THE EFFECTS OF STRUCTURAL PERTURBATIONS ON DECOUPLED CONTROL

R A Calico & R L Hinrichsen

Air Force Institute of Technology
Wright-Patterson AFB, OH 45433
USA

ABSTRACT

The effects of structural perturbations on the decoupled control of a large space structure are considered. Control of the structure is accomplished through the use of multiple sub-controllers, each of which controls a subset of the spacecraft modes. The stability of the entire system is assured by constraining the gain matricies for the individual sub-controllers such that the stability of the system is not affected by the coupling between the sub-controllers. The effect of structural perturbations is to re-introduce coupling among the sub-controllers which may lead to instability. This coupling is shown to be related to changes in the row and column spaces of the individual control and observation matrices, respectively. A simple test for the determination of the effects of these changes is presented. The use of the test is evaluated on the control of the CSDL I spacecraft using three sub-controllers.

Keywords: Control, Large Space Structures, Spacecraft Dynamics

1. INTRODUCTION

The problem of controlling large space structures has received close attention in the past several years. Several techniques have been proposed to deal with the various technical problems associated with controlling these structures. Two problems in particular have been central to the control of large space structures. The first is the infinite dimensional nature of the system dynamical model. A realistic controller will be designed for a lower order approximation to the system. The effects of the controller on the modes not considered in the control model must be accounted for. These effects are commonly referred to as spillover. Higher order dynamical models offer a partial solution but at the expense of ease of implementation. In particular, a large order control model using full state feedback and a full state observer may not be able to be implemented in real time.

The second problem is the accuracy of the dynamical model. Typically, a structural model will be obtained using a finite element approximation. This model may contain several hundred modes for a reasonably complex structure. Even with these large order approximations errors in predicted mode shapes and frequencies are common for higher modes. Higher order finite element models may remove some of the errors but much of the error is due to the idealization of the structural elements and to nonlinearities in joints. These sources of error will be present in any model. Any control scheme for such systems must therefore be able to work on a system with uncertain dynamics.

A technique for controlling a large number of modes using a number of decoupled controllers has been developed by the first author and several others (Ref. 1 and 2). The technique partitions the modes of the structure into subsets. Each subset has a separate control system designed for just that set of modes. This allows for the active control of a large number of modes without having any one sub-controller be too large. The various sub-controllers will of course interact and very likely destabilize one another. This interaction is prohibited by constraining the sensor outputs and the actuator inputs such that the stability of the entire system is that of the individual controllers. The effect of inaccuracy in the structural model is to re-introduce coupling among the individual controllers.

This paper develops a technique to evaluate the effect of perturbed structural models on the performance of a decoupled control system. The technique is an open loop evaluation which only requires knowledge of the mode shapes of the original and perturbed systems. The amount of coupling re-introduced by perturbations is related to changes in the row and column spaces of the control and output matrices, respectively. The technique is demonstrated on the CSDL I spacecraft model. Twenty-five perturbed models were defined, each of which contained random changes in the stiffness of bar elements which make up the truss like structure. These stiffness changes were between ten and twenty percent of the nominal values. Several control designs consisting of three sub-controllers using state feedback and incorporating full state observers were accomplished for the nominal model. Each was then evaluated using the open loop criteria for each perturbed case and the results compared to the results of an eigenvalue analysis for that case.

Proceedings of the Second International Symposium on Spacecraft Flight Dynamics, Darmstadt, FR Germany, 20–23 October 1986 (ESA SP-255, Dec. 1986)

2. PROBLEM FORMULATION

The matrix second order differential equations for the motion of an n degree of freedom linear system are given by

$$M\ddot{q} + Kq = Du \tag{1}$$

where M and K are the mass and stiffness matrices. The forcing function has been expressed as the product of a matrix D and a vector u. For the current study this forcing function is due to n_a point force actuators. The elements of the vector u will represent the magnitude of the actuator forces and the matrix D will describe the location and orientation of the actuators. The system of equations (1) may be put in first order model form

$$\dot{x} = Ax + Bu, \tag{2}$$

where the 2n state vector x is composed of the n modal displacements and the n modal rates, in that order. The matrices A and B are of the form

$$A = \begin{bmatrix} 0 & I \\ -[\omega_i^2] & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \phi^T D \end{bmatrix} \tag{3}$$

The matrix $[\omega_i^2]$ is a diagonal matrix of modal frequencies squared and the matrix ϕ is the modal matrix for the system (1).

The output of the system (2) will be assumed to be measured with a combination of position and velocity sensors. The output y may therefore be expressed as

$$y = Cx = [C_p\phi \vdots C_v\phi]x \tag{4}$$

where the matrices C_p and C_v define the location and orientation of the position and velocity sensors, respectively.

3. DECOUPLED CONTROL

The state vector x is of order 2n and represents the entire spacecraft structural model plus the spacecrafts rigid body motion. The controller is typically based on a much smaller number of modes. If we assume that multiple controllers are present, each controlling n_i modes, the state vector x is conveniently represented by:

$$x = \{x_1^T, x_2^T, \ldots x_N^T, x_r^T\}^T = \{x_c^T, x_r^T\}^T \tag{5}$$

where the x_i represent vectors of dimension n_i of states controlled by the ith controller and x_r is an n_r -vector of residual states.

The controlled state, x_c is that portion of the state which must be controlled in order to insure satisfactory system performance. The determination as to which modes must be actively controlled and to which of the N controllers it is assigned is at the discretion of the control designer.

Using the representation of equation (5), we may now express our state equations in the following form:

$$\dot{x}_i = A_i x_i + B_i u \quad i = 1, \ldots, N \tag{6a}$$

and

$$\dot{x}_r = A_r x_r + B_r u \tag{6b}$$

In addition, the output equation has the form:

$$y = \sum_{i=1}^{N} C_i x_i + C_r x_r \tag{7}$$

where in the above equations,

$$A_j = \begin{bmatrix} 0 & I \\ [-\omega_j^2] & [-2\xi\omega_j] \end{bmatrix} \; ; \quad B_j = \begin{bmatrix} 0 \\ \phi^T D_{aj} \end{bmatrix}, \tag{8a}$$

$$j = 1, 2, \ldots, N, r$$

$$C_j = [C_{p_j}\phi, C_{v_j}\phi], \; j = 1, 2, \ldots, N, r \tag{8b}$$

The non-zero partitions of the matrices B_j have dimensions $n_j \times n_a$ and the partitions of the C_j are of diemsion $n_s \times n_j$ where n_a and n_s are the numbers of actuators and number of sensors. In the A_j matrices modal damping has been added.

Control for N individual system of the form:

$$\dot{x}_i = A_i x_i + B_i u$$

$$i = 1, \ldots, N \tag{9}$$

$$y = C\, x$$

is now considered. It is important to note that the control u is the same control for each of the N systems and hence couples them. In addition, the output y includes not just the modes of the ith system but all of the original system modes. Hence, using these outputs directly will couple the N sub-controllers. The coupling due to u is referred to as controller spillover and that due to y as observation spillover. Our interest is in designing the N sub-controllers such that the system stability is not affected by the spillover effects. To this end, we consider the conditions first for the removal of all spillover and then only those necessary to preserve stability.

The spillover terms may be eliminated from the controlled states in the following manner. Define new control variables v_i and new outputs w_i by the following relationships:

$$u = \sum_{i=1}^{N} T_i v_i \tag{10}$$

$$w_i = \Gamma_i y \tag{11}$$

The matrices T_i and Γ_i will be chosen to eliminate spillover effects. Substituting for u in (6a) and using y from (7) in (11) we obtain:

$$\dot{x}_i = A_i x_i + B_i \left(\sum_{j=1}^{N} T_j v_j \right) \quad (12a)$$

$$w_i = \Gamma_i \left(\sum_{j=1}^{N} C_j x_j \right) \quad (12b)$$

If all spillover is to be removed, we require that:

$$B_i T_j = 0$$

$$i = 1, \ldots, N; \; j = 1, \ldots, N \quad (13)$$

$$\Gamma_i C_j = 0$$

In this case, we have N decoupled relationships of the form:

$$\dot{x}_i = A_i x_i + B_i^* v_i \quad (14)$$

with decoupled outputs:

$$w_i = C_i^* x_i \quad (15)$$

where $B_i^* = B_i T_i$ and $C_i^* = \Gamma_i C_i$. Relationships (14) and (15) may be used to design N sub-controllers which when run simultaneously will not interact. Less restrictive conditions were developed in reference (1) which allowed for coupled but stable operation on N sub-controllers. These conditions are given by:

$$B_i T_j = 0$$

$$i = 1, \ldots, N-1$$

$$j = i+1, \ldots, N \quad (16)$$

$$\Gamma_i C_j = 0$$

In reference (1) full state feedback control using full state observers was implemented observing the conditions in (16). The results of that study showed that the stability of the controlled state x_c was the same as that of the sub-controllers. It should be noted that in all of the foregoing, the residual modes have been ignored. Spillover between modes in the controlled state x_c and the residual states x_r still exists but for a sufficiently large x_c should not be of significance.

3.1 Sensor and Actuator Requirements

Before moving to an example, a few words concerning the requirements for sensors and actuations are in order. First, consider the conditions given by expressions (16). In order to satisfy these expressions, the columns of T_N must be orthogonal to the rows of B_1 thru B_{N-1}. That is, the columns of T_N must be in the span of the null space of the matrix B_{1N}, where:

$$B_{IN} = \begin{bmatrix} B_1 \\ \hline \vdots \\ \hline B_{N-1} \end{bmatrix} \quad (17)$$

Assuming that B_{IN} is of full rank, the null space of B_{IN} has dimension:

$$P_{IN} = \left(n_a - \sum_{i=1}^{N-1} n_i\right) \quad (18)$$

Hence, for the matrix T_N to exist, the number of actuators n_a must be greater than the total number of modes controlled by the first N-1 controllers. That is:

$$n_a > \sum_{i=1}^{N-1} n_i \quad (19)$$

From expressions (10), we note that the dimensions of the control v_N for the nth controller is of dimension P_{IN}. That is, the dimensions of v_N equals the dimension of the null space of B_{IN}. Without the transformation T_N, the control u is of course of dimension n_a. This loss in control dimension is the price paid for the decoupling.

It is easily shown that the other conditions involving T_1 thru T_{N-1} can be met if inequality (19) is satisfied. Similarly, determining the Γ_i which satisfy expressions (16) requires that the number of sensors be such that:

$$n_a > \sum_{i=2}^{N} n_i \quad (20)$$

In a manner analogous to the control terms the dimensions of the outputs of the individual system w_i are in general less than the dimension of y. This is due to the dimensions of the matrices Γ_i.

3.2 Controller Design

We will use a three sub-controller control design. Each of the sub-controllers will be designed using full state feedback and a full state estimator. The sub-controllers will be decoupled from one another using the techniques of the previous two sections. Both the controllers and observers were designed using linear quadratic regulator theory. In all cases considered the state weighting matrices were used which were identity matrices multiplied by a constant value of twenty. The control weightings used were identity matrices. These same weightings were used for the observer designs.

4. THE MODEL

The CSDL I model shown in Figure 1 was used in this study. The model consists of a six bar truss element supported by three pair of rods. The bars forming the truss element and supports are assumed to be massless and capable of deformation only in the axial direction. The ten nodal locations are given in Table I. Concentrated masses are located at nodes one through four. The masses at these nodes were each two units. The areas of the respective rods are shown in Table II. Youngs modulus was assumed to be unity for the units used.

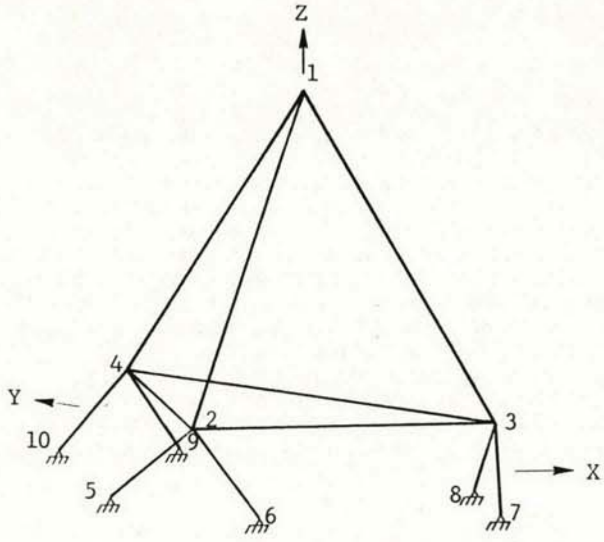

Figure 1. CSDL I Structural Model

The support legs of the truss element are assumed to contain colocated position sensors and force actuators. The twelve modal frequencies and associated mode shapes were determined using a finite element analysis. These were used along with the sensor and actuator locations and directions to form a state space control model in the form given by equations (2) through (4). Each mode of the structure was assumed to have modal damping with $\xi = .005$.

TABLE I - NODE COORDINATES

NODE	X	Y	Z
1	0.0	0.0	10.165
2	-5.0	-2.887	2.0
3	5.0	-2.887	2.0
4	0.0	5.7735	2.0
5	-6.0	-1.1547	0.0
6	-4.0	-4.6188	0.0
7	4.0	-4.6188	0.0
8	6.0	-1.1547	0.0
9	2.0	5.7735	0.0
10	-2.0	5.7735	0.0

TABLE II - ELEMENT CONNECTIVITIES AND AREAS

Element	Node 1	Node 2	Area
1	1	2	1000.
2	1	3	100.
3	1	4	100.
4	2	3	1000.
5	3	4	1000.
6	2	4	1000.
7	2	5	100.
8	2	6	100.
9	3	7	100.
10	3	8	100.
11	4	9	100.
12	4	10	100.

4.1 Structural Perturbations

Perturbations from this nominal model were obtained by randomly changing the areas of the six rods which make up the truss element. These rod areas were changed by a random amount between ten and twenty percent. Twenty-five such perturbed models were used. A finite element analysis of each perturbed model was accomplished and mode shapes and model frequencies determined. This information was then used to define the perturbed state, control and output matrices.

5. PERTURBATION EFFECTS ON DECOUPLING

The effects of structural perturbations on the decoupled control scheme are of two types. First the structural perturbations affect the individual sub-controllers. These changes to the matrices of equations (14) and (15) will cause root shifting and may cause instability in the individual controller/observer pairs. For the present analysis we will not address this problem. Our concern is with the second effect of the pertubations the re-introduction of coupling among the sub-controllers. The extent to which coupling is re-introduced can be quantified by considering changes in the control and output matrices for the individual controllers.

We recall from equations (16) that the sub-controllers are decoupled when these equations are satisfied. The transformation matrices T_j and Γ_i are determined using the nominal values for the individual control and output matrices. The columns of the T_j have the property that they are orthogonal to the rows of the appropriate B_i. Similarly, rows of the Γ_i are orthogonal to the columns of the appropriate C_j. Therefore changes in the B_i's and C_j's due to structural perturbations only re-introduce coupling to the extent that they change the row space of appropriate B_i's and the column space of the appropriate C_j's. That is, if for a perturbed case these spaces are unchanged then no coupling is introduced.

5.1 Coupling Indicators

Let a representative perturbed output matrix be given by C_p and its associated rows by c_{pi}. The extent to which these columns are contained within the column space of the unperturbed control matrix C, can be determined by considering the projection of c_{pi} onto the column space of C. Denoting this projection by $\hat{c}_{pi}$, we note

$$\hat{c}_{pi} = C(C^T C)^{-1} C^T c_{pi} \tag{21}$$

The extent to which c_{pi} has a component out of the column space of C can therefore be determined by calculating the angle between c_{pi} and $\hat{c}_{pi}$.

$$\cos \alpha_i = \frac{\hat{c}_{pi}^T \; c_{pi}}{\|\hat{c}_{pi}\| \; \|c_{pi}\|} \tag{22}$$

If the angle α_i is small the amount of coupling re-introduced by that row of C_p is small. If all the angles α_i are small then the perturbations in C should not lead to significant coupling among the sub-controllers.

In a directly analogous manner the change in the row space of the control matrix B due to perturbations may be expressed in terms of the angles B_i, where

$$\cos \beta = \frac{\hat{b}_{pi}^T \; b_{pi}}{\|\hat{b}_{pi}\| \; \|b_{pi}\|} \tag{23}$$

and

$$\hat{b}_{pi} = B^T (BB^T)^{-1} B b_{pi} \tag{24}$$

The angle β_i denotes the angle between a vector representing a row of the perturbed control matrix, B_p and the projection of that row into the row space of the unperturbed control matrix, B. Again, if the angles β_i are all zero $B_p T = [0]$ if $BT = [0]$. If the angles β_i are small the coupling introduced by the perturbations in B should be small.

The determinination of the angles β_i and α_i requires only a knowledge of the nominal and perturbed structural models. The computations required are quite simple and require no knowledge of the control design for the individual sub-controllers.

The decoupled control scheme requires that each modes be assigned to a specific sub-controller. These modal groupings have an effect on the calculation of the row and column space changes since they determine which modes are represented in a particular B_i and C_i pair. The calculation of the α_i's and β_i's for various modal groupings is a valuable aid in determining which modal groupings lead to the least sensitivity to model perturbations.

6. EXAMPLE

The control of the CSDL I model using three decoupled controllers is considered. In all cases considered eight modes are actively controlled and four modes are treated as residual modes. Full state feedback using full state observers is used for each individual sub-controller. The effects of coupling due to strutural perturbations are examined by evaluating the nominal controllers and observers on twenty-five perturbed systems.

The focus of the current work is to evaluate the coupling among the various sub-controllers which is re-introduced by the perturbations. The eigenvalues of the entire system however are changed by the pertubations not only due to this effect but also due to the coupling between the individual observer pairs and due to coupling with the residual modes. The coupling between the observer and controller in each sub-controller is noted but in the current work nothing is done in the control design of the individual sub-controllers to minimize the effects of perturbations. It should be noted however that the decoupling scheme can be used in conjunction with any gain selection method desired as it only requires designing the controller and observer for each sub-controller using the inputs defined by the various transformations. Changes in eigenvalues due to residual modes effects are also noted. However, since knowledge of the residual mode is not used in the control scheme the errors due perturbations are no more serious than those due to residuals in the nominal model. If the residual mode effects are large it is an indication that they probably should be accounted for in the control design. For example, this could lead to more modes being actively controlled or to reduced control being applied to those modes coupling heavily with the residuals.

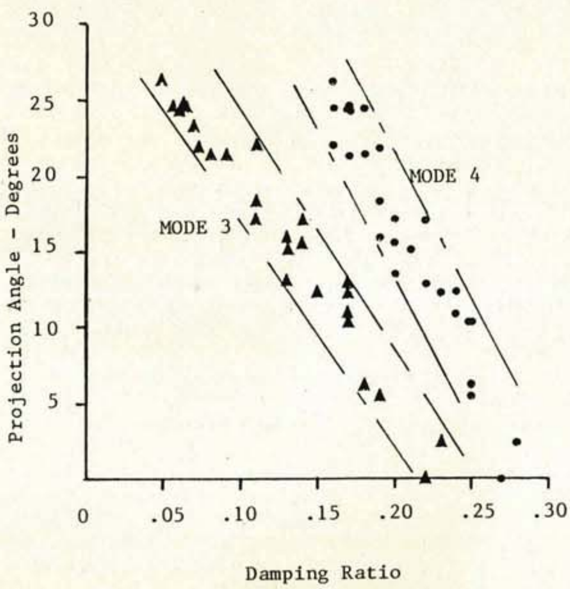

Figure 2. Modal Damping Changes due to Perturbations

A three controller system with modes 1,2 and 3 contained in sub-controller one, modes 4 and 5 in the second sub-controller and modes 6,7,8 in the third is considered first. For this controller modes 9 through 12 are residual. Figure 2 shows the damping ratio of the observer roots associated with modes three and four versus the angle that the row in the perturbed B_1 associated with mode three projections out of the row space of the nominal B_1. The figure shows strong correlation between the angle and loss in damping of modes three and four. The strong correlation between this angle and damping loss in mode four is due to the fact that the row in the perturbed B_2 associated with mode four projects out of the nominal row space of B_2 by essentially an identical amount. This damping loss is not entirely due to the intercontroller coupling but this is the primary effect. This can be seen by considering a typical perturbed case.

TABLE III

Comparative Modal Damping

Mode	Nominal	Perturbed With Residuals	Perturbed Without Residuals	Perturbed Without Inter-Controller Coupling
1c	.24	.25	.26	.25
1o	.005	-.0079	-.0070	-.0086
2c	.26	.26	.25	.32
2o	.17	.15	.18	.16
3c	.27	.32	.33	.33
3o	.22	.006	.0078	.25
4c	.41	.48	.52	.32
4o	.31	.16	.14	.32
5c	.38	.42	.41	.40
5o	.38	.41	.40	.34
6c	.38	.38	.38	.38
6o	.38	.38	.38	.38
7c	.005	.0044	.0044	.005
7o	.34	.34	.34	.34
8c	.005	.0055	.0055	.005
8o	.32	.32	.32	.32
9	.0041	.0043	—	—
10	.0045	.0043	—	—
11	.0041	.0047	—	—
12	.005	.005	—	—

Table III compares the modal damping of the entire system perturbed including residuals with that excluding the effects of residuals and that excluding the effects of both residuals and inter-controller coupling to the nominal control design. By comparing the columns in Table III we can determine the effect of the various types of coupling. The table shows the observer roots of mode 3, the control and observer roots of mode 4 and the observer root of mode 1 are the most affected by the perturbations. Observer mode 1 is in fact unstable. From consideration of the last column of Table III we note that this instability has been caused by coupling between the observer and controller roots of sub-controller one. In the nominal case mode 1 has only the light passive damping. The perturbations cause the nominally uncoupled observer/controller modes to couple and this drives observer root one unstable. By comparing the last column with the next to the last columns shows the effects of the coupling between the three sub-controllers. The damping of modes 3o and 4o are of particular interest. Both of these have lost a large part of their damping due to the inter-controller coupling. This effect has been shown for all twenty-five perturbed cases in Figure 2. While Table III shows results for only one perturbed case it is representative of the behavior for other cases.

Another three sub-controller case is considered with new modal groupings. For this case modes 3,4, and 5 are in sub-controller one, modes 1 and 2 in the second sub-controller. The residual modes and those in sub-controller three remain unchanged. These modal groupings showed much less effect due to inter-controller coupling. In no case was the change in damping due to inter-controller coupling more than 11%. This was also reflected in the projection angles out of the nominal row and columns spaces of the B_i and C_i for this controller. Table IV shows the average angles and their standard deviations for the controller for the twenty-five perturbed cases.

TABLE IV

Average Projection Angles

Matrix	Mode	Avg. Angle	Std. Deviation
B1	3	3.78°	.66°
B1	4	2.55°	.53°
B1	5	1.27°	.28°
C2,B2	1	.85°	.17°
C2,B2	2	.88°	.26°
C3	6	0°	0°
C3	7	1.13°	.21°
C3	8	1.24°	.33°

The angles in Table IV indicate that the inter-controller coupling introduced by the perturbations should be small. The eigenvalue analysis supports this result. As an example Table IV shows the eigenvalues of the nominal model compared to the entire perturbed model, to be perturbed model without residuals and to the eigenvalues of the perturbed model without residuals and inter-control coupling.

TABLE V

Comparative Modal Damping

Mode	Nominal	Perturbed With Residuals	Perturbed Without Residuals	Perturbed Without Inter-Controller Coupling
1c	.22	.26	.26	.26
1o	.13	.10	.11	.11
2c	.28	.28	.28	.29
2o	.18	.15	.17	.16
3c	.31	.36	.36	.36
3o	.044	-.0065	-.005	-.0055
4c	.35	.41	.43	.44
4o	.005	-.032	-.032	-.033
5c	.32	.28	.30	.30
5o	.41	.40	.38	.38
6c	.38	.38	.38	.38
6o	.38	.38	.38	.38
7c	.005	.005	.005	.005
7o	.34	.34	.34	.34
8c	.005	.005	.005	.005
8o	.32	.32	.32	.32
9	.0054	.003	—	—
10	.0052	.0052	—	—
11	.0038	.0052	—	—
12	.0051	.005	—	—

Comparison of the last two columns of Table V shows that very little change in modal damping due to the inter-controller coupling. This result agrees well with the small projection angles in Table IV. Again the coupling in the individual sub-controller controller/observer pairs causes instability.

Comparing the cases of Tables III and V shows that the case in Table V has much less inter-controller coupling. These two cases differ only in which modes are assigned to which sub-controller. Modal assignment then is an important determiner in the amount of coupling introduced due to perturbations. This result is also shown in comparing the magnitude of the angles calculated for each case. The perturbed rod areas for the two cases are shown in Table VI.

TABLE VI

Perturbed Rod Areas

Element	Case Table III	Case Table V
1	1161.1	1109.3
2	113.1	85.8
3	86.9	115.8
4	870.9	1150.9
5	1196.9	820.1
6	1108.2	1160.3

The strong influence of modal grouping on inter-controller coupling is explained by considering the frequencies associated with the nominal modes. Modes 3,4, and 5 are close in frequency as are those of modes 7 and 8. The eigenvectors associated with these groupings under perturbation tend to rotate as a unit. This is analogous to the behavior of eigenvectors associated with principal axes for a body with equal moments of inertia. For such a body a small perturbation in mass distribution would cause the corresponding eigenvectors to rotate through arbitrarily large angles to establish the new principal axes. While the angle rotated through could be large the space spanned by the two eigenvectors would remain nearly unchanged. Hence, modes with eigenvectors which move together (closely spaced frequencies) should be in the same sub-controller. When this is not the case, small perturbations may cause large rotations of individual rows or columns of the B_i or C_i. This leads to large angles for projections out of the appropriate row or column space and large inter-controller coupling. In the first case considered mode 3 was in the first sub-controller and modes 4 and 5 in the second. The rows associated with modes 3 and 4 in B_{1p} and B_{2p} consistantly gave rise to a large angle out of the space spanned by the rows of the nominal B_1 and B_2 hence gave rise to coupling. For the second modal grouping modes 3,4 and 5 were all in sub-controller one. Under perturbation this grouping gave rise to very small changes in the row space of B_1 and hence very little inter-controller coupling occurs.

7. CONCLUSIONS

Several conclusions can be drawn about the nature of inter-controller coupling introduced by structural perturbations. First, the coupling can lead to modal shifts which may be destabilizing. However, even for the relatively large perturbations considered, in the current study no instabilities were caused by this coupling. Second, the amount of coupling introduced is greatly affected by modal assignments to the various sub-controllers. Proper modal assignment can lead to a major reduction in the effect of perturbations. Finally, the open loop angle criteria presented is a very good indicator of coupling.

8. REFERENCES

1. Calico R A & Miller W T 1982, Decentralized control for a flexible spacecraft, AIAA/AAS Astrodynamics Conf. Paper No. 82-1401.

2. Calico R A & Thyfault D V 1984, Decoupled large space structure control, Proceedings AIAA Dynamics Specialists Conf. Palm Springs, CA 17-18 May 1984, 390-395.

3. RADC-TR-80-78 Vol II 1980, Acossfour (active control of space structures) theory appendix, Charles Stark Draper Laboratory.