ACTIVE VIBRATION DAMPING OF FLEXIBLE STRUCTURES USING THE TRAVELLING WAVE APPROACH

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ABSTRACT

A traveling wave approach is presented for the vibration control of networks of slender flexible structural components. The performance of the resulting controller is evaluated and compared with a "classical" modal controller by use of a simple model of a prestressed string. Both controllers are therefore tested in numerical simulations. The traveling wave approach is demonstrated to have significant advantages. In particular, it is insensitive to a change in the system's boundary conditions.

Keywords: active vibration damping, traveling wave approach, modal controller, robustness.

1. INTRODUCTION

Active vibration damping is being seriously considered for large flexible space structures, and to some extent also in the more traditional branches of engineering. It has been studied intensively during the last decade almost exclusively with regard to potential applications in aeronautics and astronautics.

In the traditional approach the structure is discretized, via finite elements or some other technique, and the problem of controlling a distributed parameter system is thus substituted by a new problem described by a system of ordinary differential equations. The powerful techniques developed for discrete control problems are then applied, methods such as pole placement or optimal control being used.

Recently, a new approach has been proposed (Ref.1,2). In this new technique the structure is not discretized but split up into structural elements, such as beams, cables, etc., each of these elements being a simple continuous system. The boundary and transition conditions are then dropped in the first stage of the control design. The motion in the structural elements is described by traveling waves and the controls are designed with intent to absorb as much of the energy of these traveling waves as possible. Instead of solving a control problem for a large system of ordinary differential equations, the local control of an unbounded medium described by partial differential equations is thus examined and a number of local and uncorrelated control problems has to be solved. It turns out that the solution of these local problems is relatively simple, at least for bars, cables (Ref.3) and beams, although it requires a new measurement technique to divide the motion of the elements into contributions coming from waves traveling into different directions. This technique is closely related to intensity measurements used in acoustics, but has not yet been implemented for the application in vibration control (Ref.4). A further analysis shows that controls designed in this manner for the unbounded medium will also work for a finite medium with arbitrary boundary conditions.

The present paper describes the main results obtained during the last year at the Institut für Mechanik in Darmstadt concerning the control of bars and strings. In a model problem of a fixed-fixed string with 2 control forces it is shown how the active damping device can be designed using traveling waves. The performance of this new vibration control is then compared to a vibration control designed via a "classical" modal analysis technique. The analysis and the numerical simulations show that the new approach presents considerable advantages in several aspects. This is illustrated by changing the boundary conditions for the model problem. While the modal controller designed for the fixed-fixed string fails, the proposed controller is robust against these changes.

2. STRING MODEL

In this paper, a prestressed string is used to evaluate the performance of different controller designs. The transverse vibrations of the string are described by the wave equation

$$\ddot{w}(x,t) - w''(x,t) = q(x,t) \quad , \tag{1}$$

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where x denotes the space coordinate, t the time and $w(x,t)$ the string's transverse displacement. The wave speed has been normalized to one by an appropriate choice of the time scale. As depicted in Fig. 1, the string is fixed at $x = 0$ and elastically supported at $x = \pi$ by a spring of stiffness $(1-\rho)/\rho$. The constant ρ can be chosen arbitrarily from the interval $[0,1]$, $\rho = 0$ corresponding to a fixed and and $\rho = 1$ to a free end. These boundary conditions can be written as

$$w(0,t) = 0 \quad , \tag{2}$$

$$(1-\rho)\, w(\pi,t) + \rho\, w'(\pi,t) = 0 \quad . \tag{3}$$

For vibration control two force actuators are attached to the string at $x = a_1$ and $x = a_2$ respectively, $0 < a_1 < a_2 < \pi$. The time histories of the control forces are denoted by $u_1(t)$ and $u_2(t)$. The external load $q(x,t)$ in (1) is then given by

$$q(x,t) = \sum_{\ell=1}^{2} \delta(x-a_\ell)\, u_\ell(t) \quad , \tag{4}$$

where $\delta(.)$ stands for the DIRAC function. Additionally, two sensors are located at $x = s_1$ and $x = s_2$ respectively, $0 < s_1 < s_2 < \pi$, measuring the transverse displacements

$$m_k(t) := w(s_k,t) \quad , k = 1,2 \quad , \tag{5}$$

at these locations. If the sensors are located near to each other, the measurements determine $\dot{w}(\tilde{x},t)$ and $w'(\tilde{x},t)$ for $\tilde{x} = (s_1 + s_2)/2$, through simple formulae, as we shall see in 3.1.2.

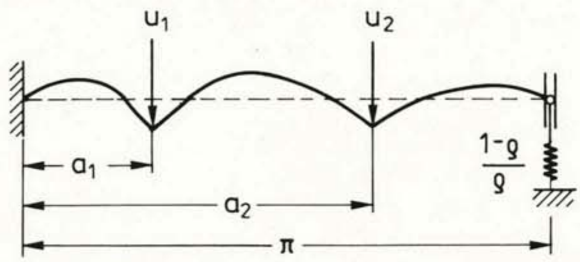

Figure 1. String model

3. CONTROLLER DESIGN

3.1 Traveling wave approach

3.1.1 Determination of the control laws. It is well-known that every solution of the wave equation (1) with zero excitation term $q(.,.)$ can be written in the form

$$w(x,t) = f(x-t) + g(x+t) \quad , \tag{6}$$

with appropriate functions $f(.)$ and $g(.)$ (d'ALEMBERT's formula), representing waves traveling to the right and left respectively. For an infinite string it can furthermore be shown that a concentrated force $u(t)$ applied at $x = a$ generates the waves:

$$w(x,t) = \begin{cases} \frac{1}{2} \int_0^{t+(x-a)} u(\tau) d\tau \;, & x < a \;, \\ \frac{1}{2} \int_0^{t-(x-a)} u(\tau) d\tau \;, & x > a \;. \end{cases} \tag{7}$$

In (7) the integrals are replaced by zero if the upper bound is smaller than the lower bound. With (6) and (7) we are now able to express any solution of (1), (4) in the form

$$w(x,t) = \begin{cases} f(x-t) + g(x+t) \;, & a_1 < x < a_2 \;, \\ r(x-t) + g(\tilde{x}+(t+x-\tilde{x})) + \frac{1}{2} \int_0^{t+(x-a_1)} u_1(\tau)\, d\tau \;, & x < a_1 \;, \\ l(x+t) + f(\tilde{x}-(t-x+\tilde{x})) + \frac{1}{2} \int_0^{t-(x-a_2)} u_2(\tau)\, d\tau \;, & x > a_2 \;. \end{cases} \tag{8}$$

$\tilde{x}$ is selected from (a_1,a_2) and corresponds to the location of the measurement unit. The functions $r(.)$ and $l(.)$ stand for the waves traveling from the boundaries towards the section between the two actuators.

For the determination of the control forces we calculate the energy carried away from the section $[a_1,a_2]$ per unit of time. In dimensionless notation it is given by:

$$\begin{aligned} P(t) &= w'(a_1^-,t)\dot{w}(a_1^-,t) - w'(a_2^+,t)\dot{w}(a_2^+,t) \\ &= [g'(\tilde{x}+(t+a_1-\tilde{x})) + \tfrac{1}{2} u_1(t)]^2 \\ &\quad - [r'(a_1-t)]^2 - [l'(a_2+t)]^2 \\ &\quad + [f'(\tilde{x}-(t-a_2+\tilde{x})) - \tfrac{1}{2} u_2(t)]^2 \quad . \end{aligned} \tag{9}$$

This expression is now minimized by choosing the controls such that the two terms with positive sign vanish identically. We thus obtain the control laws

$$\begin{aligned} u_1(t) &:= -2\, g'(\tilde{x}+(t+a_1-\tilde{x})) \quad , \\ u_2(t) &:= \;\; 2\, f'(\tilde{x}-(t-a_2+\tilde{x})) \quad . \end{aligned} \tag{10}$$

3.1.2 The measurement problem. The control laws (10) require the knowledge of the traveling waves $f(.)$ and $g(.)$ at the location $x = \tilde{x}$ and at the times $t - (\tilde{x}+a_1)$ and $t - (a_2-\tilde{x})$, respectively. Suppose that we have measurements

$$\begin{aligned} m_1(t) &= w(s_1,t) \quad , \\ m_2(t) &= w(s_2,t) \end{aligned} \tag{11}$$

in the vicinity of $\tilde{x} = (s_1+s_2)/2$. We can then approximate the velocity and the slope of the string at $\tilde{x}$ by

$$\dot{w}(\tilde{x},t) \approx \frac{\dot{m}_1(t) + \dot{m}_2(t)}{2} \quad ,$$
$$w'(\tilde{x},t) \approx \frac{m_2(t) - m_1(t)}{s_2 - s_1} \quad . \tag{12}$$

Better approximations can be derived by use of higher order derivatives of $m_1(.)$ and $m_2(.)$. They are discussed in Ref.4 . From d'ALEMBERT's formula (6) we further obtain

$$g'(\tilde{x}+t) = \frac{1}{2}\,[w'(\tilde{x},t) + \dot{w}(\tilde{x},t)] \quad ,$$
$$f'(\tilde{x}-t) = \frac{1}{2}\,[w'(\tilde{x},t) - \dot{w}(\tilde{x},t)] \tag{13}$$

which together with (12) gives $u_1(t)$ and $u_2(t)$ in terms of the measurement:

$$u_1(t) = -\left[\frac{m_2-m_1}{s_2-s_1} + \frac{\dot{m}_2+\dot{m}_1}{2}\right]\Bigg|_{t+(a_1-\tilde{x})} \quad ,$$
$$u_2(t) = \left[\frac{m_2-m_1}{s_2-s_1} - \frac{\dot{m}_2+\dot{m}_1}{2}\right]\Bigg|_{t-(a_2-\tilde{x})} \quad . \tag{14}$$

Note that due to $a_1-\tilde{x} < 0$, $a_2-\tilde{x} > 0$, the control law has a delay character. No boundary conditions have been used for determining (14).

3.2 "Classical" modal approach

The "classical" controller design starts with determination of eigenfunctions for the system under consideration. In our model problem we choose $\rho = 0$ as the reference system so that the eigenfunctions are

$$\Phi_k(x) = (1/\sqrt{2})\,\sin kx \;,\; k\in\mathbb{N} \quad . \tag{15}$$

The solution of (1) is then expanded in the form

$$w(x,t) = \sum_{k=1}^{\infty} z_k(t)\,\Phi_k(x) \tag{16}$$

and after projecting (1) on the eigenfunctions, the infinite system of ordinary differential equations

$$\ddot{z}_k(t) + k^2 z_k(t) = \sum_{\ell=1}^{2} \Phi_k(a_\ell)\,u_\ell(t) \;,\; k\in\mathbb{N} \quad , \tag{17}$$

is obtained.

Similarly, the measurements can be written as

$$m_i(t) = \sum_{k=1}^{\infty} \Phi_k(s_i)\,z_k(t) \;,\; i = 1,2 \quad . \tag{18}$$

Truncating the infinite sum in (16) at some finite N and writing the differential equations as a first order system gives

$$\dot{\mathbf{y}}(t) = \mathbf{A}\,\mathbf{y}(t) + \mathbf{B}\,\mathbf{u}(t) \tag{19}$$

with

$$\mathbf{y}(t) := [z_1(t),..,z_N(t),\dot{z}_1(t)/1,..,\dot{z}_N(t)/N]^T \in \mathbb{R}^{2N} \;,$$
$$\mathbf{A} := \begin{bmatrix} 0 & \Lambda \\ -\Lambda & 0 \end{bmatrix} \in \mathbb{R}^{2N\times 2N} \;,$$
$$\Lambda := \mathrm{diag}\,(1,2,..,N) \in \mathbb{R}^N \;, \tag{20}$$
$$\mathbf{B} := \begin{bmatrix} 0 \,..\, 0 & \Phi_1(a_1)/1 \,..\, \Phi_N(a_1)/N \\ 0 \,..\, 0 & \Phi_1(a_2)/1 \,..\, \Phi_N(a_2)/N \end{bmatrix}^T \in \mathbb{R}^{2N\times 2} \;.$$

This system is called "design system" for the present control problem. Optimal linear state feedback

$$\mathbf{u}(t) = -\mathbf{F}\,\mathbf{y}(t) \;,\quad \mathbf{F} \in \mathbb{R}^{2\times 2N} \quad , \tag{21}$$

will be used in the present example. The constant gain matrix $\mathbf{F}$ is obtained from the solution of the well-known algebraic matrix RICCATI equation (Ref.5). In the quadratic integral criterion the matrix

$$\mathbf{R}_1 = \mathrm{diag}(0,..,0,7,..,7) \in \mathbb{R}^{2N\times 2N} \tag{22}$$

is used as weighting matrix for the state variable $\mathbf{y}$. The corresponding matrix for the controls is set equal to

$$\mathbf{R}_2 = \mathbf{I} \in \mathbb{R}^{2\times 2} \quad . \tag{23}$$

The number of sensors typically being smaller than the number N of modes occcuring in (19), the measurements $m_1(.)$ and $m_2(.)$ must be fed to an observer for obtaining an estimate $\hat{\mathbf{y}}(t)$ of the state vector. Here a linear optimal observer (KALMAN-BUCY filter)

$$\dot{\hat{\mathbf{y}}}(t) = \mathbf{A}\hat{\mathbf{y}}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{K}\,[\mathbf{m}(t) - \mathbf{C}\,\hat{\mathbf{y}}(t)] \tag{24}$$

is used, $\mathbf{m}(.)$ and $\mathbf{C}$ being defined by

$$\mathbf{m}(t) := [m_1(t),\, m_2(t)]^T \in \mathbb{R}^2 \;,$$
$$\mathbf{C} := \begin{bmatrix} \Phi_1(s_1) \,..\, \Phi_N(s_1) & 0 \,..\, 0 \\ \Phi_1(s_1) \,..\, \Phi_N(s_2) & 0 \,..\, 0 \end{bmatrix} \in \mathbb{R}^{2\times 2N} \quad . \tag{25}$$

The optimal observer gain matrix $\mathbf{K} \in \mathbb{R}^{2N\times 2}$ is obtained from the solution of the corresponding algebraic observer RICCATI equation (Ref.5). The covariance matrices occuring in this equation are chosen to be

$$\mathbf{V}_1 = (0,.,0,2,..,2) \in \mathbb{R}^{2N\times 2N} \tag{26}$$

for the noise term in the equation of motion (19) and to be

$$\mathbf{V}_2 = \mathbf{I} \in \mathbb{R}^{2\times 2} \tag{27}$$

for the noise term in the corresponding measurement equation. The observer equation has to be integrated numerically during the operation of the controller and the state variable $\hat{\mathbf{y}}(t)$ is then used as an approximation to the system's state $\mathbf{y}(t)$. The feedback law (21) is thus replaced by

$$\mathbf{u}(t) = -\mathbf{F}\,\hat{\mathbf{y}}(t) \quad . \tag{28}$$

It should be pointed out that in this short review only the basic principles of the "classical" modal approach can be listed, completeness not being sought.

4. NUMERICAL SIMULATIONS

4.1 Approximations

The performance of the two types of control is now compared via numerical simulations. The evaluation model used consists of eqs. (1)-(3), discretized by finite differences.

4.1.1 Difference scheme. The wave equation (1) is discretized using the simplest explicit difference scheme

$$\frac{W^{j,n+1}-2W^{j,n}+W^{j,n-1}}{(\Delta t)^2} - \frac{W^{j+1,n}-2W^{j,n}+W^{j-1,n}}{(\Delta x)^2} = 0 \ , \tag{29}$$

where the abbrevation

$$W^{j,n} = W(j\,\Delta x, n\,\Delta t) \tag{30}$$

holds. The boundary conditions and the transition conditions at the actuator locations are approximated by skew differences having the same accuracies as those in (29). In all simulations

$$\Delta x = \pi/100 \ , \qquad \Delta t = \Delta x/2 \tag{31}$$

are fixed.

4.1.2 Measurements. Determination of controls by the traveling wave approach according to (14) requires the knowledge of $\dot{m}_1(.)$ and $\dot{m}_2(.)$. In the simulations they are approximated by use of differences

$$\dot{m}_i(t) := \frac{m_i(t+\Delta t) - m_i(t-\Delta t)}{2\,\Delta t} \quad , \ i = 1,2 \quad , \tag{32}$$

which are of second order accuracy with respect to Δt.

4.1.3 The observer. Integration of the observer equation (24) is performed with the scheme

$$\hat{\mathbf{Y}}(t+\Delta t) = \hat{\mathbf{Y}}(t) + \Delta t\ [(\mathbf{A}-\mathbf{KC})(\mathbf{I}+ \frac{\Delta t}{2}\,(\mathbf{A}-\mathbf{KC}))]\ \hat{\mathbf{Y}}(t) + \Delta t\ [\mathbf{B}\,\mathbf{u}(t) + \mathbf{K}\,\mathbf{m}(t)] \tag{33}$$

where $\hat{\mathbf{Y}}(.)$ denotes the approximation to $\hat{\mathbf{y}}(.)$.

4.2 Systems under consideration

A first comparison is done for our reference system ($\rho = 0$), i.e. for the string with both ends clamped. For the traveling wave approach actuators are located at

$$a_1 = .55\,\pi \ , \qquad a_2 = .6\,\pi \ , \tag{34}$$

and sensors at

$$s_1 = .57\,\pi \ , \qquad s_2 = .59\,\pi \ . \tag{35}$$

For the "classical" modal approach actuators and sensors are placed at

$$a_1 = s_1 = .18\,\pi \ , \quad a_2 = s_2 = .9\,\pi \ . \tag{36}$$

The number of modes included in the modal aproach is set equal to

$$N = 8 \ . \tag{37}$$

In both cases initial data are chosen as

$$w(x,0) = \sin(3\pi\,(x/\pi)^2) \ , \qquad \dot{w}(x,0) = 0 \ . \tag{38}$$

The string's motion with acting controls is depicted in Fig. 2a and 3a. Corresponding time histories of the total energy contained in the system are plotted in Fig. 2b and 3b.

The same control laws determined for the nominal "design system" are now used for the system with $\rho = .5$. In this case, due to the elastic support the eigenfunctions are different from the nominal ones. These simulations do therefore represent a test for robustness of the control law with respect to changes in the boundary conditions. All actuator and sensor locations and initial data correspond to the first case. The results of this simulation are given in Fig. 4a,b and Fig. 5a,b.

4.3 Comparison of results

We begin the discussion of results examining the performance of the newly proposed controller. Figures 2 and 4 clearly show that the controller designed via the travelling wave approach works well in both examples. In the first case ($\rho = 0$) all waves are completely anihilated after a finite time t_1. This time is exactly twice the time required by a wave to travel from the measurement location $\tilde{x}$ to the boundary farthest away from $\tilde{x}$, i.e. to the left boundary. In the second case ($\rho = .5$) a qualitatively similar behavior can be observed. However, due to the spring at the right boundary, a small amount of energy is still contained in the string between the actuator at $x = a_2$ and the right boundary for times greater than t_1. Calculating the motion of the string under action of the controls (10) for boundary conditions (3) with $\rho = .5$ we obtain

$$w(x,t) = (f_0+l_2)\,\exp\left\{-\,\frac{\rho}{1-\rho}\,(x+t-(\pi+t_2))\right\} \ , \tag{39}$$

where $x \in (a_2,\pi]$ and $x+t > \pi+t_2$, t_2 being the time required by a wave to travel from $\tilde{x}$ to the right boundary. Furthermore the definitions

$$f_0 := f(\tilde{x} - 0) \quad ,$$
$$l_2 := l(\pi + t_2) \qquad (40)$$

hold. Therefore, the amount of energy contained in the string for $t > t_1$ decays exponentially.

It must be pointed out that all the simulations were carried out by replacing the "exact" control law (10) by the approximation (14). Nevertheless, the controller worked very efficiently and looks promising for practical implementations in future.

Finally, it should be remarked that the controller using only two actuators shows a certain bandlimitation. It can only control harmonic waves with wavelength λ satisfying

$$\frac{\lambda}{2} > a_2 - a_1 \quad . \qquad (41)$$

From this point of view it seems advisable to choose a small spacing between the actuators. From the mathematical point of view the difficulty can be circumvented by adding a third actuator at $x = a_3$, $a_1<a_3<a_2$, in such a manner that $(a_3-a_1)/(a_2-a_3)$ is an irrational number.

Next, we discuss performance of the "classical" modal controller. As shown in Fig. 3 the modal controller works in a satisfying manner for the nominal system ($\rho = 0$). During the simulations it was also tried to obtain a faster decay of the total energy by "increasing" the weighting matrix $\mathbf{R}_1$ in the design criterion. In these cases, however, the controller applied to the evaluation model became unstable, probably due to spill-over effects in the observation and the control problem. In the second case ($\rho = .5$, Fig.5) the controller is unstable. This behavior is due to the fact that in our modal model the functions $\phi_k(.)$ given by (15) no longer represent the eigenmodes of the evaluation model. Modal controllers always require exact knowledge of these eigenfunctions!

To conclude our discussion we shortly compare the computational requirements for both controller designs. For the traveling wave approach evaluation of (14) together with (32) is required at each instant. Additionally, an intermediate storage must be used to take care of the time shifts as given in the arguments in (14). All these operations can be implemented in a very efficient way on a digital computer. For the modal controller much more efforts are to be made. In the design stage large eigenvalue problems are solved for determining the feedback and the KALMAN matrix. (Our design model with eight modes already requires solution of two 32x32 eigenvalue problems.) In the operational stage of the modal controller integration of the observer equation and an additional matrix multiplication for determination of the controls according to (28) are necessary at each instant.

Summarizing, it can be said that the design of active vibration damping devices via the traveling wave approach seems to have certain advantages which justify its further study and related practical experiments. In Darmstadt this method is presently being developed for dispersive waves (bending vibrations in beams).

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TRAVELING WAVE APPROACH

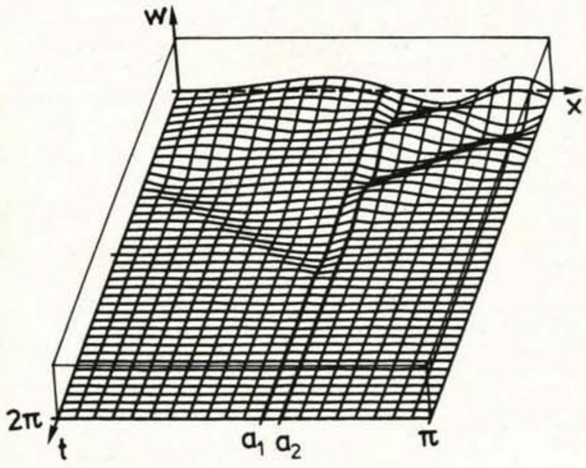

Figure 2a. w(x,t) for ρ = 0

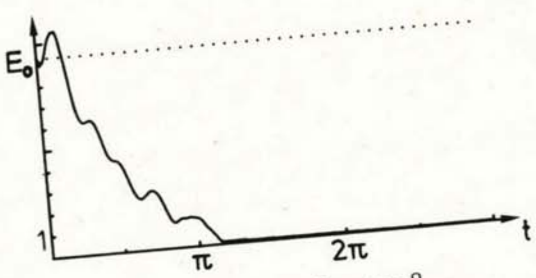

Figure 2b. Total energy for ρ = 0

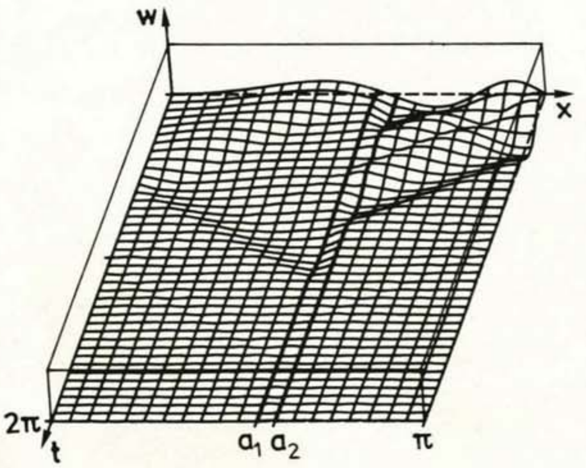

Figure 4a. w(x,t) for ρ = .5

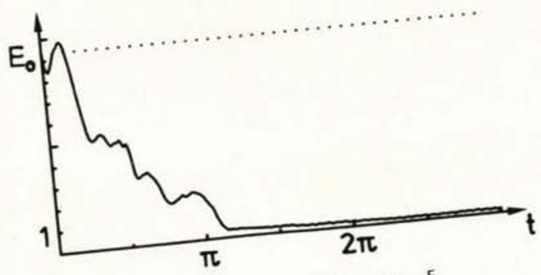

Figure 4b. Total energy for ρ = .5

MODAL CONTROLLER

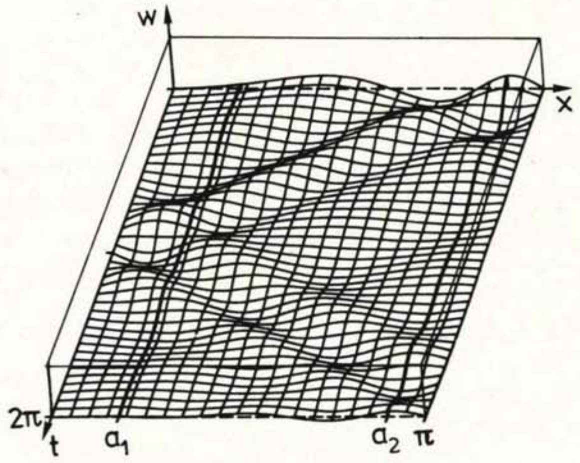

Figure 3a. w(x,t) for ρ = 0

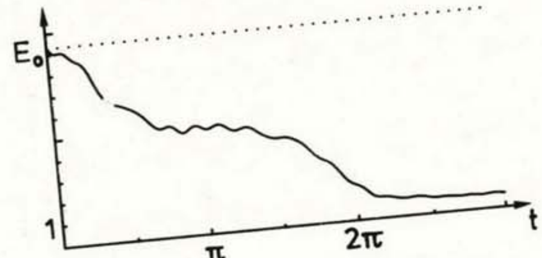

Figure 3b. Total energy for ρ = 0

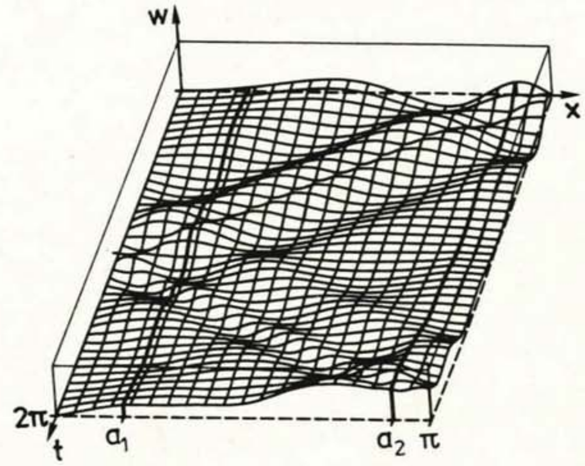

Figure 5a. w(x,t) for ρ = .5

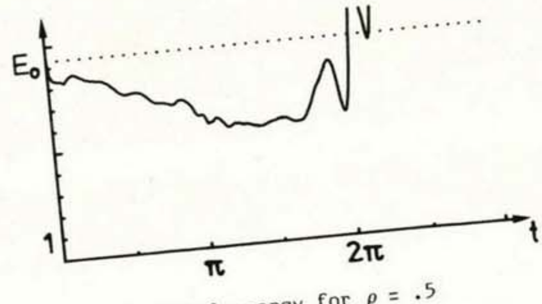

Figure 5b. Total energy for ρ = .5