A FORMULATION FOR STUDYING STEADY STATE/TRANSIENT DYNAMICS OF A LARGE CLASS OF SPACECRAFT AND ITS APPLICATION

A M Ibrahim & V J Modi

Department of Mechanical Engineering
The University of British Columbia
Vancouver, B.C., Canada V6T 1W5

ABSTRACT

A relatively general formulation for studying dynamics of a system, consisting of n connected flexible deployable members forming a topological tree or a closed configuration, is presented. The mathematical description of the system can be, in general, a combination of discrete and distributed coordinates. Joints, elastic and dissipative, permit relative rotation and translation between bodies. The elastic deformations can be discretized using admissible functions, finite elements or lumped mass method. Rotations of the members, as well as of the entire system, can be described using a set of orientation angles, Euler parameters or Rodrigues vectors. The formulation accounts for: the presence of momentum or reaction wheels; thrusters distributed over the flexible and rigid portions; and any prescribed forms of energy dissipation mechanisms. The formulation is valid for orbiting as well as ground based and marine systems. Application of the formulation is illustrated through an example, in spacecraft dynamics, which is of contemporary interest.

Keywords: Dynamical Formulation; Flexible Transient Systems; Application to Diverse Earthbound, Offshore and Space-Based Systems.

1. INTRODUCTION

Flexibility effects on satellite attitude motion and its control have become topics of considerable importance. Over the years, a large body of literature pertaining to the various aspects of satellite system response, stability and control has appeared (Ref. 1). A recent issue of the Journal of Guidance, Control, and Dynamics published by the AIAA (American Institute of Aeronautics and Astronautics) contains a series of articles reviewing the state of the art in the general area of large space structures (Ref. 2).

Attention is also directed towards planning of in-orbit experiments such as SCOLE (Satellite COntrol Laboratory Experiment), the Orbiter Mounted Large Platform Assembler Experiment, NASA/Lockheed Solar Array Flight Experiment (SAFE) and a host of others to check, calibrate and improve algorithms. It is generally concluded that in-orbit information acquired during the construction phase of a space station is the only dependable procedure for its overall design. With the U.S. commitment to a space station in early 1990's, the need for understanding structural response and control characteristics of such time varying, highly flexible systems is further emphasized. This as background, the paper presents a relatively general Lagrangian matrix formulation of the nonlinear, nonautonomous, and coupled equations of motion, describing dynamics of a large class of systems characterized by flexible interconnected structural members (Fig. 1). Essential features of this highly versatile formulation may be summarized as follows:

- arbitrary number, type (tether, membrane, beam, plate, shell) and orientation of flexible members, connected so as to form a topological configuration, open or closed, deploying independently at specified velocities and accelerations;
- the appendage is permitted to have variable mass density, flexural rigidity and cross-sectional area along its length;
- governing equations account for gravitational effects, shifting center of mass, changing inertia and appendage offset together with transverse, axial, and torsional oscillations;
- appendage as well as system rotations can be described using Euler or Rodrigues parameters, or any of the orientation angles;
- elastic deformations can be discretized using modal representation or admissible functions, finite elements, or lumped mass method;
- in general, joints between the flexible members are taken to be elastic and dissipative permitting relative rotation and translation between bodies;
- the system may contain momentum or reaction wheels, gimballed or fixed, as well as thrusters;
- the equations are applicable to earth-bound underwater and space-based systems;
- in spacecraft dynamics studies, the generalized coordinates corresponding to librational degrees of freedom can be so chosen as to make the governing equations applicable to both spin stabilized and gravity gradient orientations;
- the equations are programmed in nonlinear as well as linearized forms to permit the study of:
  (i) large angle maneuvers;
  (ii) nonlinear effects.

The program is written in a modular fashion to help isolate the effects of flexibility, deployment, character and orientation of the appendages, inertia and orbital parameters, number and type of admissible functions, etc. Environmental effects due to solar radiation pressure, aerodynamic forces, Earth's magnetic field interaction, etc., can be incorporated easily through generalized forces. The same is true with internal and external dissipation mechanisms.

2. APPROACH TO FORMULATION OF THE PROBLEM

As can be expected, development of such a general formulation presented a challenging task and involved efforts spanning over several years. Obviously, details of the development, which form a part of the Ph. D. thesis in preparation, are beyond the scope of this concise presentation (Ref. 3). Emphasis is on methodology, philosophy of approach, and physical insight during development of the formulation.

Proceedings of the Second International Symposium on Spacecraft Flight Dynamics, Darmstadt, FR Germany, 20–23 October 1986 (ESA SP-255, Dec. 1986)

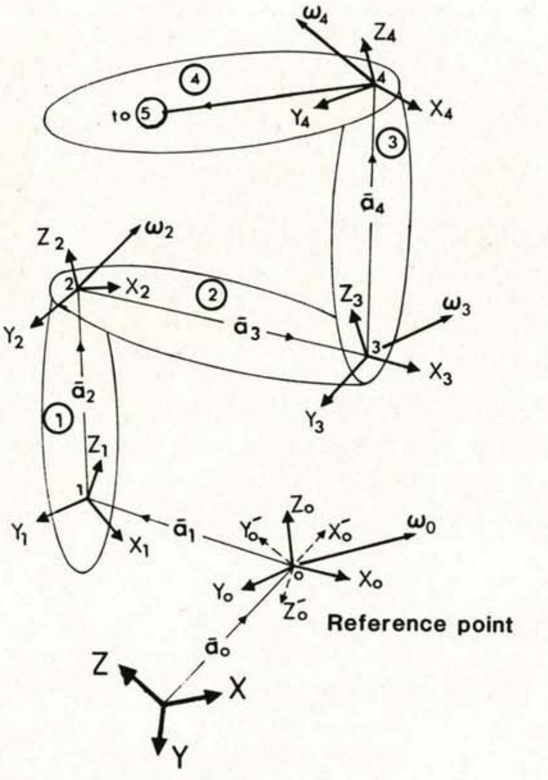


Figure 1 System description and coordinate system. Note the reference point will be usually c.m. of the system in space dynamics application.

2.1 Kinematics

Objective here is to obtain mathematical expressions for position and velocity of an arbitrarily located mass element. This is achieved by approaching the element in the direct way as against isolating each body and imposing reactions and constraints.

Consider a system of n elastic bodies connected arbitrarily to form a branched or closed loop topological geometry as shown in Fig. 1. Let X, Y, Z be an inertial coordinate system and x_0, y_0, z_0 with origin at O, a reference frame of coordinates. Now let body 1 be referred to as the main body. Fixed to it, at a convenient location, is the coordinate system x_1, y_1, z_1. On the successive bodies, at their respective hinge points on the direct path connecting i and $i-1$ members, are located body coordinates x_i, y_i, z_i. Thus coordinates x_2, y_2, z_2 are fixed to body 2 at the hinge connecting bodies 1 and 2. Similarly, O acts as a hinge for body 1 . The hinges are permitted to have three dimensional translational as well as rotational degrees of freedom.

Consider a mass element dm_i on link i defined by a position vector $\bar{r}^i$ with reference to the inertial coordinate system X, Y, Z, (Fig. 2). The geometry of the system is described by a matrix S_i^j hence numbering of bodies in sequence is not necessary,

$$\bar{r}^i = \sum_{j=0}^{n} S_{i+1}^j C_{j-1} \bar{a}_j \qquad \text{with} \qquad \bar{a}_{i+1} = \bar{\zeta}_i . \tag{1}$$

Here:

$$S_i^j = \begin{cases} 0, & \text{if } j \text{ is not in the direct path to } i; \\ 1, & \text{if } j \text{ is in the direct path to } i; \end{cases}$$

$C_j =$ relative rotation matrix relating coordinates j to the inertial set;

$n =$ number of connected bodies;

$\bar{a}_j =$ position of coordinates j with respect to coordinates $j-1$ on the direct path;

$\bar{\zeta}_i =$ position vector to the element dm_i with respect to coordinates x_i, y_i, z_i, (Fig. 2), $\bar{\rho}_i + \phi_i \delta_i$;

$\bar{\rho}_i =$ position vector to the element dm_i in nominal undeflected position;

$\phi_i =$ matrix of admissible functions;

$\delta_i =$ generalized coordinates; $\phi_i \delta_i$ represent deflection of dm_i from nominal equilibrium position.

Differentiating with respect to time, the expression for inertial velocity can be written as,

$$\dot{\bar{r}}^i = \check{\bar{r}}^i + \sum_{j=0}^{n} S_i^j C_{j-1} \tilde{\omega}_j \, {}^j\bar{r}^i , \tag{2}$$

where:

$$\check{\bar{r}}^i = \sum_{j=0}^{n} S_{i+1}^j C_{j-1} \check{\bar{a}}_j ;$$

$\tilde{\omega}_j =$ skew symmetric matrix representing rotation of coordinates j with respect to coordinates $j-1$;

${}^j\bar{r}^i =$ position of dm_i with respect to coordinates j. ${}^1\bar{r}^i$ is shown in Fig. 2.

Note, the first term on the righthand side represents translational velocity contributions due to deformations, deployment, hinge translation, etc., while the second term contains contributions due to rotations of the hinges.

2.2 Kinetics

For the Lagrangian approach, the next logical step would be to obtain expressions for kinetic, potential and elastic energies of the system. As can be expected, because of the complex and quite general character of the system, derivation of the expressions demands considerable time, space, and effort. For conciseness only more important steps relevant to the development are indicated here.

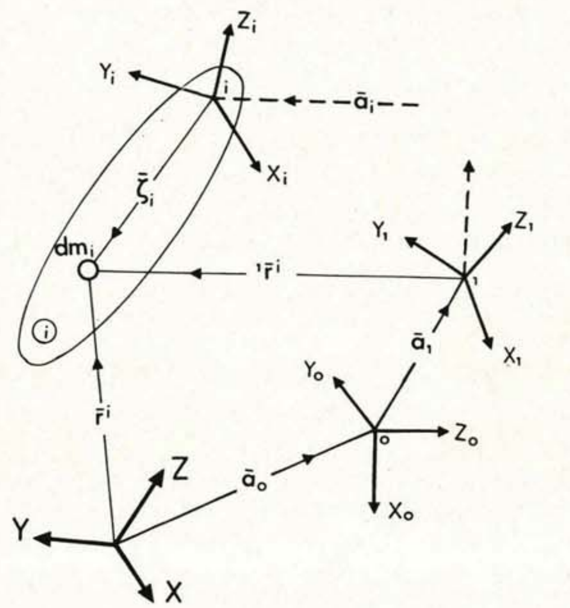


Figure 2 Geometry of position vectors defining location of the mass element dm_i on body i

2.2.1 Kinetic energy.

$$T = \frac{1}{2}\sum_{i=1}^{n}\int_{m_i} \dot{\bar{r}}^i \cdot \dot{\bar{r}}^i dm_i \,. \tag{3}$$

Substituting from equation (2), the kinetic energy can be shown to have a form,

$$T = \frac{1}{2}v^T E v + \omega^T G v + \frac{1}{2}\omega^T H \omega = \frac{1}{2}\dot{x}^T M^* \dot{x}\,, \tag{4}$$

where: $\dot{x} = \begin{Bmatrix} v \\ \omega \end{Bmatrix}$; $\quad M^* = \begin{bmatrix} E & G^T \\ G & H \end{bmatrix}$;

and v is defined by an array of all the linear velocities appearing in the kinetic energy expression, $(\dot{m}_i, \dot{a}_i, \dot{e}_i, \dot{\delta}_i)$.

From the consideration of physical appreciation, it is important to recognize that E has the dimensions of mass; G the dimensions of the first mass-moment; and H the dimensions of the second mass moment. Thus E represents mass associated with deployment, ejection, translation and vibration, while G corresponds to their first moment about the hinges. H matrix has more direct meaning. Let H^{jk} represent a submatrix. Then for $j = k$, H^{jj} represents a 3×3 matrix corresponding to inertias about the axes at j. For $j \neq k$, the submatrix has elements involving distances to joints j and k with common mass elements affected by rotations of both the hinges.

The next logical step is to introduce holonomic constraints and modify $\{\dot{x}\}, [M]$ making them consistent with the constraints. The kinetic energy can then be rewritten separating contributions from specified, s, and generalized coordinates, q, as follows:

$$T = \frac{1}{2}\begin{Bmatrix} \dot{s} \\ \dot{q} \end{Bmatrix}^T \begin{bmatrix} N & L^T \\ L & M \end{bmatrix} \begin{Bmatrix} \dot{s} \\ \dot{q} \end{Bmatrix} = \frac{1}{2}\dot{q}^T M \dot{q} + \dot{s}^T L^T \dot{q} + \frac{1}{2}\dot{s}^T N \dot{s}$$

$$= \frac{1}{2}\dot{q}^T M \dot{q} + \Gamma^T \dot{q} + T_0 \,. \tag{5}$$

Note here both $\dot{s}$ and $\dot{q}$, in general, will be arrays of the form $(\dot{m}_i, \dot{a}_i, \dot{e}_i, \dot{\delta}_i, \omega_i)$. It is important to recognize that this general expression for kinetic energy consists of quadratic as well as linear terms in generalized velocities and a term independent of the generalized velocity. The quadratic term represents contribution from the generalized coordinates while T_0 is that from the specified coordinates. The linear term is due to a coupling between the system of two coordinates.

2.2.2 Potential energy. Contribution to the potential energy arises from two sources: gravitational field and strain energy due to flexibility. Gravitational contribution, U, is given by,

$$U = -\frac{\mu_e m}{a_0} + \frac{\mu_e}{a_0^2}\{l\}^T\{L^0\} - \frac{\mu_e}{2a_0^3}\mathrm{tr}[H^0] + \frac{3\mu_e}{2a_0^3}\{l\}^T[H^0]\{l\}\,, \tag{6}$$

where:

$m =$ mass of the system, $\sum_{i=1}^{n} m_i$;

$\{L^0\} = \sum_{i=1}^{n}\sum_{j=1}^{n} m_i S_i^j C_{j-1}\bar{a}_j + \sum_{i=1}^{n} C_i \int_{m_i} \varsigma_i dm_i$;

$H^0 =$ inertia of the system about the reference point O (associated with ω_0);

$\mu_e =$ universal gravitational constant;

$\bar{a}_0 =$ position vector from the earth's center to the instantaneous center of mass;

$\{l\} =$ direction cosines of the unit vector along $\bar{a}_0$ w.r.t. x_0, y_0, z_0, coordinates.

Note, the first term represents gravitational energy due to the system treated as a point mass; the second term appears due to separation between the reference point and the center of mass and vanishes when they are coincident; while the remaining two terms are due to finite size of the system.

Strain energy for a given flexible body, V, is given by,

$$V = \frac{1}{2}\int_\tau \sigma^T \epsilon d\tau = \frac{1}{2}\int_\tau \epsilon^T C \epsilon d\tau \,. \tag{7}$$

Considering the strain to have linear and nonlinear contributions from deformations δ, it can be written as:

$$\{\epsilon\} = \{B^0\delta\} + \{\delta^T B^j \delta\}\,,$$

where B^0 and B^j depend on the chosen admissible functions ϕ. Substituting this expression for ϵ in equation (7), the strain energy expression can be written as

$$V = \delta^T\left[V_1 + 2\delta^T V_2^j + \delta^T V_3^{jk}\delta\right]\delta\,,$$

where:

$$V_1 = \sum_{j=1}^{6}\sum_{k=1}^{6} C_{jk}\int_\tau B_{j-}^0 (B_{k-}^0)^T d\tau \,;$$

$$V_2^j = \sum_{k=1}^{6}\sum_{l=1}^{6} C_{kl}\int_\tau B_{kj}^0 B^j d\tau \,;$$

$$V_3^{jk} = \sum_{l=1}^{6}\sum_{m=1}^{6} C_{lm}\int_\tau B_{-j}^l B_{-k}^m d\tau \,.$$

Now the elastic force is given by,

$$\frac{\partial V}{\partial \delta} = V_{,\delta} = K\delta\,, \tag{8}$$

where:

$K =$ stiffness matrix $= K_1 + K_2 + K_3$;

$K_1 = V_1 + V_1^T$;

$K_2 = 2\left[\delta^T V_2^j + (V_2^j)^T\delta + \sum_{j=1}^{6}\delta_j V_2^j\right]$;

$K_3 = \left[\delta^T\left[V_3^{jk} + (V_3^{jk})^T\right]\delta\right] + \sum_{j=1}^{6}\sum_{k=1}^{6}\delta_j\delta_k\left[V_3^{jk} + (V_3^{jk})^T\right]$.

Note, the notation for partial differentiation used above for brevity. Global stiffness matrix can now be assembled with δ as generalized coordinates or nodal displacements.

EQUATIONS OF MOTION

Using the Lagrangian procedure in conjunction with kinetic and potential energy, expressions (5), (6), and (8),

gives $$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial (U+V)}{\partial q} = Q + A^T\lambda$$

$$M\ddot{q} + [M_{,t} + G]\dot{q} + \{\dot{q}^T\gamma_j\dot{q}\} + \Gamma_{,t} + Kq + (U - T_0)_{,q} = Q + A^T\lambda\,, \tag{9}$$

where:

$M =$ square symmetric matrix of generalized mass, Eq. (5)

$G = \left[\{\Gamma_{j,q}\}^T - \Gamma_{,q_j}^T\right]$, skew symmetric matrix;

$\gamma_j = \left[M_{j-,q} - (1/2)M_{,q_j}\right]$, square matrix associated with the Coriolis force contribution;

$\Gamma =$ coefficient associated with the linear contribution in the kinetic energy, Eq. (5);

$K =$ stiffness matrix, Eq. (8);

$U =$ gravitational potential energy, Eq. (6);

$T_0 =$ kinetic energy contribution independent of generalized velocities, Eq. (5);

$Q =$ generalized forces.

Note, the Lagrangian character of the formulation is amenable to point transformation making it possible to obtain governing equations using alternate standard procedure of Hamilton. More importantly, the equations clearly isolate contribution of forces from different sources thus retaining physical insight into the problem. Furthermore, the form is ideally suited for implementing control strategies. It should be emphasized that most available formulations of multibody systems do not possess the above mentioned features. Terms representing contributions from various sources are explained next:

$M\ddot{q} =$ inertia forces;
$M_{,t}q =$ reaction forces due to deployment and mass expulsion;
$Gq =$ gyroscopic forces;
$\{\dot{q}^T\gamma_j\dot{q}\} =$ Coriolis and centrifugal forces arising from generalized coordinates;
$\Gamma_{,t} - T_{0,q} =$ centrifugal forces arising from specified coordinates;
$Kq =$ elastic forces;
$U_{,q} =$ gravitational force;
$A^T\lambda =$ nonholonomic constraint forces as well as holonomic constraints not accounted for earlier.

4. AN ILLUSTRATIVE EXAMPLE

Consider a spacecraft with central rigid body, and a flexible deployable appendage attached to it, in a specified arbitrary orbit (Fig. 3a). Let the reference point O be the instantaneous center of mass of the system and the inertial coordinate X, Y, Z located at the center of the Earth. Thus O is the hinge point of body 1 . Now in this special case, $\overline{a}_0$ represents position vector from the center of the Earth to the instantaneous center of mass and hence specifies the trajectory. Let 1 be the center of mass of the rigid body. Thus $\overline{a}_1$ represents instantaneous position of the moving center of mass with respect to point 1. $\overline{a}_2$ defines position of the appendage attachment point taken stationary in this case. The system variables now become, $\{\dot{\overline{x}}\}^T = (\dot{\overline{a}}_0, \dot{\overline{e}}_2, \dot{\overline{\delta}}_2, \overline{\omega}_0)$. Note, $\dot{m}_1$ and $\dot{m}_2$ do not appear explicitly because of the constraint relations between $\dot{m}_2$ and $\overline{e}_2$. For numerical results, the rigid body is taken to be the Orbiter and the appendage a flexible beam (Fig. 3b).

Case 1: The Rigid Orbiter Without Any Flexible Appendage

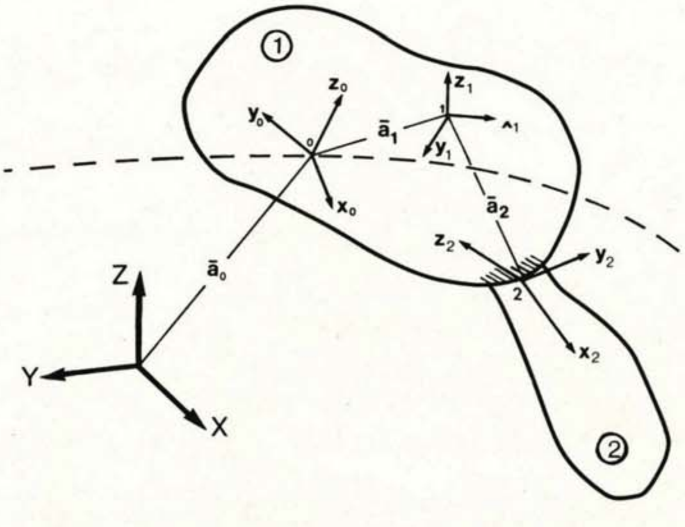
$$\{\dot{\overline{x}}\}^T = (\dot{\overline{a}}_0, \overline{\omega}_0) .$$

Here $\overline{a}_0$ is the specified coordinate defining the Keplerian motion and the generalized co-ordinate $\overline{\omega}_0$ describes the attitude motion α (pitch), β (yaw), and γ (roll).

Case 2: The Orbiter Having a Deployable Flexible Beam with Libration Degrees of Freedom Held Zero

$$\{\dot{\overline{x}}\}^T = (\dot{\overline{a}}_0, \dot{\overline{e}}_2, \dot{\overline{\delta}}_2, \overline{\omega}_0) .$$

Here $\dot{\overline{q}} = \dot{\overline{\delta}}_2$ and $\dot{\overline{s}} = \dot{\overline{a}}_0, \dot{\overline{e}}_2, \dot{\overline{\omega}}_0 = 0$.



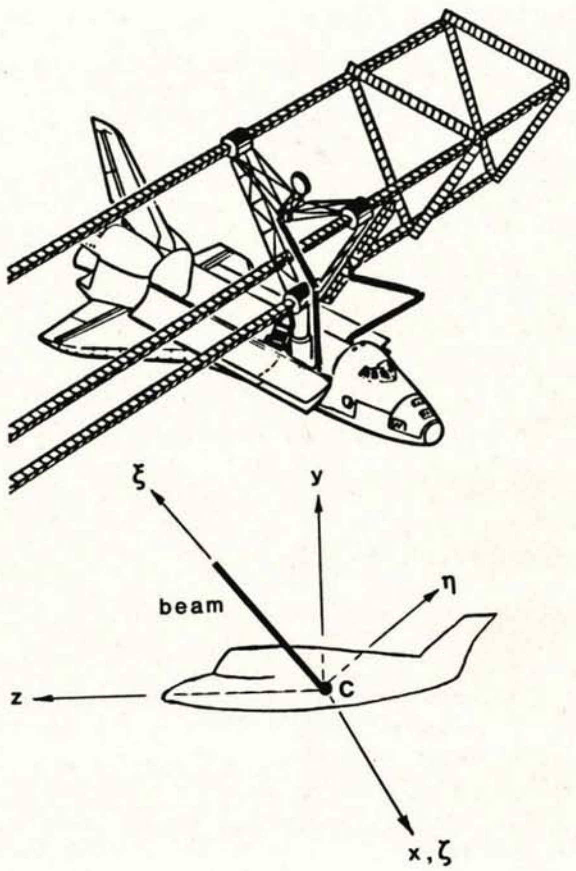


Figure 3 A partiular case showing application of the general formulation:

(a) an orbiting rigid body with a flexible deployable appendage attached to it;
(b) artist's view of the Orbiter based manufacture of the beam type structural components for construction of space platforms. Principal body coordinates x, y, z with their origin at the instantaneous center of mass and beam coordinates ζ, η, ξ with the origin at the attachment point are also indicated. In general the two origins are not coincident.

4.1 Results and Discussions

For analysis, the flexibility and deployment rate parameters were taken to be of the same order of magnitude as used or likely to be employed in practice. In the diagrams e represents orbital eccentricity; Ψ, Λ, Φ (roll, yaw, pitch, respectively) are the librational angles; EI is the beam flexural rigidity, assumed constant over the length in this particular example; and $\dot{L}$ corresponds to the deployment rate. λ_{in} and λ_{out} denote beam inclinations to the local vertical in and normal to the orbital plane, respectively. The perigee was taken to be 331 km. The truss or beam vibrations were represented by a maximum of the first four modes, Ψ_i, of a cantilever. P_l, Q_l represent generalized coodinates associated with the admissible functions used to represent beam-type appendage oscillations in the lth mode in ς and η directions, respectivly. $\overline{P}_l$ and $\overline{Q}_l$ represent transverse generalized coordinates normalized with respect to the total length.

Numerical values for some of the more important parameters used in the computation are given below:

Orbiter: Mass $= 79,710\ kg$;
$I_{xx} = 8,286,760\ kgm^2$; $I_{xy} = 27,116\ kgm^2$;
$I_{yy} = 8,646,050\ kgm^2$; $I_{yz} = 328,108\ kgm^2$;
$I_{zz} = 1,091,430\ kgm^2$; $I_{zx} = -8,135\ kgm^2$.

Here x, y, z are the principal body coordinates of the Orbiter with the origin coinciding with the center of mass. In the nominal configuration x is along the orbit normal, y coincides with the local vertical and z is aligned with the local horizontal in the direction of motion (Fig. 3b). γ (roll), β (yaw), and α (pitch) refer to rotations about the local horizontal, local vertical, and orbit normal, respectively.

Beam: Mass $(M_b) = 129\ kg$;
Length $(L) = 33\ m$;
Flexural Rigidity $(EI) = 436\ kgm^2$.

To get some appreciation as to the system dynamics during transition to instability, the Lagrange configuration was subjected to pitch, yaw, and roll disturbances separately (Fig. 4). With a pitch disturbance as large as 30° (Fig. 4a), the roll and yaw remain unexcited and the system is stable. The same is essentially true with a yaw disturbance (Fig. 4b). However, even with a relatively small roll disturbance (Fig. 4c), the diverging yaw oscillations set-in tending towards instability. Thus roll control seems to be a key to ensure stability of the Orbiter in the Lagrangian configuration.

Effects of beam deployment on the tip dynamics is studied in Fig. 5. Initial tip deflection is 4% of the beam length. Two time histories with the same duration of deployment are considered. As can be expected, the frequency of oscillation in and out of the orbital plane gradually decreases with deployment finally attaining a steady state value upon its termination. It is of interest to recognize that they reach the same steady state amplitude although it is much larger during deployment compared to the deployed case.

5. CONCLUDING REMARKS

The formulation presented here will prove useful to design engineers involved in planning of future communications satellites. It will also assessing dynamical, stability, and control considerations associated with the Orbiter based construction of spaceplatforms.

6. REFERENCES

1. Modi V J 1974, Attitude dynamics of satellites with flexible appendages - a brief review, J. Spacecraft and Rockets, 11, 743-751.
2. AIAA 1984, Large space structure control: early experiments, J. Guid., Cont., and Dyn., 7, 513-562.
3. Ibrahim A M 1986, Mathematical modelling of flexible multibody dynamics with application to orbiting systems, Ph. D. Thesis, (in preparation), Univ. of British Columbia.

ACKNOWLEDGEMENT

The investigation reported here was supported by the Natural Sciences and Engineering Research Council of Canada, Grant No. G-1547.

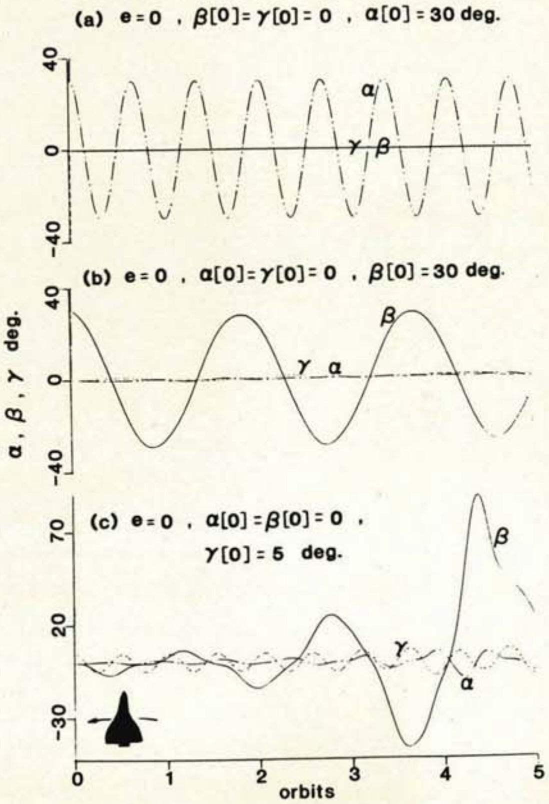


Figure 4 Librational response of the Orbiter to an independent excitation in pitch, yaw, and roll. Note the pitch and yaw disturbances lead to essentially uncoupled motions. The system appears to become unstable in yaw through its coupling with roll.

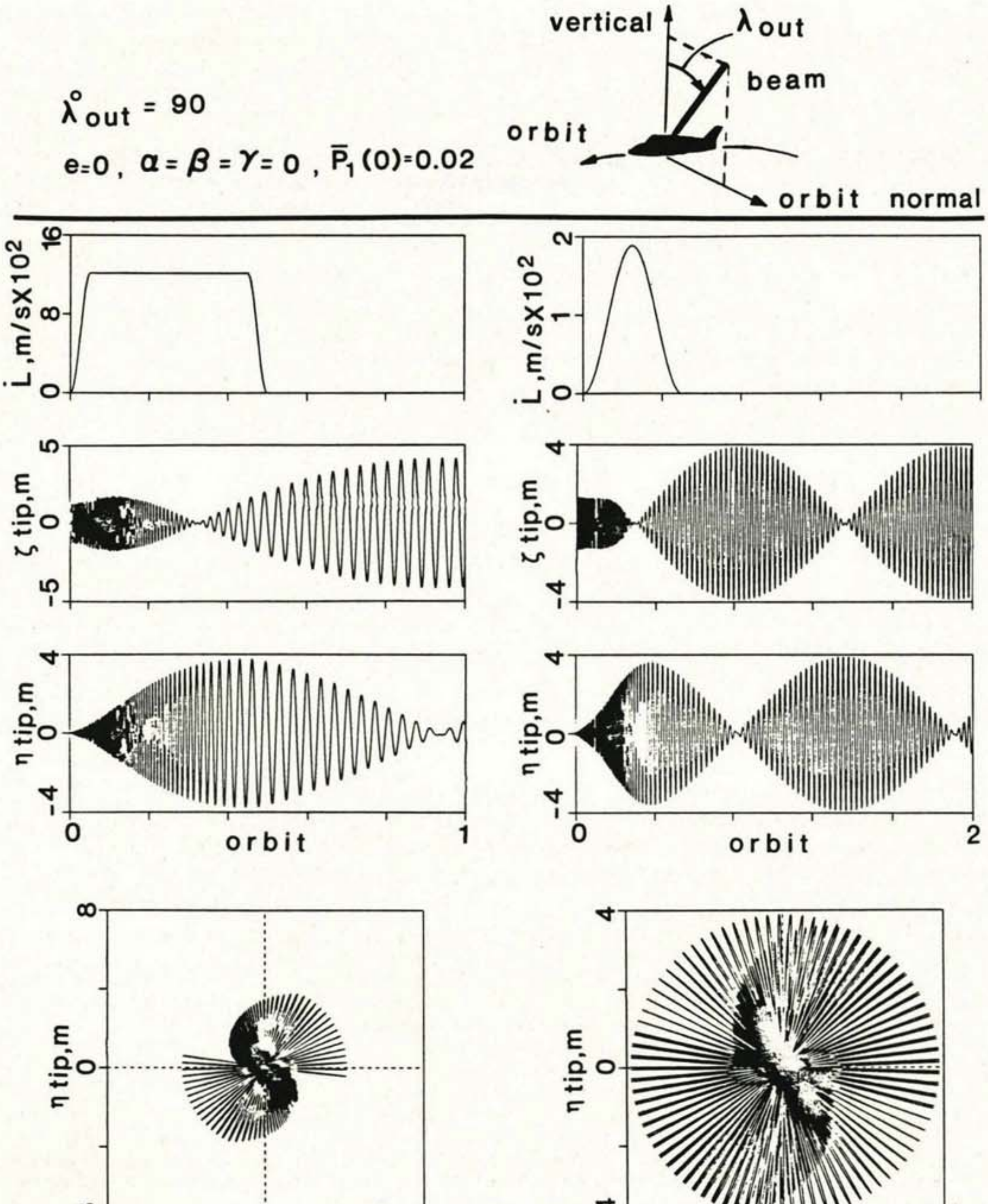


Figure 5 Effect of deployment strategies on tip response of a beam deploying normal to the orbital plane. Note a reduction in beam frequency during deployment. The steady state amplitude is essentially independent of the strategy for a given time of deployemnt.