SAMPLED NONLINEAR CONTROL FOR LARGE ANGLE MANEUVRES OF FLEXIBLE SPACECRAFT

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ABSTRACT

This paper deals with large angle attitude maneuvers of flexible satellites. It is outlined the design of a feedback control law, based on partial informations of the state variables, which gives very interesting results when implemented into a sampled data control scheme.

Keywords: Large angle maneuvering, flexible spacecrafts,nonlinear control, sampled-data schemes.

1. INTRODUCTION

In many missions the attitude operation plan frequently requires reorientation maneuvers in order to point or scan certain areas of interest.

Many attitude control systems for satellites are currently based on a sequence of rotations about each of the three principal axes separately because of the consequent linearity of the equations of motion; hence the relative simplicity of the online command generation problem. Arbitrary attitude maneuvers can be so executed as sequences of single axis rotations. Such an approach can give unsatisfactory results if particular attitude constrains are required.

A different approach will be followed hereafter; on the bases of the ideas developed in [1-2] some results of nonlinear control theory will be applied to the problem under study. A good reference book for the study of the more recent results in nonlinear control is [3]; in [4,5,6] other interesting applications have been developed.

It is considered a satellite which consists of flexible appendages attached to a main body; the controls are assumed to act uniquely on the main body and to be generated either by gas-jet or reaction-wheel actuators. A sampled-data control scheme will be proposed which makes use of the outputs coming from a rate gyro-package located on the main body.

The design of the feedback control law is based on the "Control-model" (C-model) introduced in section 2. The C-model consists of the kinematic equations, in the quaternions parametrization, together with the dynamical equations which describe the behaviour of the discretized structure. The design of the control law is developed in section 3. It comes from: the application of input-output linearization and stabilization techniques, [7-8], the study of the corresponding sampled-data control scheme, [2], and the use of a compensation technique which allows not to take into account of the model displacements and velocities in the evaluation of the control law.

Simulation results are, finally, presented and discussed in section 4. The real behaviour of the flexible satellite is simulated by the "S-model" introduced in section 2; that is a mathematical model which takes into account also of centrifugal, and Coriolis effects (neglected in the C-model) and is based on a more accurate representation of the flexible parts.

The proposed control law leads to improvements which are also apparent from the simulation results. The main difficulties w.r. to the control law proposed in [1], where the same control problem is studied, stand in:

- "partial compensation" of the sampling effects allowing to increase the "sampling and hold" interval;
- design of a control law based on partial state informations: angular rates and quaternions.

2. DYNAMICS AND KINEMATICS MODELLING

In this section we introduce the mathematical model which will be used in the simulations and a simplified mathematical model which will be used in the design of the control law. The first one is denoted by "S-model" and will be assumed to represent the "real" behaviour of the spacecraft. The second model, denoted by "C-model", is obtained from the previous one by neglecting, as usual, Coriolis and centrifugal effects and using a "less accurate" representation of the flexible appendages. The C-model will be used in the design of the control law. Once for all we note that the same results can be obtained by using a hybrid coordinate approach.

It is well known that an appropriate mathematical

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model for studying attitude control problems of a satellite with flexible appendages, w.r. to an inertial coordinate frame, can be obtained by considering the kinematic equations together with the dynamical equations.

We recall that the kinematics, i.e. the dynamics of the attitude error, can be expressed as:

$$\dot{\vartheta} = \frac{1}{2} S(\omega)\vartheta = \frac{1}{2}\begin{pmatrix} 0 & -\omega_1 & -\omega_2 & -\omega_3 \\ \omega_1 & 0 & \omega_3 & -\omega_2 \\ \omega_2 & -\omega_3 & 0 & \omega_1 \\ \omega_3 & \omega_2 & -\omega_1 & 0 \end{pmatrix}\vartheta \qquad (1)$$

where the ω_i's, i = 1,2,3, denote the body rates and the ϑ_i's, i = 0,1,2,3 , denote the unitary quaternions subject to the constraint $\sum_{i=0}^{3} \vartheta_i^2 = 1$.

The dynamics of the body rates can be obtained by the well known Euler theorem:

$$\dot{L}_{tot} = -\omega \wedge L_{tot} + u_e \qquad (2)$$

once that the torques due to the reaction wheels and the flexible appendages have been modelled; in (2) u_e denotes the vector of external torques acting on the main body, L_{tot} the total angular momentum and "$\wedge$" denotes the usual vector product.

The dynamics of the reaction wheels is described by:

$$\dot{\Omega} = -\dot{\omega} + J_R^{-1}u_r \qquad (3)$$

where:

Ω: (3×1) vector of angular rates of the satellite reaction wheels;

u_r: (3×1) vector of reaction torques;

J_R: (3×3) diagonal matrix of the moments of inertia of the reaction wheels.

The dynamics of the flexible appendages is assumed, hereafter, to be described by:

$$M\ddot{\xi}+K\xi+C\dot{\xi} = H(\xi,\dot{\xi},\omega,\dot{\omega}) + F \qquad (4)$$

where:

ξ : (3N×1) vector representing the physical displacement of the particles which constitute the discretization of the flexible appendages;

M : (3N×3N) mass matrix;

k : (3N×3N) stiffness matrix;

C : (3N×3N) damping matrix;

H : (3N×1) vector representing the intertial forces (centrifugal, Coriolis and forces due to non uniform angular rate variation) due to the main body rotation;

F : (3N×1) vector of the external forces.

For our purposes we will assume that the external forces act uniquely on the rigid body; hence:

$$F = 0 \qquad (5)$$

Moreover:

$$L_{tot} = J\omega + J_R\Omega + L_{flex}(\xi,\dot{\xi}) \qquad (6)$$

in (6) the angular momentum due to the flexible appendages, without taking into account of the spin effect, is given by:

$$L_{flex}(\xi,\dot{\xi}) = \sum_{i=1}^{N_e} m_i(r_{oi} + \xi_i)\wedge \dot{\xi}_i \qquad (7)$$

where:

m_i : the mass of the i-th particle;

r_{oi}: (3×1) vector representing the position, w.r. to the satellite axis, of the particle in the undeformed structure.

Finally, denoting by H_i, i = 1,...,N , the i-th (3×1) subvector of H representing the inertial forces due to the main body rotation acting on the i-th particle, the general expression of H can be obtained recalling that

$$H_i = -m_i\{(r_{oi}+\xi_i)\wedge \dot{\omega} + [2\dot{\xi}_i+\omega\wedge(r_{oi}+\xi_i)]\wedge \omega\} \qquad (8)$$

The equations (1)÷(4) together with (5)÷(8) constitute the mathematical model which will be used in the simulations (S-model): it will be assumed to represent the "real" evolution of the system under study.

Our C-model is now obtained by considering a less accurate discretization of the flexible appendages, with $\bar{N} < N$ particles, and by neglecting the Coriolis and centrifugal effects in (4), i.e.

$$H = \Delta\dot{\omega} \qquad (9)$$

where:

Δ : $(3\bar{N}\times 3)$ matrix of coupling coefficients.

Simple manipulations on equations (1)÷(4) taking into account of (5),(6),(7) and (9) give a mathematical model of the form:

$$\dot{x} = f(x) + \sum_{i=1}^{m=3} G_i u_e + \sum_{i=1}^{3} \bar{G}_i u_r \qquad (10)$$

where the state x is the $[(10+6\bar{N})\times 1]$ vector:

$$x^T = (\vartheta^T\ \omega^T\ \Omega^T\ \xi^T(z = \dot{\xi})^T)^T$$

One has:

$$\begin{aligned} \dot{\vartheta} &= \frac{1}{2} S(\omega)\vartheta \\ \dot{\omega} &= -P1J^{-1}N(\omega,\Omega,z)-J^{-1}P2^T(C1z+K1\xi)+P1J^{-1}(u_e-u_r) \\ \dot{\Omega} &= -\dot{\omega} + J_R^{-1}u_r \\ \dot{\xi} &= z \\ \dot{z} &= -P2J^{-1}N(\omega,\Omega,z)-P3(C1z+K1\xi)+P2J^{-1}(u_e-u_r) \end{aligned} \qquad (11)$$

where:

$$\Delta 1 = M^{-1}\Delta,\ C1 = M^{-1}C,\ K1 = M^{-1}K$$

$$P1:=(I-J^{-1}\Delta 1^T\Delta 1)^{-1}$$

$$P2:=\Delta 1 P1 \qquad (12)$$

$$P3:=(I+\Delta 1 J^{-1}P2^T)$$

$$N(\omega,\Omega,z)=N_1(\omega)+N_2(\omega,\Omega)+N_3(\omega,z) =$$
$$= \omega\wedge J\omega + \omega\wedge J_R\Omega + \omega\wedge\Delta 1^T z$$

and:

J : (3×3) inertia matrix of the undeformed structure.

We will refer, hereafter, to the system of equations (11) as the C-model when both gas-jets and reaction wheels actuators are present; by putting $u_r = 0$, $u_e \neq 0$ the gas-jets control mode is reproduced, with $u_e = 0$, $u_r \neq 0$ the reaction-wheels control mode. We recall that the controls are assumed to act on the rigid part of the satellite.

3. THE SAMPLED NONLINEAR CONTROL LAW

In this section we will firstly recall the design of a continuous state feedback control law, acting on the system (11), proposed in [1]. The digital implementation of such a control is then discussed and an improved control law is proposed following the ideas developed in [2]. Finally, on the bases of those arguments it will be proposed a control low which makes use of partial information on the state (does not make use of the measures of the physical displacements and velocities $\xi,\dot{\xi}$) and whose digital implementation gives satisfactory results.

3.1. The continuous control law

Given a system of the form (10), fixed a set of m state dependent variables, y_i, is well known the condition under which it is possible to find a state feedback of the form

$$u(x) = \alpha(x) + \beta(x)v$$

in such a way that each y_i does depend only on the corresponding v_i (decoupling problem). Namely the following (m×m) matrix, A(x), must be invertible:

$$A(x) = \{\langle \frac{\partial}{\partial x} L_f^{d_i} y_i,\ G_j \rangle\}$$

where d_i is the smallest integer k such that

$$\langle \frac{\partial}{\partial x} L_f^k y_i,\ G_j \rangle \neq 0$$

for at least one j. Under that hypothesis the feedback

$$u(x) = A^{-1}(x)(v - \Gamma(x))$$

$$\Gamma(x) = \begin{pmatrix} L_f^{d_1+1} y_1 \\ \vdots \\ L_f^{d_m+1} y_m \end{pmatrix} \qquad (13)$$

is well defined on U the open subset of the state space over which A(x) is nonsingular. That control law not only provides a solution on U to the posed decoupling problem, but, regarding the variables y_i as outputs, the feedback system is diffeomorphic to a system which has a linear input-output dynamics and a possibly nonlinear, unobservable part [8]; in particular each input-output decoupled linear channel results to be an open loop chain of d_i+1 integrators. Hence, if the unobservable part of the system is asymptotically stable, the stabilization of the linear dynamics ensures the stabilization of the whole system. In conclusion, denoting by K a suitable (m×n) gain matrix, and by T(x) a coordinate transformation under which the input-output dynamic of the feedback system is linear the stabilization over U can be achieved under the control law (13) with:

$$v = KT(x) \qquad (14)$$

Such a control law can be used to close a prefixed maneuver by choicing $y_i = \vartheta$, i = 1,2,3, as discussed in [1].

Let us consider, firstly, the gas-jets control mode, $u_e \neq 0$, $u_r = 0$; it is an easy matter of computation to verify that one has:

$$d_1 = d_2 = d_3 = 1$$

$$A_{GJ}(x) = \frac{1}{2}\begin{pmatrix} \vartheta_o & -\vartheta_3 & \vartheta_2 \\ \vartheta_3 & \vartheta_o & \vartheta_1 \\ \vartheta_2 & \vartheta_1 & \vartheta_o \end{pmatrix} M2\cdot J^{-1} = \frac{1}{2} R(\vartheta)P1J^{-1} \quad (15)$$

$$\Gamma_{GJ}(x) = -\frac{1}{4}(\bar{\omega})^2\begin{pmatrix} \vartheta_1 \\ \vartheta_2 \\ \vartheta_3 \end{pmatrix} + \frac{1}{2}R(\vartheta)\begin{pmatrix} f_5 \\ f_6 \\ f_7 \end{pmatrix}$$

$$v = k_1\chi + k_2\dot{\chi} \qquad (\chi:=(\vartheta_1\ \vartheta_2\ \vartheta_3)^T)$$

where $|\vec{\omega}|^2 = \Sigma\omega_i^2$ and f_i, i = 5,6,7 denotes the components of the drift term in (11).(13),(14) and (15) together with an appropriate choice of the gain coefficients k_1 and k_2 characterize the control law which is defined on $U = \{x;\ \Sigma\vartheta_i^2 = 1$ and $\vartheta_o \neq 0\}$ ([1]).

It is a matter of computation to verify that the feedback system of Fig. 1 is diffeomorphic to the following system (16) under the coordinate transformation, on U:

$$T(x) = (\vartheta_o\ \ x^T\ \ \dot{x}^T\ \ \xi^T\ \ \hat{z}^T = (z-\Delta 1\omega)^T)$$

One has:

$$\dot{\vartheta}_o = -\frac{1}{\vartheta_o}\chi^T\dot{\chi}$$
$$\ddot{\chi} = k_1\chi + k_2\dot{\chi} + v' \qquad (16)$$
$$\dot{\xi} = \hat{z} + 2\Delta 1 R^{-1}(\vartheta_o,\chi)\dot{\chi}$$
$$\dot{\hat{z}} = -K1\xi - C1\hat{z} + 2C1\cdot\Delta 1 R^{-1}(\vartheta_o,\chi)\dot{\chi}$$

where v' is an auxiliary external input. The maneuvering problem is now reduced to the stabilization of the free evolution of the system (16) by fixing k_1 and k_2. Since χ and $\dot{\chi}$ goes exponentially to zero and $R^{-1}(\vartheta_o,\chi)$ to the identity matrix, the convergence to zero of ξ and z follows

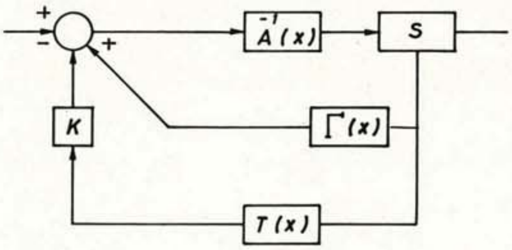

Fig. 1

It must be noted that the proposed continuous control law results to be the superposition of three control actions, say u_1,u_2 and u_3, which act as follows: u_1 is a direct compensation of the gyroscopic torques and the ones induced by the flexible appendages; u_2 introduces a nonlinear dynamics on ω in such a way that the links between $\ddot{\chi}$ and χ be a decoupled double chains of integrators; u_3 assignes the dynamics to each chain of double integrators. In fact, by developing the computations, from (13),(14) and (15) one has:

$$u(x)=u_1(x)+u_2(x)+u_3(x)=N(\omega,z)+JP1^{-1}J^{-1}P2^T(C1z+K1\xi)+$$
$$+\frac{\vec{\omega}^2}{2\vartheta_o}\chi+(\frac{2k_1}{\vartheta_o}\chi+k_2\omega)$$

The same computations can be developped in the reaction-wheels control mode ($u_e = 0$, $u_r \neq 0$); one has:

$$AR_W(x) = -A_{GJ}(x) \qquad \Gamma_{RW}(x) = +\Gamma_{GJ}(x)$$

We stress that the implementation of the designed control law is based on the knowledge of the whole state vector; moreover devices which work continuously w.r. to time are needed. It will be discussed hereafter the implementation of a sampled control law which is based on the availability of the sampled values of the angular rates.

3.2. The proposed sampled-data control scheme

Let us regard to the torques induced by the flexible appendages as disturbances. As before noted, the proposed control law acts by compensating those disturbances and we will denote by $u_f(x)$ such a control action. One has:

$$u(x)=(\frac{\vec{\omega}^2}{2\vartheta_o}+\frac{2k_1}{\vartheta_o})\chi+k_2\omega+\omega\wedge J\omega+$$
$$+u_f(x):=u_r(x)+u_f(x) \tag{17}$$

Let us now assume that a control action based on a rigid model of the spacecraft have been implemented; with the notations introduced this corresponds to the implementation of the control law $u_r(x)$. The corresponding dynamics of the angular rates is given by:

$$\dot{\omega}=[(\frac{2k_1}{\vartheta_o}+\frac{\vec{\omega}^2}{2\vartheta_o})\chi+k_2\omega]-P1J^{-1}u_f(x) \tag{18}$$

as can be easily verified.

From (18) the knowledge of $\dot{\omega}$ should be needed, for the computation of $u_f(x)$, while just the knowledge of the angular rates is assumed. As we look for a digital implementation of the whole control law we can use (18) to evaluate $u_f(x)$ at time $t = k\delta$ from the knowledge of ω over $[(k-1)\delta,k\delta]$ and $u_f(x)$ at $(k-1)\delta$.

We recall now that the desired behaviour of ω (corresponding to the application of u(x)), say ω_d, should satisfy:

$$\dot{\omega}_d=(\frac{2k_1}{\vartheta_o}+\frac{\vec{\omega}_d^2}{2\vartheta_o})\chi+k_2\omega_d \tag{19}$$

Equation (19) be sollved by means of a fast integrators over each time interval $[(k-1)\delta,k\delta]$ starting from $\omega_d((k-1)\delta)=\omega((k-1)\delta)$.

A good evaluation of $u_f(x)$ can kow be obtained from (18); one has:

$$u_f(x(k\delta))=u_f(x(k-1)\delta))+$$
$$+\frac{1}{\delta}JP1^{-1}(\omega_d(k\delta)-\omega(k\delta)) \tag{20}$$

with $u_f(x(0))=0$, since we assume $\xi(0)=z(0)=0$, and where $\omega(k\delta)$ denotes the measure of ω at time $k\delta$. The performance of the overall control system depend on the performance ensured by the digital implementation of the control law $u_r(x)$. The direct implementation of $u_r(x)$ into a sampled data control scheme does not give a satisfactory solution as evidentiated by the simulation (Fig. 4). In order to improve the whole digital control system we need to recall some fact ([2]).

Let us denote by F a fixed decoupling and linearising control law of the previous kind. It is clear that a piecewise constant control, obtained by the holding of the values computed by means of F from the sampling of the state evolution, does not give a precise solution to the control problem; more precisely the implementation of F into a sampled data control scheme gives a solution whose "quality" decreases by increasing the amplitude of the sampling (and holding) interval, δ. Such a problem was studied in [2] where an "extended feedback", $\tilde{F}$, was proposed to improve the performance of the control scheme. We recall brefly, hereafter, the main results of that study.

Definition: An approximated solution at order ℓ of a given control problem is achieved when the truncated Taylor series, at the order ℓ w.r. to δ, of the output satisfies the design properties.

Proposition: The implementation of the control law

(13)-(14) into a sampled data control scheme gives an approximated solution at order d_i+1 w.r. to any output y_i, $i = 1,\dots,m$, to the decoupling and linearization problem. □

Remark. each approximated output at the order d_i+1, results to be characterized by a linear dynamics with transfer function:

$$W_i(z) = \frac{\delta^{d_i+1}}{(d_i+1)!}\;\frac{(z-1)^{d_i}+(d_i+1)(z-1)^{d_i-1}\dots+(d_i+1)!}{(z-1)^{d_i+1}} \qquad (21)$$

Such a dynamics coincides with the exact sampling of d_i+1 integrator; for it is enough to note that:

$$\xi_1(k+1) = \xi_1 + \delta\xi_2 + \frac{\delta^{d_i+1}}{(d_i+1)}\, v_i(k)$$
$$\vdots$$
$$\xi_{d_{i+1}}(k+1) = \xi_{d_{i+1}} + \delta v_i(k) \;;\quad y_i = \xi_1$$

has the transfer function (21). Hence the approximated output at order d_i+1 has exactly the same value of the output of the continuous feedback control system at time $t = k\delta$.

The order of approximation can be increased by considering sampled-data control schemes with feedback control laws which depend on products of inputs. A feedback, $\tilde{F}$, which enables to get approximations at the order d_i+2 was proposed in [2]; it has the form:

$$\tilde{u}(x,v)=u(x,v)+\mathrm{Col}\left(\frac{\delta}{d_i+2}\, L_{f+\sum_{j=1}^{m} g_j u_j(x,v)}\, u_i(x,v)\right) \qquad (22)$$

it will be referred to as "extended control". The computation of the extended feedback, from $u_r(x)$, is developped in the Appendix.

In conclusion the proposed sampled-data control scheme that we propose is depicted in Fig. 2. On that scheme are based the simulations implemented which are discussed in the next section. As far as the functions of the blocks in Fig. 2 are concerned we have:

SM is the simulation model which represents the real dynamics of the spacecraft;

EC represents the computation of the extended control law $\tilde{u}_r(x(k\delta))$;

I1 represents the integration of the kinematic equations;

I2 represents the integration of the desired angular rates ω_d;

S and H represent sampling and holding of δ amplitude.

To conclude we note that the same arguments here developped can be used to characterize the control law when reaction-wheels actuators are used. In fact the simulations discussed in the next section use reaction wheels actuators.

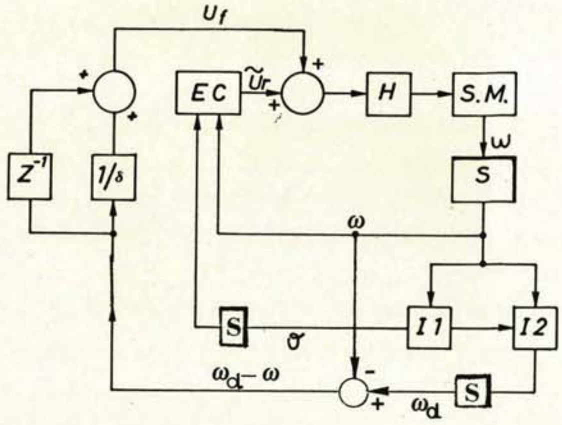

Fig. 2

4. SIMULATION RESULTS

The simulation model described in section 2 has been implemented on a VAX-780 digital computer. A test vehicle (Fig. 3) has been selected; it consists of a main rigid body with attached four booms; the structural parameters are reported in Table 1 together with the other simulation parameters. We note that the S-model takes into account of eight vibration modes while in the C-model only the antisymmetric vibration modes have been considered. Some user defined subroutines have been purposedly written to implement control laws, sensors and actuators.

Figures 4 to 7 illustrate the results of four simulations referred to an attitude maneuver of 120°; the variables plotted for each simulation are: the quaternion ϑ_o, the first flexible coordinate, z_1, the three control torques u_i, $i = 1,2,3$.

Fig. 4 shows the results obtained by a direct digital implementation of the control law u_r, together with u_f; the control torques are updated each 0,1 sec. The result obtained is not satisfactory since the amplitude of z_1 decreases slowly.

Fig. 5 shows the results obtained by means of the digital implementation of the extended control $\tilde{u}_r$; with the same updating of the control torques, each 0,1 sec., the results can be considered satisfactory since the attitude error and z_1 goe to zero in about 10 sec. By comparing the results figures 4 and 5 it is evident the improvement even if the holding time is small.

The implementation of the extended control $\tilde{u}_r$ gives very good results also by considerably increasing the holding time; Fig. 6 is referred to a simulation in which $\delta = 1$ sec. It must be noted that the direct implementation of the control law u_r should bring to instability after 5 seconds.

Finally Fig. 7 shows the results obtained by means of the combined control action of gas-jet and reaction-wheel actuators. The extended control $\tilde{u}_r$ is computed and a positive (negative) impulsive torque is applied just on the basis of the sign of $\tilde{u}_r$: a dead-band of 0,2 radiants has been considered. The zero error is achieved by using reaction wheels.

TABLE 1

Moment of inertia Jxx	= 1.0 $Kg.m^2$
Moment of inertia Jyy	= 1.52 $Kg.m^2$
Moment of inertia Jzz	= 1.98 $Kg.m^2$
length of booms 1 and 3; ℓ	= 1.4 m
lenght of boomb 2 and 4; ℓ	= 0.7 m
mass of booms m	= 0.45 Kg.
mass of main body M	= 2.0 Kg.
moment of inertia of R.Wheels	= 0.001 $Kg.m^2$
Elements of K matrix	= k_{i1} = k_{i2} =0.1
integration step d	= 0.1 sec
initial conditions:	
ϑ_o	= 0.5404
ϑ_i (i=1,2,3)	= -0.4858
ω_i (i=1,2,3)	= 0.0 rad/sec
z_i (i=1, N)	= 0.0 m.

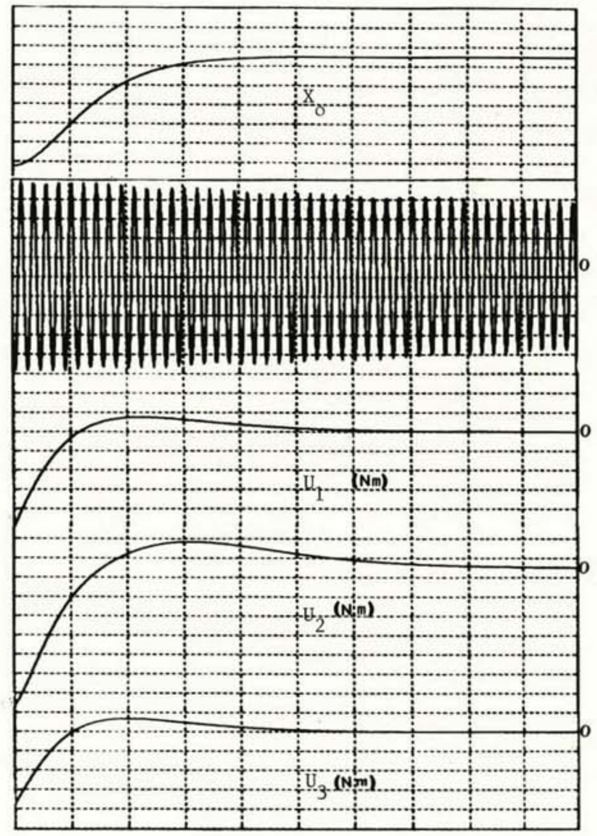

Fig. 4.

50 sec

5. CONCLUSIONS

A method for the design of attitude control systems for flexible spacecrafts has been presented. The design procedure employs input-output linearization and stabilization techniques; computation of the sampled-data control laws and compensation techniques have been applied in such a way to reduce the influence of the flexible part on the control system design. An idealized test vehicle subject to a variety of control laws has been simulated. The improvements obtained by using a sampled-data scheme with an extended control are evident from the results of the simulations.

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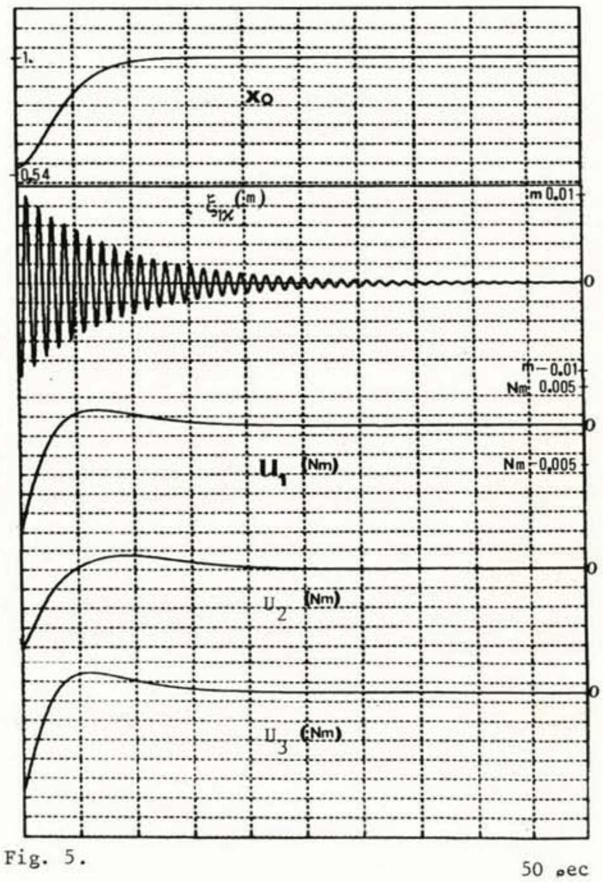

Fig. 5.

50 sec

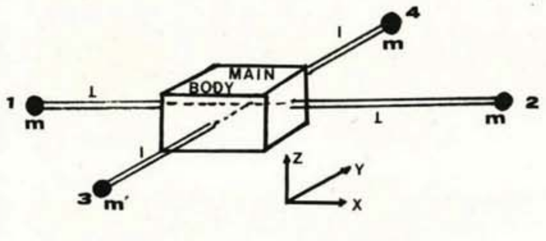

Fig. 3.

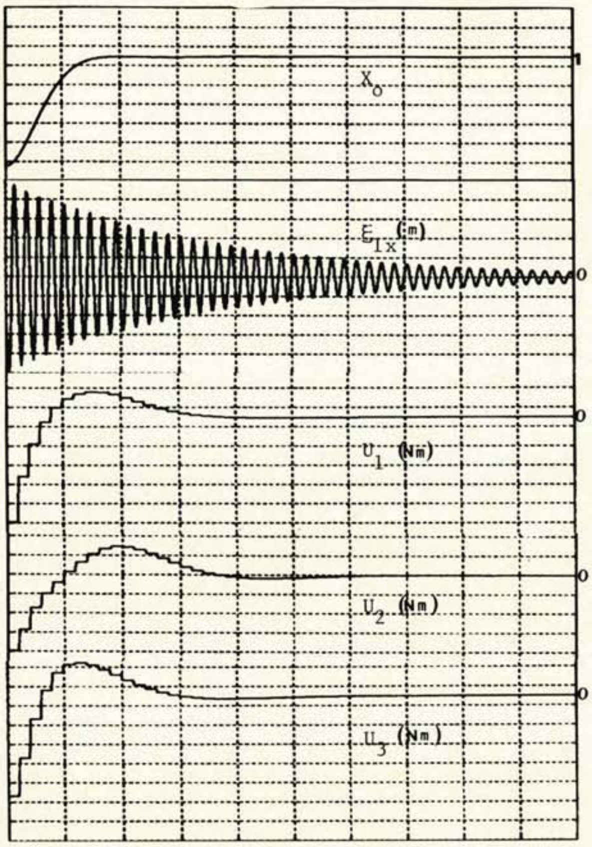

Fig. 6. 50 sec

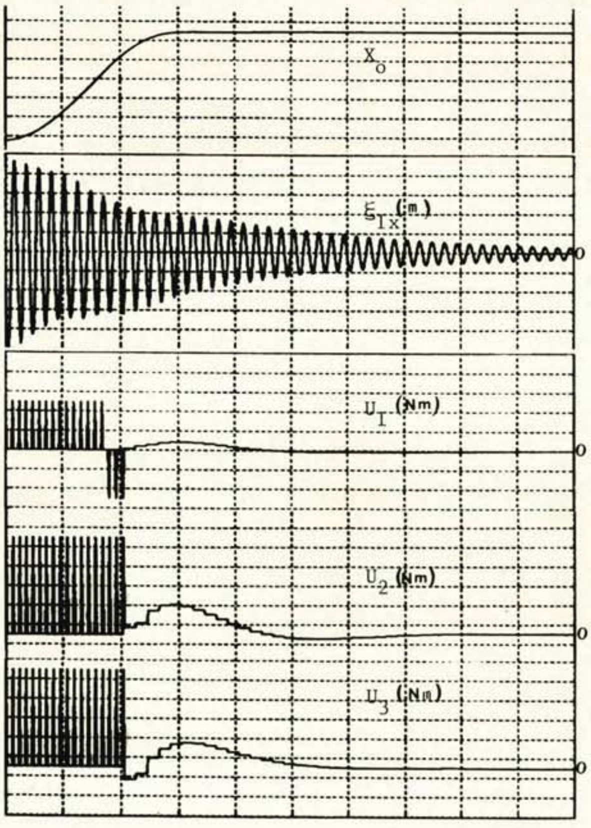

Fig. 7. 50 sec

REFERENCES

[1] S. Monaco and S. Stornelli, A nonlinear attitude control law for a satellite with flexible appendages, Proc. of the 24-th C.D.C. pp. 1654-59, Fort Landerdale, December 1985.

[2] S. Monaco, D. Normand-Cyrot and S. Stornelli, On the linearizing feedback in nonlinear sampled-data control schemes, submitted to the 25-th C.D.C., Athens december 1986.

[3] A. Isidori, Nonlinear control systems: an introduction, Lect. Notes on Contr. and Inf. Sci., n. 72, Springer-Verlag, 1985.

[4] T.A.W. Dwayer, Exact nonlinear control of large angle rotational maneuvers, IEEE Trans. on A.C., Vol. AC-29, n. 9, pp. 769-774, 1984.

[5] A. De Luca, A. Isidori and F. Nicolò, Control of robot arm with elastic joints via nonlinear dynamic feedback, Proc. of 24-th C.D.C., Fort Landerdale, pp. 1671-79, 1985.

[6] D. Dande, A. Glumineau and C.H. Moog, Nonlinear decoupling and inversion techniques applied to a single point moornig of a Tanker, Proc. of 24-th C.D.C., Fort Landerdale, pp. 1660-65, 1985.

[7] A. Isidori, A.J. Krener, C. Gori Giorgi and S. Monaco, Nonlinear decoupling via feedback: a differential geometric approach, IEEE on A.C. , Vol. AC-26, pp. 331-345, 1981.

[8] P.E. Crouch, N. Carmichael, A. Isidori and S. Monaco, Application of linear analytic control theory to attitude control, E.S.A. Contract Report from Applied System Study, Coventry U.K., 1981.

APPENDIX

The computation of the extended control associated to $u_r(x)$ in (17), for the system (11),(12) in the gas-jets control mode will be developed hereafter. Since $d_1 = d_2 = d_3 = 1$, one has:

$$\tilde{u}_r(x) = u_r(x) + \frac{\delta}{3} L_{f(x)+Gu\ (x)}\ u_r(x)$$

where:

$$u_r(x) = \omega \wedge J\omega + JP1^{-1}((\frac{2k_1}{\vartheta_o} + \frac{\omega^T\omega}{2\vartheta_o})\,\chi + k_2\omega)$$

We have:

$$\tilde{u}_r(\ x) = u_r(x) + \frac{\delta}{3}\frac{\partial u_r}{\partial x}(f(x) + Gu\ (x) \qquad (1A)$$

where:

$$\frac{\partial u_r}{\partial x} = (\frac{\partial u_r}{\partial \vartheta_o} \vdots \frac{\partial u_r}{\partial \chi} \vdots \frac{\partial u_r}{\partial \omega} \vdots \frac{\partial u_r}{\partial \xi} \vdots \frac{\partial u_r}{\partial z})$$

and:

$$\frac{\partial u_r}{\partial \vartheta_o} = -(\frac{2k_1}{\vartheta_o^2} + \frac{\omega^T\omega}{2\vartheta_o^2})JP1^{-1}\chi$$

$$\frac{\partial u_r}{\partial \chi} = (\frac{1}{2\vartheta_o}\omega^T\omega + \frac{2k_1}{\vartheta_o})JP1^{-1}$$

$$\frac{\partial u_r}{\partial \omega} = \begin{pmatrix} J_3-J_2 & 0 & 0 \\ 0 & J_1-J_3 & 0 \\ 0 & 0 & J_2-J_1 \end{pmatrix}\begin{pmatrix} 0 & \omega_3 & \omega_2 \\ \omega_3 & 0 & \omega_1 \\ \omega_2 & \omega_1 & 0 \end{pmatrix} + k_2 JP1^{-1} +$$

$$\frac{1}{\vartheta_o} JP1^{-1}(\omega_1\chi \,\vdots\, \omega_2\chi \,\vdots\, \omega_3\chi)$$

$$\frac{\partial u_r}{\partial \xi} = \frac{\partial u_r}{\partial z} = \underline{0}$$

Moreover:

$$f(x)+Gu(x) = \begin{pmatrix} \frac{1}{2}\, s(\omega)\vartheta \\ (\frac{2k_1}{\vartheta_o} + \frac{\omega^T\omega}{2\vartheta_o})\chi + k_2\omega \\ * \\ * \end{pmatrix}$$

Finally from (1A), one has:

$$\tilde{u}_r(x) = u_r(x) + \frac{\delta}{3}\{(\frac{\vec{\omega}^2}{2\vartheta_o} + \frac{2k_1}{\vartheta_o})JP1^{-1}[\frac{3}{2\vartheta_o}\chi^T\omega\chi + \frac{1}{2}R(\vartheta_o,\chi)\omega]$$

$$+ k_2 JP1^{-1}[(\frac{3\vec{\omega}^2}{2\vartheta_o} + \frac{2k_1}{\vartheta_o})\chi + k_2\omega]$$

$$+ [(\frac{2k_1}{\vartheta_o} + \frac{\vec{\omega}^2}{2\vartheta_o})\chi + k_2\omega] \wedge J\omega\}.$$