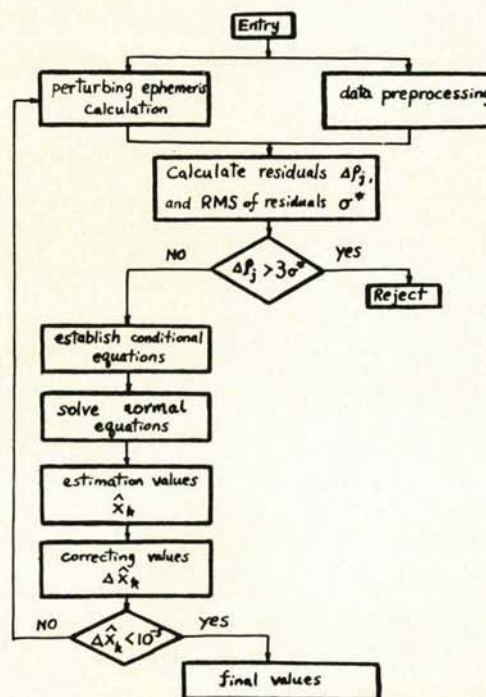


Fig. 1. The block diagram of differential correction program

## 3. Orbit Correction

The differences between the initial and nominal orbit are shown in Fig. 2 and 3, where  $r_n$  is the radius of nominal circular orbit.



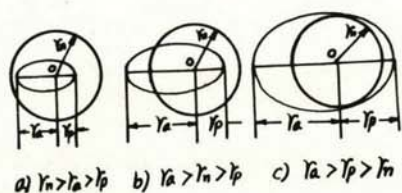


Fig. 2

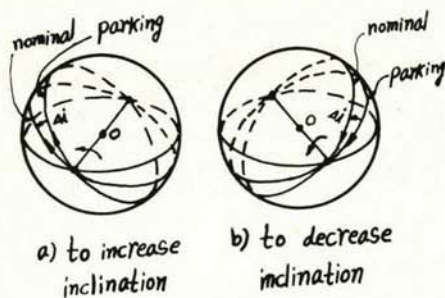


Fig. 3

According to the concept of Hohmann transfer and Lagrange's planetary equations, assuming the initial orbit eccentricity is small (say  $e_0 \leq 0.01$ ) and the duration of thrust operation is short, and assuming the arc upon which the thrust operates is symmetrical with respect to the perigee (or apogee), we can derive the control equations for co-planar transfer as follow:

$$\begin{cases} \Delta r_a = \frac{T}{n_0} \sqrt{\frac{1+e}{1-e}} \left[ \left(2 - \frac{3}{2}e\right) \Delta t \pm \frac{4}{n_0} \sin \frac{n_0 \Delta t}{2} + \frac{3e_0}{2n_0} \sin n_0 \Delta t \right] \\ \Delta r_p = \frac{T}{n_0} \sqrt{\frac{1-e}{1+e}} \left[ \left(2 + \frac{3}{2}e\right) \Delta t \mp \frac{4}{n_0} \sin \frac{n_0 \Delta t}{2} - \frac{3e_0}{2n_0} \sin n_0 \Delta t \right] \end{cases}$$

where the upper sign is used for the maneuver at perigee and the lower sign for the maneuver at apogee,  
 $\Delta r_a$  — error in apogee altitude  
 $\Delta r_p$  — error in perigee altitude  
 $T$  — thrust acceleration in transverse direction  
 $n_0$  — mean initial angular velocity of the satellite  
 $\Delta t$  — duration of thrust operation  
 $e_0$  — eccentricity of initial orbit and for non-coplanar transfer:

$$\begin{cases} \Delta i = \pm \frac{2W}{n_H^2 a_H} \sin \frac{n_H \Delta t}{2} \\ \Delta \Omega = 0 \end{cases}$$

(in the case of symmetrical operation with respect to ascending or descending node). where the "+" sign indicates the maneuver at ascending node and "-" sign indicates the maneuver at descending node,  $n_H$  and  $a_H$

are the mean angular velocity and the semi-major axis of the nominal orbit respectively.  
 $W$  — thrust acceleration in normal direction  
 $\Delta i$  — error in orbital inclination  
 $\Delta \Omega$  — net change of argument of ascending node.  
 If the duration of operation,  $\Delta t$ , is large, the perigee altitude is effected by apogee correction (or vice versa) for co-planar orbit correction and  $\Delta \Omega$  oscillates with larger amplitude for non-coplanar correction. Hence, long duration of thrust operation is not preferred for this orbit correction model. If the initial orbit is corrected on a series of successive orbits with small thrust and short operating time, the total characteristic velocity required will be much smaller. Table 1 and 2 show their comparison and Fig. 4 and 5 illustrate an example of three—successive—orbit correction, where assume  $\Delta r_p = -10 \text{ km}$ ,  $e_0 = 0.01$ ,  $H = 723 \text{ km}$ ,  $i = 98.22^\circ$  and thrust-to-weight ratio = 0.00133.

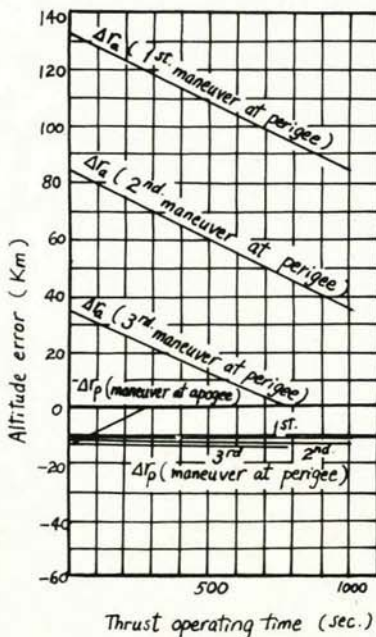


Fig 4. co-planar transfer on small arcs.

Table 1.

correction method	full correction	correction on small arcs	Hohmann transfer
number of maneuvers	at perigee	3	1
	at apogee	1	1
total characteristic velocity (m/sec.)	58.12	38.89	37.30



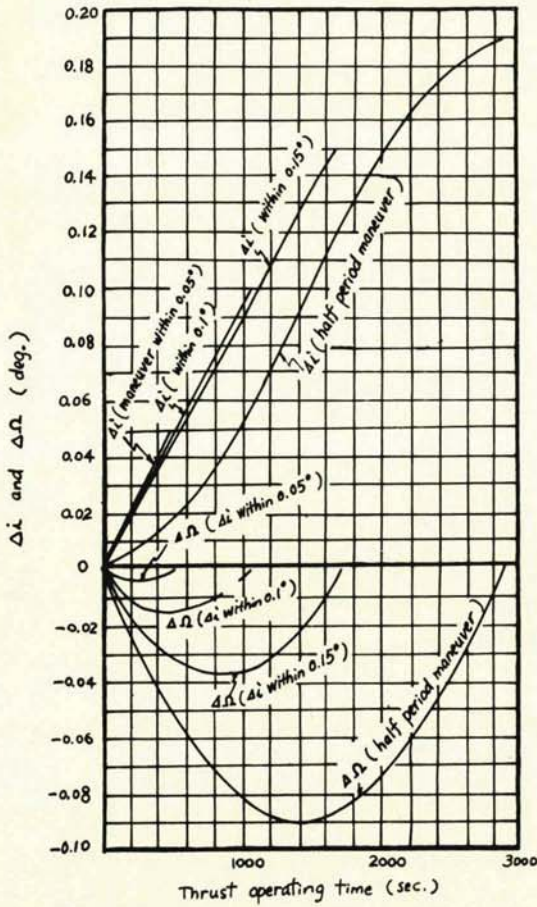


Fig 5. Non-coplanar transfer Table 2. (total correcting quantity is 0.5)

correction method	full half period	0.1° each	0.05° each	single-impulse
number of maneuvers	3	5	10	1
total characteristic velocity (m/sec.)	95.14	68.93	66.11	65.41

From these results we see that, the total characteristic velocity required for successive maneuvers with small thrust and short operating duration is comparable with that of Hohmann coplanar transfer and non-coplanar single impulse transfer. Attitude errors may exist during maneuvers, among these the yaw angle is the most effective in this correction. Let  $\bar{\tau}$  and  $\bar{w}$  denote the thrust accelerations produced by transversal and normal thruster respectively. In the case of co-planar transfer, the change of inclination caused by yaw angle error  $\psi$  can be derived as:

$$|\Delta i| \approx \frac{\psi \bar{\tau}}{n^2 a_n} \Delta t$$

and in the case of non-coplanar transfer, the change of orbital semi-major axis is given as:

$$|\Delta a| \approx \frac{2\psi}{n} \bar{w}$$

Fig 6 and 7 illustrate their effects under various values of  $\psi$ .

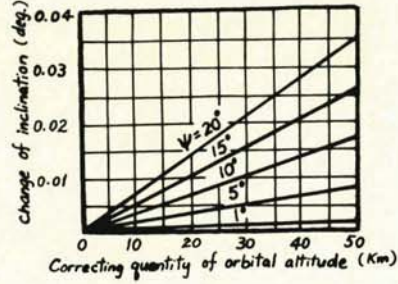


Fig 6. Change of inclination due to various  $\psi$

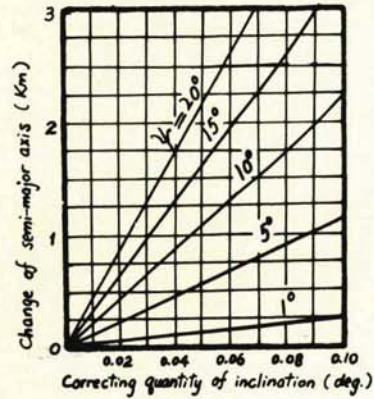


Fig 7. Change of semi-major axis due to various  $\psi$

4. REFERENCES

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