OPERATIONAL HALO ORBIT MAINTENANCE TECHNIQUE FOR SOHO

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ABSTRACT

This work is concerned with the study of orbits in the proximity of the L1 and L2 libration points of systems similar to the ones of the Earth-Sun or the one of the Earth-Moon. In particular, it presents the method of orbit maintenance proposed in the feasibility study of the SOHO mission. This method is based on the computation of a reference orbit by solving a simple constrained minimisation problem. This orbit has a very small residual acceleration and can be used as the reference orbit for a control of the spacecraft by means of the well known linear-quadratic control theory. This type of control not only minimises the manoeuvres used for the orbit maintenance but stabilises the operational orbit as well. The simplicity, robustness and flexibility of the method makes it very well suited for its direct application in the operational flight dynamics software for the control of the SOHO mission.

KEYWORDS

Three Bodies Problem; Halo Orbits; Control; Station Keeping; Orbit Maintenance.

1. INTRODUCTION

The study of the motion of a point of negligible mass subject to the gravitational attraction of two primary bodies of masses $m1 > m2$ that, under their own gravitational forces, move along circular orbits around their centre of mass, is the subject of the so-called Restricted Problem of Three Bodies, RPTB. The study of the RPTB is simplified by introducing a synodical system of coordinates, defined as a rotating system with origin at the centre of mass; the x-axis in the direction m2 to m1; the z-axis parallel to the momentum vector of the system m1, m2, and the y-axis to complete the right handed system, Fig. 1.1. The equations governing the motion of the point mass are (Szebehely, 1967):

$$\ddot{x} - 2\dot{y} = S_x$$

$$\ddot{y} + 2\dot{x} = S_y \qquad (1.1)$$

$$z = S_z$$

where

$$S = (x^2 + y^2)/2 + (1-u)/r1 + u/r2$$

$$r1^2 = (x-u)^2 + y^2 + z^2$$

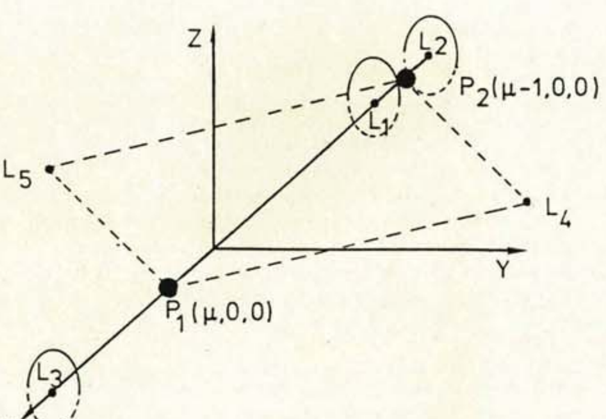
$$r2^2 = (x-u+1)^2 + y^2 + z^2$$

$$u = m2 / (m1+m2)$$

The unit of length is taken as the distance between primaries and the unit of time such that the angular velocity of the circular motion is equal to one.

Euler and Lagrange discovered more than two centuries ago that the system of equations (1.1) has five equilibrium points. Three of these points (denoted L1, L2, L3) are collinear with the primaries, with the point L1 between m1 and m2; the points L2 and L3 on opposite sides with m1 and m2 in between them and L2 close to m2. The points L4 and L5 are on the x-y plane and form equilateral triangles with the primaries and the centre of mass.

Fig. 1.1 RPTB and Libration Points



The distance g of the libration points L1, L2 to the primary of mass m2 is given by the solutions of the Euler's quintic equations:

$$r^5 \mp (3-u)r^4+(3-2u)r^3-ur^2 \pm 2ur - u = 0 \qquad (1.2)$$

where the upper sign is used for L1 and the lower one for L2. A similar equation exists for L3.

The RPTB has occupied a central position in the fields of celestial mechanics, dynamics of systems and applied mathematics where it has been a fertile ground for the development and testing of theories and methods. Several examples of primary bodies and associated libration points are found in the solar system. The Trojan family of asteroids is one case of natural bodies trapped near the triangular equilibrium points of the Jupiter-Sun system.

The importance of space sciences in our days has renewed the interest of the study and possible use of the RPTB. Farquhar (1966) suggested to use the L2 point of the Earth-Moon system for a communication relay satellite with the far side of the Moon. This pioneering work led to several proposals for application of the libration points: solar observatory at the L1 point of the Earth-Sun system (ISEE-C, SOHO); optical interferometry with free flying spacecraft at the triangular points (TRIO).

In order to use those libration points and in particular the collinear ones the following problems have to be solved:

a) the RPTB is an idealisation of the real world and the libration points do not exist in the real planetary system;
b) the motion of a particle near the points L1 and L2 is very unstable, therefore a spacecraft will leave the neighbourhood of the libration point;
c) the exact collinearity will prevent ground telemetry and tracking of the spacecraft.

An analysis of the motion near the libration point of the RPTB (linear theory) shows that the motion along the x, y and z contains an exponentially growing part and a periodic oscillation. The x and y motion are coupled and have the same frequency and the z motion is uncoupled and has a nearby frequency. It is therefore possible to select initial conditions such that only the periodic part is excited and the resulting motion remains in the neighbourhood of the quilibrium point and away from collinearity. As the orbit evolves with time collinearity will occur since the motion projected onto the x-z plane is a Lissajous curve. For orbits further away from the libration point, the non-linear effects will introduce a coupling on the motion along x, y and z such that periodic orbits can be obtained. Such three dimensional periodic orbits are called a Halo Orbit. A spacecraft following that type of orbit will stay away from collinearity with the primaries. The instability of the motion will produce an amplification of any residual error and if the spacecraft is not controlled it will depart from the halo orbit and in fact go very far away from the libration point. For a system with the mass and distance parameters of the Sun-Earth or Earth-Moon system the amplification factor is about 1800 per revolution. In these cases the spacecraft will escape in less than 180 days or 14 days, respectively.

The full planetary system can be considered as a mild perturbation of the theoretical RPTB, and local geometrical libration points can be defined at each instant of time by the same equations (1.2) of a RPTB with parameters as the real world at that given time (mass, distances, angular velocity). The motion of a spacecraft near these points will have the same basic properties as the motion in the RPTB. The ISEE-C was successfully maintained near the geometrical libration point L1 of the Sun-Earth system for almost four years (Farquhar et al., 1979, Muhonen, 1983) by means of 15 manoeuvres with an average value of 2 m/s each.

The SOHO mission of the ESA scientific programme will be injected into a similar orbit. However, the high sensitivity of the instrument for spectrography requires that the orbital control manoeuvres be as small as possible. Hence it is not possible to use the same control strategy as for ISEE-C and different new techniques have been developed to reduce the magnitude of these manoeuvres to a minimum, (Rodriguez, 1984), (Gomez et al., 1985). Both techniques have in common the idea of defining a reference orbit with a very small residual error accelaration, i.e., in the absence of errors in the modelling and control manoeuvres, the spacecraft will follow the orbit using very small manoeuvres (typically less than 1 mm/s per revolution).

Both methods have the same performances with respect to the total manoeuvre capability required, but the techniques used are very different. The required manoeuvres are only due to inavoidable errors of: injection manoeuvre into the halo orbit; orbit determination errors; execution of manoeuvres errors; the planetary model used; the spacecraft model used; etc. This paper presents the method of halo orbit maintenance proposed for SOHO in the feasibility study. The simplicity, robustness and flexibility of the method make it very well suited for its direct application in the operational flight dynamics software for the orbit maintenance of SOHO.

2. ANALYTICAL THEORIES OF HALO ORBITS

The fundamental work of Farquhar (1970) and Farquhar et al. (1977) on halo orbits around the L1, L2 libration points was followed by the work of Richardson (1980a, 1980b) with a third order analytical theory for the description of halo orbits in RPTB. The theory is formulated in a synodical adimensional coordinate system centred at the libration point under consideration. A Lindstedt-Poincaré method is applied to obtain the solution. The small parameter of the theory is d/g, where d is the distance of the spacecraft to the libration point and g is the distance of the libration point to the nearest primary, given by equation (1.2). The theory was used for the control of the halo orbit of the ISEE-C mission Heuberger (1977), Farquhar et al. (1977), Muhonen (1983). Since the theory is of order 3, the errors to be expected for a halo orbit like the one of the ISEE-C spacecraft are of several thousand kilometers in position and several meters per second for the velocity.

Using the same methodology Gomez et al. (1985) have succeeded in developing a semianalytical theory valid up to any order.
The theory computes the position and velocity of the spacecraft by Fourier series of the form:

$$x = \sum a_{ijk} \; \alpha^i \; \beta^j \; \varphi^k$$
$$y = \sum b_{ijk} \; \alpha^i \; \beta^j \; \varphi^k \qquad (2.1)$$
$$x = \sum c_{ijk} \; \alpha^i \; \beta^j \; \varphi^k$$

where the summations are over i, $j > 0$, $k \in Z$, $\varphi = \exp(\sqrt{-1}\; f\; t)$; α and β are the amplitudes of the halo orbit along the x and z axis, respectively.

The theory is semianalytical in the sense that given the mass parameter u = m2/(ml+m2), the coefficients a, b, c and f are computed numerically by recursive relations. Those coefficients are independent of the halo orbit size, hence, once fixed the z-amplitude of the halo orbit, the x-amplitude is computed and the position and velocity at any time can be obtained by equations (2.1). The amplitudes α , β obey a relation of the form $\Delta = \Delta(\alpha,\beta)$. This relation has a solution for $\Delta = \Delta(\alpha, \; 0)$, therefore, there is a minimum x-amplitude α below which a halo orbit cannot exist.

The theory is implemented in a computer program available at ESOC. Table 2.1 gives the computer time required on a Siemens 7880 to compute the coefficients of the theory for different orders.

Table 2.1 Comuter Time vs Order of the Theory

ORDER	3	5	7	9	11	13	15
TIME (sec)	0.07	0.3	2.1	11.5	47	159	461

The accuracy of the theory can be assessed by comparison with the numerical results obtained by a numerical integration with initial point in the plane y = 0 and refinement of the initial condition x_0, z_0, Vy_0, to obtain a periodic orbit. Table 2.2 shows the difference between the theory for different orders and the numerical results obtained for a system of bodies with mass and dimension properties like the Sun-Earth, and for a halo Orbit similar to the one of SOHO (Gomez et al., 1985).

Table 2.2 Values at y=0. Different order of the analytical theory versus numerical solution.

CASE	Δ X(Km)	Δ Z(Km)	$\Delta \dot{Y}$(mm/s)	ΔPeriod(s)
A11-A10	2.499	0.264	-4.544	10.874
A11-A9	2.030	1.417	-44.777	"
A11-A8	23.600	4.144	-42.356	49.169
A11-A7	24.526	10.728	-43.156	"
A11-A6	250.093	-16.874	452.638	-875.130
A11-A5	-190.220	-99.387	438.237	"
A11-A4	-3817.41	-554.727	6836.76	-13108.2
A11-A3	-5442.82	-1602.12	6677.25	"
A11-Num	-0.090	0.048	0.192	0.076

Even with these very accurate initial values a numerical integration of the equation of motion of the RPTB will produce an orbit that will escape of the vicinity of the libration point in less than 2 revolutions.

When the values of the RPTB theory are used as initial conditions for the numerical integration of an orbit in the real planetary system, the resulting orbit will escape after having completed half a revolution but before completing one full orbit. That behaviour is already good enough to permit the use of those initial conditions as starting values to be refined by the method presented in section 3.

3. NUMERICAL GENERATION OF QUASI-PERIODIC ORBITS

The libration points of the RPTB do not exist in the complex field of forces of teh real planetary system. Near the L3, L4 and L5 points of the Sun-Earth system, the effect of Mars, Venus and Jupiter are at least as important as the attraction of the Earth. However, for orbits near the L1, L2 points of the Sun-Earth or the Earth-Moon systems, the influence of other bodies is not so important. These are the systems and points that will be considered in the following, with the understanding that for the Sun-Earth system, the Earth is replaced by the Earth-Moon barycentre.

We can define the geometrical libration points at a given time, by the same Euler's equation with the mass, length and angular velocity as those of the real main bodies at that instant of time. The dynamical behaviour near these points will be very similar to the one in the case of the RPTB. The concept of periodic halo orbits is replaced by the one of quasi-periodic orbits, i.e., orbits that are like the halo orbits from the point of view of geometrical and dynamical properties but not strictly periodic. In a loose manner, we shall refer to these geometrical points and to the quasi-periodic orbit by the same terminology used for the RPTB.

Gomez et al. (1985) have developed a very refined semianalytical theory for quasi-periodic orbits in the real planetary system. The theory uses Fourier development similar to the one used for the RPTB. For orbits of the size of the one of SOHO (Az = 120000 km), there are 132, 165 and 57 terms to be used in the analytical development of x, y and z, respectively. But even with that very refined theory, a numerical integration, using the initial conditions given by the theory, will produce an orbit that will escape in less than 2 revolutions.

The highly unstable character of the motion near L1, L2 and the unavoidable numerical errors make it impossible to devise a method to compute some initial conditions such that the numerical integration of the equation of motion produces the desired quasi-periodic orbit. The only possibility is to define some initial conditions and a set of orbital manoeuvres (dVi) such that by integrating numerically the equation of motion and applying the prescribed manoeuvres, the orbit stays close to the geometrical libration points for as long as required. Ideally the manoeuvres dVi should be zero but due to the numerical errors they have a very small value (numerical noise); typically they are less than 1 mm/s per revolution.

The proposed algorithm has two main steps:

i) Values given by an analytical theory are numerically refined to obtain a numerical orbit that is almost periodic for N revolution (N is a function of the computing precision and typically N = 2).

ii) Small manoeuvres are computed such that by execution of these manoeuvres at time T_0 + Tman(i), the new orbit is almost periodic for N revolutions after T_0 + Tman(i). The interval between manoeuvres is half a revolution.

The first step is based on the fact that analytical theories give values that can be used to compute a numerical orbit that crosses again the plane y=0 before going away from the libration point. The algorithm refines this value to produce an orbit that crosses twice the plane y=0. Thereafter, the values are refined to cross the plane three times and so on. The condition used for the refinement of the values is based on the fact that the errors in position are not very important (because we are interested in a quasi-periodic oribt); the errors in the velocity are bigger in the y component than in the x and z component; and the velocity at the next y=0 plane crossing should be almost perpendicular. The second step was suggested for the RPTB in the final report of the DISCO phase A study (BAe, 1982).

The algorithm is as follows.

1. Initialise T0 (epoch), and NCMAX (maximum number of desired crossing);
2. Compute X0 (T0), Z0 (T0) and VY0 (T0) (given by the analytical theory for the y=0 plane);
3. Set NCROSS = 1
4. Do steps 5 to 9 until NCROSS NCMAX
5. Do steps 6 to 8 while $(VC_x{}^2 + VC_x{}^2)^{1/2} > eps$
6. Integrate numerically the full equations of motion up to the NCROSSth crossing of the y=0 plane;
7. Compute DV0 as a solution of:

   min $(DV0_x{}^2 + DV0_z{}^2)$

   subject to the constraints:

   $VC_x = 0$ and $VC_z = 0$

   where VC is the velocity at the NCROSSth crossing;
8. Set V0 to V0 + DV0;
9. Set NCROSS to NCROSS + 1.

The algorithm performs very efficiently and is very robust. At every new crossing of the y=0 plane the norm of the correction DV0 decreases. Typically, the correction for the first crossing is of several m/s, while the correction for the fourth crossing is of the order of 0.001 mm/s. A maximum of three iterations at each crossing is enough to obtain such a solution.

The constants eps have to be chosen in the range of 0.5 to 1.5 m/s and convergence has to be tested for cases where after several iterations the velocity VC stays non-orthogonal to the y=0 plane. This might happen since the quasi-periodic orbit is not symmetrical with respect to the y=0 plane.

Other methods based on the minimisation of different magnitudes or components than the ones recommended here have been tested and failed to converge to a quasi-periodic orbit valid for two revolutions.

Fig. 3.1 presents the difference between the initial condition given by the theory of Richardson and the numerical values obtained by the above algorithm for initial epoch throughout one year. The error of the V0y is of 4 to 12 m/s. The sinusoidal component is due to the eccentricity of the Earth's orbit. Fig. 3.2 gives the same values when the nine order theory of Gomez et al. (1985) is used. The mean error is negligible, the sinusoidal component with 4 m/s amplitude is due to the Earth's eccentricity and the small sinusoidal component is due to the effect of the Moon; the effect is not present after the date Feb. 1 because the step used for computation after that date is too big.

Fig. 3.1 Difference between Richardson analytical theory and numerical quasi-periodic orbit

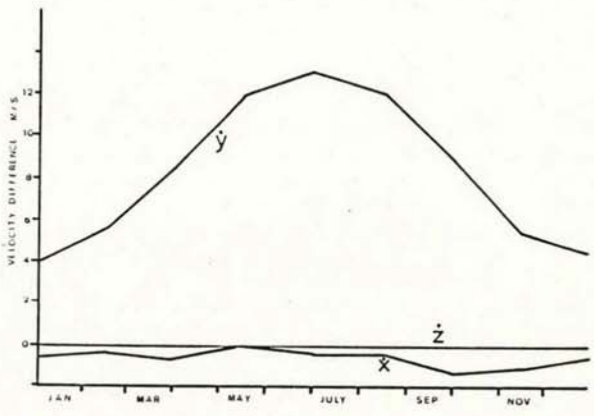
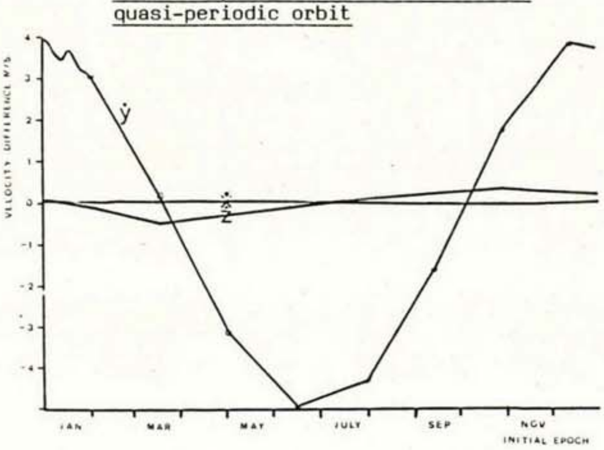


Fig. 3.2 Difference between Gomez et al.analytical theory of order 9 and numerical quasi-periodic orbit

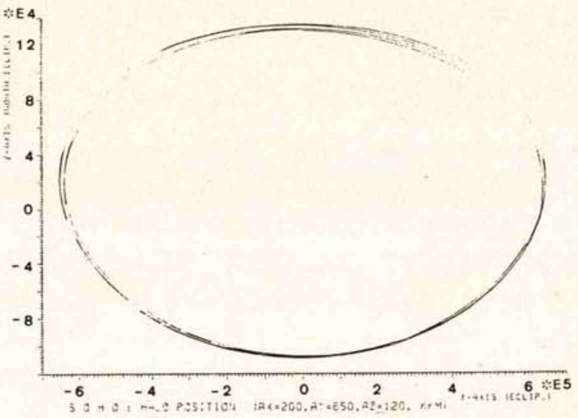


It is clear that after a certain number of crossings the finite computer word length will produce numerical problems. Thus the proposed method cannot produce a reference orbit for more than 4 crosses of the y=0 plane or, equivalently, two revolutions. However, a small modification of the method will produce an orbit for as long as desired. After an orbit that crosses four times the y=0 plane has been obtained, it is possible to modify the velocity of that orbit at the first y=0 plane crossing in such a form that the resulting correction required is of the order of tenth of milimeters per second. With this newly obtained orbit it is possible to modify the velocity at the second crossing in such a form as to cross the y=0 plane up to the sixth cross, and so forth.

With this method a reference orbit will be obtained with discontinuities in the velocity of less than 1 mm/s at each y=0 plane crossing. The spacecraft could be maintained in the quasi-periodic orbit for a very long period of time with a negligible cost, if: a) the physical model of the planetary system and of the spacecraft could be known with enough accuracy; b) the spacecraft could be injected with high precision; c) the orbit could be well known and d) the small manoeuvres were executed properly.

Fig. 3.3 presents the evolution of the reference orbit of the SOHO mission, obtained by the described method.

Fig. 3.3 SOHO: Proposed operational orbit (6 years)



The algorithm to generate a reference orbit can be formulated such that it is valid for the different phases in the generation of a quasi-periodic orbit, i.e. refinement of the values given by an analytical theory and extension of the validity of a quasi-periodic orbit.

The general algorithm is as follows:

1. Initialise T0
2. Initialise X0 (T0), Z0 (T0), VY0 (T0)
3. Initialise NB, NBSTEP, NC, NCSTEP and IC
4. Do step 5 to 12 until NB+MC IC
5. Set NB to NB + NBSTEP
6. Set NC to NC + NCSTEP
7. Propagate state for NB crossing applying all computed corrections, DV(1), DV(2), ... DV(NB-1), and obtain V(NB)
8. Set DV(NB) = 0
9. Do step 9 to 12 until conditions at cross NB+NC are satisfied
10. Propagate state from cross NB to cross NB+NC
11. Compute DV at cross NB to satisfy the conditions at cross NB+NC
12. Set V(NB) to V(NB + DV
13. Set DV(NB) to DV(NB) + DV

where NB, NBSTEP, NC and NCSTEP are parameters of the algorithm and IC is the requested number of crossings. To refine an initial condition given by an analytical theory to obtain an orbit that crosses the y=0 plane four times the combination NB=0, NC=1, NCSTEP=1, IC=4 should be used, while to obtain a reference orbit valid for six years (12 crosses) the following values have to be used: NB=0, NBSTEP=1, NC=4, NCSTEP=0, IC=12.

The method has been applied to obtain quasi-periodic orbits near the L1 point of the Sun-Earth system with sizes of up to 500000 km in the amplitude of the motion along the z-axis. The stability of the method has been demonstrated by producing an orbit of the L1 point of the Earth-Moon system up to more than 50 revolutions. The robustness of the method with respect to errors of the initial values X0, Z0, VY0 given by the analytical theories have been tested by starting with values X0+DX0, Z0+DZ0. The results of the test show that, for orbits like the SOHO one, it is possible to obtain a quasi-periodic orbit with DX0 and DZ0 of more than 5000 km each.

4. CONTROL OF QUASI-PERIODIC ORBITS

Let Xr(t) be the state vector (position and velocity) at time t of a quasi-periodic orbit generated with the method described in section 3 and using a particular planetary system, say the JPL-DE 119 ephemerides tape. Let X(T) be the state vector of a spacecraft moving along a trajectory in the real solar system and controlled such that X(t) = Xr(t) for all times t. It is obvious that some forces will have to be applied to the spacecraft to make it follow the trajectory which is not a natural trajectory in the real planetary system. That force is called the residual acceleration and it is only due to the differences between the planetary models and the errors of the numerical calculations. By using a good planetary model and precise numerical methods of integrating the differential equations of motion, the residual acceleration can be reduced to negligible levels (of the order of mm/s when the effect is integrated over one revolution). Of course, it is neither possible to inject the spacecraft into the given orbit $Xr(t_0)$, nor to know the exact state vector of the spacecraft at a given time, nor to execute precisely the control forces required to cancel the residual accelerations, nor to avoid other modellisation errors.

Nevertheless, the idea of a reference orbit with very small residual acceleration to be followed by the spacecraft by a suitable control is very attractive.

The well known theory of optimal Linear-Quadratic Control (Bellman, 1967) together with the stochastic separation principle (Bucy, 1968) provide the required method for the control and stabilisation of a quasi-periodic orbit in the real planetary system. The proposed control method applies to the following linear dynamical system:

$$\delta X(t_{i+1}) = \Phi(i,i+1)\,\delta X(t_i) + G(i)\begin{vmatrix}\Delta V_x\\ \Delta V_y\\ \Delta V_z\end{vmatrix} \qquad (4.1)$$

with

$$G(i) = \Phi(i,\, i+1)\begin{vmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ Cx & 0 & 0\\ 0 & Cy & 0\\ 0 & 0 & Cz\end{vmatrix}$$

where Cx, Cy and Cz are 1 if it is desired to have manoeuvres along the x, y and z axis, respectively, and zero otherwise.

$\Phi(i,i+1)$ is the orbital transition matrix from time t_i to t_{i+1}. δx are the differences X-Xr, where X is the spacecraft estimated estate and Xr is the reference state computed with the method of section 3 and using the most accurate model available.

The quadratic function to be minimised is:

$$J = \left|\left|\delta X_N\right|\right|^2_{\Gamma} + \sum_{i=0}^{N-1}\left(\left|\left|\delta X_i\right|\right|^2_{H_i} + \left|\left|\Delta v_i\right|\right|^2_{R_i}\right) \quad (4.2)$$

where H_i and R_i are weighting matrices semidefinite, positive and positive semidefinite, respectively. The norm of the a vector X with respect to the matrix A is defined as

$$\left|\left|X\right|\right|^2_A = X^T A X$$

Equation (4.2) expresses that the weighted deviation between the real orbit and the reference orbit and the total cost of the manoeuvres to be executed should be minimised. The solution to that problem is given by computing the so-called control gain:

$$K(i) = B(i)\ \Phi(i,i+1)$$

where

$$B_i(i) = P_{k+1}\ G_k(R+G^T P_{k+1} G)^{-1}$$

and P is the solution of a discrete Riccati equation

$$P_k = H_k + \Phi^T((I-BG^T)P_{k+1}(I-GB^T)+BRB^T)\ \Phi$$

with initial conditions $P_N = \Gamma$

At each time step the manoeuvre to be executed is given by:

$$\Delta V = K\,\delta X.$$

The proposed algorithm for the control of the quasi-periodic orbit can be summarised in the following steps:

1. Compute a reference orbit with the best available model for the spacecraft and for the planetary system.
2. Compute and store the gain matrices for the selected reference orbit.
3. Form the differences between the estimated state (provided by the orbit determination) and the reference state.
4. Compute the manoeuvres by means of the gain matrices.
5. Execute the manoeuvres.

5. SIMULATION AND PERFORMANCES

The methods presented have been implemented in a suite of computer programs to perform simulation of the full halo orbit maintenance process. The structure of the software allows to simulate all the functions that have to be executed in an operational software. The simulation have the structure defined in Fig. 5.1

Fig. 5.1 Structure of the simulation software

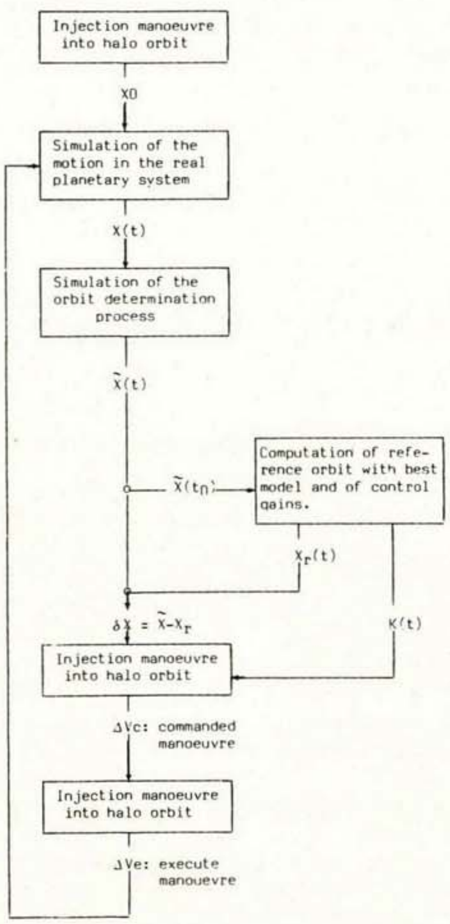


Table 5.1 presents the results of simulations performed to show the effect of the different error sources on the control of a quasi-periodic orbit like the one of SOHO. The initial injection errors are supposed to be of 100 km for each position coordinate and of 1m/s for each velocity.

VTRANS is the total manoeuvre used to correct the transient effect of the injection errors; VOPER is the total manoeuvre used to control the orbit for a period of six years after the transient initial errors have settled down; VMAX is the maximum single manoeuvre executed.

Table 5.1 Effect of single error source

Error source	VTRANS (m/s)	VOPER (m/s)	VMAX (mm/s)
Solar radiation pressure			
5 % model error	4.2	0.5	8
15 % " "	4.2	1.6	22
Orbit determination error			
bias in position : 100Km	6.7	10.5	16
50Km	5.5	5.3	8
random in position: 100Km	26.6	16.2	540
50Km	17.9	9.0	240
bias in velocity : 50mm/s	4.5	12.7	210
10mm/s	4.3	2.5	42
random in velocity: 50mm/s	5.0	16.8	422
10mm/s	4.5	3.5	75

A covariance analysis of the orbit determination process of SOHO shows that the orbit can be estimated to the values given in Table 5.2, where 'baseline' indicates the values that were taken for the mission analysis study of SOHO and 'refined' means that some errors are supposed to be estimated by ground testing of the spacecraft and by in-orbit calibration.

Table 5.2 SOHO. Orbit determination errors (1-σ)

CASE		position (km)		velocity (mm/s)	
		bias	random	bias	random
baseline	x	6	9	1.2	7.5
	y	12	12	3.3	12.0
	z	12	12	4.0	4.5
refined	x	4.5	1.5	1.2	0.6
	y	4.5	1.5	3.0	0.9
	z	6.0	1.5	2.4	0.9

With the orbit determination errors given by Table 5.2, the solar radiation modelling error of 5% and manoeuvre execution error of 5% in the modulus, the result of simulating the control of the orbit for a period of six years give the following results: for the baseline case: VOPER = 3.7 m/s, VMAX = 11.2 mm/s; and for the refined case: VOPER = 1.44 m/s, VMAX = 2.7 mm/s.

The manoeuvres are executed every 20 days if the computed value is greater than 1 mm/s, otherwise they are delayed by 20 days.

For the SOHO orbit and for the conditions of the refined case mentioned above, Fig. 5.2 shows the total delta-V used and the components of the manoeuvres. Fig. 5.3 shows the differences between the components of the reference orbit and the one of the controlled orbit. It is clear that the control produces a very stable orbit where the control cost increases linearly with time.

Fig. 5.2 SOHO. Maintenance manoeuvre history

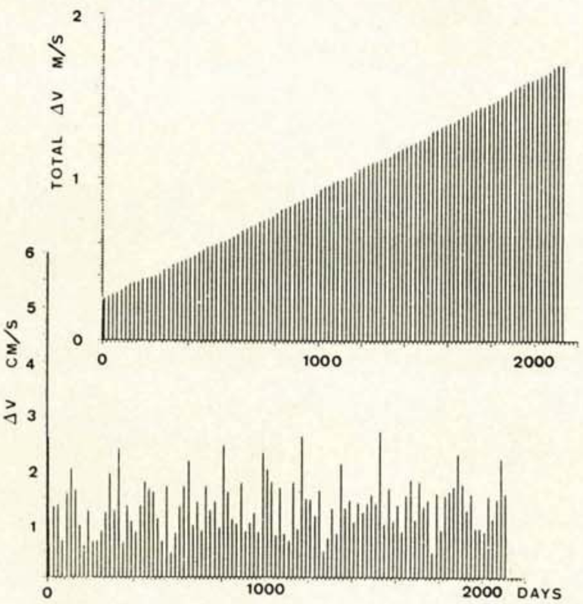
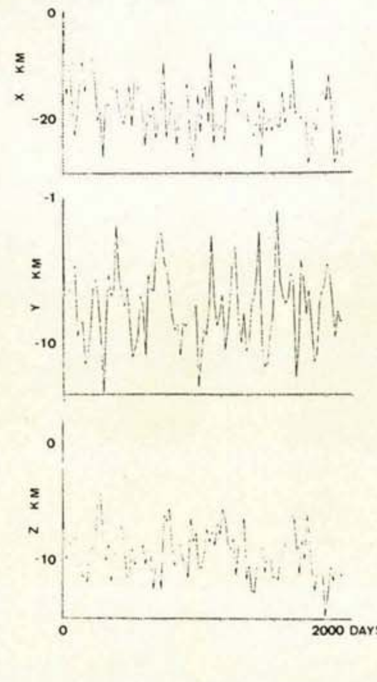
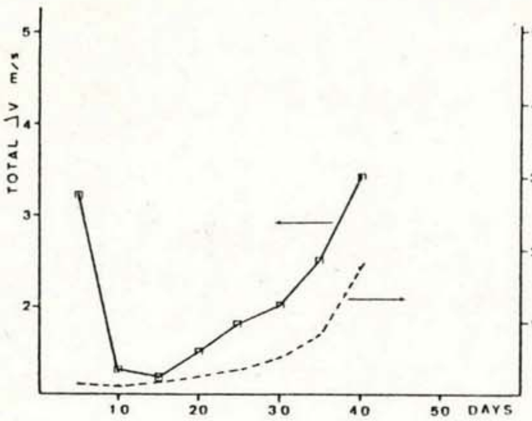


Fig. 5.3 SOHO. Difference between the reference orbit and the controlled orbit



If the manoeuvres are executed very frequently, the orbit determination errors might mask the real differences between the estimated state and the reference one, and the control performance will be rather poor. If on the other hand a big interval is taken between manoeuvres, the cost o correcting the differences increases due to the high instability of the motion. The effect of executing the manoeuvres at different intervals of time is presented in Fig. 5.4.

Fig. 5.4 Total delta-V cost for different inter vals between manoeuvres



6. CONCLUSIONS

A review has been made of the Restricted Problem of the Three Bodies and of the available analytical theories. An orbit started with initial conditions like those given by the analytical theory will escape very quickly from the neighbourhood of the libration point. A method has been developed to compute a quasi-periodic orbit that will have a small residual acceleration for very long periods of time. The method finds the solution by solving a constrained optimisation problem and it will provide a solution even in the most complex cases like the collinear points of the Earth-Moon system.

After a reference orbit is available, the well devloped theory of Linear-Quadratic Control provides the method to control and stabilise the motion of the spacecraft along the quasi-periodic orbit. The results of full scale simulations of the SOHO operational orbit is presented.

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