LOW THRUST NAVIGATION FOR A COMET-NUCLEUS SAMPLE-RETURN MISSION

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ABSTRACT

The use of electric propulsion at very low thrust levels for rendezvous missions to asteroids and comets requires special techniques, both for the generation of the nominal trajectories and for navigation in the vicinity of such trajectories. Furthermore, the orbit determination and manoeuvres in the vicinity of primitive bodies gives rise to additional difficulties, e.g. sensors and mass uncertainty. This paper is concerned with navigation (orbit determination and guidance) on (a) the heliocentric transfer trajectories, (b) approach for rendezvous with a comet, (c) estimation of the gravitation constant in a coast arc near the comet and, (d) manoeuvres within the irregular gravitational field of a comet.

Keywords:

Electric Propulsion, Feedback Guidance, Orbit Determination, Autonomous Navigation, Comet Nucleus, Optimal Control.

1. INTRODUCTION

The propulsion requirements of advanced space mission to primitive bodies such as asteroids or comets are such that low thrust electric propulsion is an attractive option. Thrusting occurs for extended periods of the missions giving rise to important differences in computing nominal trajectories and realising navigation in the operational situation, i.e. determination of the orbit and implementing guidance corrections. ESOC placed a first study contract with Hatfield Polytechnic (1982-84) to compute optimised transfer orbits; this paper summarises a second study by British Aerospace on navigation (Ref 1), also funded by ESOC.

The first study generated optimised transfer orbits to rendezvous with an asteroid but they can apply with minor changes to comet rendezvous. The 'Comet-Rendezvous - Sample - Return - Mission' (CNSR) was in fact defined as the baseline for this study of navigation, although it is restricted to the following phases: (a) cruise or transfer to the vicinity of the comet, (b) approach, and (c) manoeuvres near the comet. It does not include landing on the nucleus, return (similar to the outward cruise phase), and recovery of a capsule on Earth, possibly involving aerocapture.

The cruise phase is assumed to be ground-based, using orbit determination by radiometric measurements and commands to modify the thrust vector of the spacecraft. The continuous nature of thrusting creates an essential difference compared to ballistic trajectories interrupted by virtually impulsive corrective manoeuvres. The use of additional spacecraft measurements is assumed from within one million km of the comet.

In order to provide overlap with the ground-based system, approach navigation starts at about twenty thousand km, based entirely on spacecraft measurements as inputs to an autonomus navigation system. Changeover to the latter is inevitable shortly before rendezvous due to the communication and ground processing times. The gravitational field of the comet is still neglibible in this phase so the kinematics of relative motion are fairly simple, but note that relative speeds are a few m/s compared to the 68 km/s of Giotto's encounter with Halley. The introduction of range measurement appears to be an important parameter due to uncertainties in the ephemeris of the comet.

A coast phase is postulated to permit autonomous determination of the gravitation factor of the comet e.g. between 200 and 100 km. After this, approximate orbit determination and manoeuvring strategies have been tested but the corrections could be accomplished equally well by conventional small thrusters. The nucleus has been modelled as a conglomerate mass and a representation has been attempted of the corruption of measurements associated with an irregularly shaped nucleus.

2. CRUISE PHASE NAVIGATION

2.1 ORBIT DETERMINATION

2.1.1 State Variables and Measurements

The basic set of state variables comprises the six rectangular heliocentric coordinates of position and velocity together with the mass of the spacecraft. The following variables were optionally treated as unestimated bias terms or as estimated additional state variables.

a) correction factor to nominal thruster acceleration

b) two angular coordinate errors in pointing the thrust vector.

Variable (a) could be held constant or made to vary as a random process with a given exponential autocorrelation time constant. In addition an error factor in the gravitational parameter of the Sun could be simulated, also representing uncertainty in the force due to solar pressure on the spacecraft. In summary, there were 7 to 10 state equations which were normally subject to a substantial stochastic disturbance by means of the random fluctuations in thruster acceleration.

The ground-based measurements were taken to be two-way radiometric measurements of range and range-rate from the ESA tracking stations in Spain and W. Australia, but (corresponding to one spacecraft transponder) only one station at a time could be employed. Typical errors simulated were in range 9 m (random rms), 5 m (bias) and in range-rate 1 mm/s (random rms). Possible systematic errors in station latitude and longitude were taken to be 5 m and 2.5 m respectively, all the above numbers being agreed with ESOC.

2.1.2 Estimation Techniques

In order to put our experience into context, let us note three principal approaches to the computer-orientated determination of spacecraft orbits as follows:

(a) Least-squares fitting using a batch of data to estimate the state $x(t_0)$ at the beginning of an interval. This epoch state can be propogated forward (to correspond to the start of the next batch) by numerical integration of the trajectory.

(b) Sequential epoch state estimation of $x(t_0)$ by treating data sequentially throughout an interval. The state can again be propogated forward by integration of deterministic state equations and the error-covariance matrix is stepped forward using either the Kalman or Square-Root type of estimator.

(c) Sequential estimation of $x(t_1)$, $x(t_2)$ ... $x(t_n)$ as measurements become available at times, t_1, t_2 ... t_n, using the Kalman or Square-Root estimator.

Despite Bierman's publications [Ref 2] it is understood that JPL have relied heavily on batch processing, i.e. method (a). It is suitable for handling large amounts of radiometric data with poor first estimates and there is no problem of stability. ESOC have used all three methods for orbit determination on the GIOTTO mission. We have used method (c) because $\dot{x}(t_0)$ would appear to vary due to stochastic variations on the thrust magnitude; it is the only method appropriate to the estimation of dynamical states subject to significant stochastic disturbances such as might arise on a low thrust trajectory.

The computer programs included the option of using Kalman convential covariance filtering (KAL) or the Square-Root covariance algorithm (Ref 3) denoted SQKAL. They were checked out at an early stage to confirm (at least for one or two iterations) that they give exactly the same estimates and covariance matrices, and that the one-at-a-time treatment of measurements is equivalent to the more common processing of vector measurements in the conventional Kalman Filter. Despite the use of double precision, small numerical differences develop between KAL and SQKAL and all results quoted below are with SQKAL because of its greater accuracy. Round-off errors can (and did eventually) lead to covariance matrices with KAL which are not positive definite: this is impossible with the square-root decomposition. The original Householder routine used in SQKAL was 'home-made' but its accuracy was found to be inadequate because pivoting had not been included; it was replaced by a routine from the NAG library (Oxford, U.K.).

Filter divergence was experienced in early tests when the initial states were assumed known to an accuracy of not better than 15000 km in position and 100 m/s in velocity. The divergence arose due to nonlinear measurement functions and the difficulty of estimating components perpendicular tc the line-of-sight (LOS) from Earth to spacecraft. Nevertheless, these difficulties of divergence disappeared when good first estimates (agreed with ESOC) were adopted. Such first estimates can always be generated by preliminary batch fitting. In the absence of bias terms, the convergence of simulated errors was then consistent with the computed theoretical variances. Friedland's bias or Bierman's 'consider' matrices were not computed because bias terms were not estimated as such, even though the simulated measurements were affected by the bias terms mentioned above.

2.1.3 Illustrative Results

The baseline low thrust trajectory was supplied by ESOC and refined by N.O.C., Hatfield Polytechnic. It is a 1464 day (4.0 years) trajectory to the comet 'Bus' with a coast arc from 562 to 882 days, i.e. a middle 22 per cent of the total outward journey.

The first exercise was to confirm that the simulated orbit determination in the absence of thrusting on the coast arc gave realistic results, i.e. consistent with ESA and NASA experience. The results of Table 1 suffice to illustrate this point, although the root-sum-squares (RSS) figures quoted are approximate because they are randomized over only 3 simultations. The subinterval of 0.1 day between estimation updates is short enough for these purposes since further reduction gives only a small improvement; it is related to the number of points needed to infer indirectly the declination and right ascension from the daily sinusoidal variation of the ground Doppler tracking signal (Ref 4).

(682-702 days, 1 AU from Earth)

BIAS TERMS	RSS POSITION km	RSS VELOCITY m/s
None	64	0.064
5m lat and 2.5m long site error	130	0.064

TABLE 1: SIMULATED ORBIT DETERMINATION ON THE COAST ARC

Corresponding representative numbers supplied by ESOC were 50-150 km and 0.1 m/s and therefore this agreement was satisfactory.

Orbit determination was simulated early in the first thrusting interval (denoted E) from 0 to 20 days (0.93 to 4.66 million km from Earth) and, in the second thrusting interval from 1422 to 1442 days (42.3 to 22.3 days before rendezvous denoted L), when the spacecraft is 3.9 AU from Earth and 346 to 94 thousand km from the comet. In case E the spacecraft velocity vector relative to Earth is approximately along the LOS to Earth, but in case L it is almost perpendicular, i.e. the worst case for detecting directly variations of thrusters acceleration by means of the Doppler measurement.

A realistic slow random variation of thruster performance was taken to be a one sigma (r.m.s.) fluctuation with an exponential autocorrelation time constant of 100 days; this is supposed to represent the slow drift in the power regulating system. Even though feedback can ensure that the net thrust vector of the spacecraft points through the centre of mass, uncertainty in the latter may result in a significant thrust vector pointing error.

It was simulated as a constant in two angles and set illustratively at one degree. The results of numerous simulations can be summarised as follows.

(a) Typical errors (as above) in magnitude and direction of the thrust vector give rise to unacceptably large errors in the orbit detemintion.

(b) Such errors can be reduced to acceptable values by treating the three errors as additional state variables and including them in the estimation process; see Table 2

(Case E = near Earth, Case L = near comet, see text)

CASE	BIAS TERMS	RSS POSITION km	RSS VELOCITY m/s
E	5m lat and 2.5m long site errors	5.8	0.008
L		597	0.64

TABLE 2: SIMULATED ORBIT DETERMINATION WHILE THRUSTING

These estimation errors are acceptable, bearing in mind that case L is without any spacecraft-to-comet camera data which would be available at that stage of the mission. It is recalled that a 1975 JPL study (Ref 5) proposed the use of ballistic arcs to permit sufficiently accurate orbit determination but they assumed only very coarse regulation of motor power.

2.2 GUIDANCE

2.2.1 Options

Four approaches have been considered as follows.

(a) Repeated re-optimisations in flight using already developed ESOC/Hatfield computer programs (variable rendezvous time).

(b) Optimal perturbation guidance by varying pointing, thruster on-off and rendezvous times.

(c) End-point guidance by varying pointing, thruster on-off and rendezvous times.

(d) End-point guidance by varying pointing and throttling the motor, with fixed rendezvous time.

Option (a) is clearly feasible provided the chosen operational trajectory (allowing for other constraints and trade-offs) is an optimal trajetory, i.e. computed to deliver maximum spacecraft mass at a rendezvous with the comet. With the help of N.O.C. Hatfield Polytechnic, we have merely confirmed that re-optimizations as frequent as every 10 days would be necessary due to the stochastic nature of the thrust vector.
The algorithmic formulation of each of the approaches (b), (c) and (d) start with a computed optimum trajectory. In case (b) it was the baseline trajectory supplied by ESOC/Hatfield to comet 'Bus' already mentioned, i.e. optimum rendezvous with respect to final mass. In cases (c) and (d) the 'optimum' is defined as a trajectory which minimizes squared deviation of final position and velocity of the spacecraft with respect to the comet. Therefore any rendezvous trajectory can be employed and guidance seeks merely to minimize terminal deviations at a given time, hence the term 'end-point guidance'. It is noted now that a rapidly convergent algorithm for re-computing an optimum solution (due to perturbations) in the vicinity of an optimal trajectory is by the method of Differential Dynamic Programming or equivalently the 'Sweep Method' (Refs 6, 7). If delta denotes differences from a nominal starting trajectory then an iterative correction is of the form

$$\Delta u(k) = -C(k)^{-1}\left[H_u(k) + B(k)\Delta x(k)\right] \quad (1)$$

for each discrete interval indexed as k. The state vector is x and u is the control vector; the other terms are matrices computed on the nominal trajectory.

If the nominal trajectory is an optimum (e.g. with respect to mass or end-point deviations) then the vector Hu(k) in eqn (1) is zero and the result simplifies to

$$\Delta u(k) = -D(k)\,\Delta x(k) \quad (2)$$

where the matrix D(k) can be pre-computed (once-and-for-all) on the nominal optimum trajectory. This is the basis of perturbation guidance derived from modern control theory. It is a stable fedback law with the well known benefits of feedback, viz insensitivity to model errors, disturbances, etc.

No results on optimal perturbation guidance (case b) are presented here because of the following difficulties. An optimum trajectory was provided in terms of the state and control variables but no values were available for the 6 Lagrangian multipliers which adjoin the terminal constraints of rendezvous. As a result, eqn (1) gives rise to large iterative corrections along with adjustment of the mutipliers. The changes in pointing angles became so large (because they are very weak control variables) that the expansions on which the algorithm is based became invalid. It is mentioned that JPL (Ref 8) have used the method successfully in earlier studies but with the important difference that their optimal solution for rendezvous was restricted to a subset in which coast arcs or reduced thrusting was precluded.

In addition to the above difficulties, neither of options (a) nor (b) is considered attractive because it is expected that an operational choice of nominal trajectory will not be an optimum due to compromises on mission duration, viewing angles, communication distances, etc. Approaches (c) and (d) can apply to any trajectory and they are further discussed below.

2.2.2 Guidance by Pointing and Timing

This is option (c) in which the guidance criterion is to minimize end-point deviations between spacecraft and comet (in position and velocity) by adjusting two pointing angles, the start of coasting t_1, the end of coasting t_2, and the time of rendezvous t_f.

Adjustments of the 3 parameters (t_1, t_2, t_f) are very effective in correcting the 6 terminal conditions but of course these are insufficient degrees of freedom. Corrections to the pointing angles of the thrust vector throughout the whole of the non-coast parts of the trajectory must provide the added degrees of control but unfortunately a change in pointing angle is a weak control variable. The computation of eqn (2) included weighting on Δu (k) squared at each interval k, in order to inhibit large changes which would invalidate the method. The result was that typical initial dispersions (28 m/s at 930,000 km from Earth) were corrected to final accuracies of 27000 km and angular changes of up to 60 degrees occurred. The final relative velocity was also unacceptably large at 17 m/s, although the additional propellant required was negligible. This method was consequently rejected on the grounds of inadequate controllability.

2.2.3 Guidance by Pointing and Throttling

This is option (d) in which the timing of the coast arc and the duration of the missions is unchanged, but it is postulated that the nominal trajectory is such that only small percentage increases or decreases in thuster magnitude are possible by throttling the motor, i.e. on top of the secular change arising from dwindling solar power as a function of distance from the Sun. Small angular corrections to the thrust vector are also permitted.

In this case the controllability (largely due to throttling) was excellent. Some results are presented in Table 3 where the propellant changes are percentages of the total consumed for this mission, viz. 1110 kg for a total initial spacecraft mass of 4388 kg.

Only the first is a non-random simulation; the others depend on priming of a random number generator. However the fourth and fifth rows differ from the second and third rows (respectively) only in that orbit determination errors have also been simulated (see Table 2). It confirms that the

CASE	MAXIMUM CHANGES ANGLE, DEG	MAXIMUM CHANGES MAGNITUDE %	EXTRA PROPELLANT %
Large initial speed error of -160 m/s.	11	3.7	-0.39
Random (2%) thruster variations plus 1 deg const pointing error, but estimated perfectly.	8	3.0	-0.057
	11	7.1	0.28
As above but including orbit (state) esimation errors.	9	3.2	-0.059
	12	7.7	0.28

TABLE 3: END-POINT GUIDANCE BY POINTING AND THROTTLING

accuracy of that estimation is more than adequate. Final errors have not been quoted because (within limits) they are not relevant due to the subsequent use of camera sightings and the comet-centred approach navigation. They were in fact of the same order as typical a priori uncertainties in the position and velocity of a comet, viz. at the one sigma level about 1600 km and 0.35 m/s.

3. APPROACH NAVIGATION

3.1 RELATIVE MOTION IN THE APPROACH PHASE

The approach phase is taken to start at about one million km for the comet with say 70 days to rendezvous. Camera sightings of the comet against a star background can be assumed; they would augment the ground-based orbit determination, especially to reduce errors with respect to the comet. A modified guidance law is proposed which is suitable for on-board implementation and is indexed by distance from the comet.

It is helpful to bear in mind the relative importance of different accelerations, hence Table 4. Where relevant the distance from the sun has been taken as 2 AU and the range of numbers for the field of the comet correspond to a radius of 1-10 km and a specific gravity of unity.

SOURCE	DISTANCE FROM COMET, KM 10^2	10^3	10^4	10^5
Thrusting	10^{-4}	10^{-4}	10^{-4}	10^{-4}
Solar pressure	$2x10^{-7}$	$2x10^{-7}$	$2x10^{-7}$	$2x10^{-7}$
Heliocentric diff. field	$6x10^{-10}$	$6x10^{-9}$	$6x10^{-8}$	$6x10^{-7}$
Field of comet.	$3x10^{-5}$	$3x10^{-7}$	$3x10^{-9}$	$3x10^{-11}$
	$3x10^{-8}$	$3x10^{-10}$	$3x10^{-12}$	$3x10^{-14}$

TABLE 4: ACCELERATIONS IN THE APPROACH PHASE (m/s^2)

It will be observed that thrusting is the dominant source of acceleration until within about 100 km of the comet. Note that relative motion between spacecraft and comet is affected by the differential (not total) gravitational field of the Sun. The approach phase is defined to end at about 200 km, hence the approximation that the field of the comet can be neglected, and even the Sun is only a perturbation to the relative motion. Approach trajectories are consequently almost straight lines and they were computed in these studies from differential equations arising from a first-order expansion about the comet. Thrusting for the last one million km on the nominal baseline trajectory is less than 8 deg. from the spacecraft-to-comet vector, i.e. virtually retro-thrusting along the negative relative velocity vector. The angle between the nominal thrust vector and the line-of-sight to Earth varies from 88 to 94 degrees.

3.2 STATE ESTIMATION

3.2.1 Augmented Ground-Based Orbit Determination

In the early approach phase, camera data of the comet against a star background would be employed to supplement radiometric measurements. At one million km a typical nucleus subtends an angle the order of 10 microradians but it would appear larger than this if it were approached near perihelion due to out-gassing. The Halley multicolour camera had a resolution of 22 microradians (Ref 8) although it was not designed for navigation on the Giotto mission. The camera for the NASA Galileo mission has a 0.5x0.5 deg. field of view with a CCD imaging system which can detect stars as faint as magnitude 9 and will be employed for navigation. Such data can therefore be assumed and its importance is that refinement is permitted of the orbit of the comet itself. Quoting the example again of the Giotto mission, such a priori errors might be 1600 km and 0.35 m/s.

The 6 coordinates of position and velocity of the comet must be added to the orbit/state estimation computations carried out on the ground. In our case this would have increased the number of state variables from 10 to 16 but this was not implemented in these computational studies. The trajectory relative to the comet is approximately a straight line in the approach phase so the effect of camera data should be principally to reduce the components of comet-ephemeris errors perpendicular to this approach velocity vector, or line-of-sight from spacecraft to comet. Using the above example of ephemeris errors, this would still leave one sigma components of errors along the velocity vector of 1500 km and 0.1 m/s although this may be pessimistic.

3.2.2 Onboard Autonomous State Estimation

Within 10000 km of the comet nucleus, the visibility of stars is regarded as doubtful because the spacecraft enters the coma of the comet, although this will depend on the activity of the comet as a function of distance from the Sun. At such a distance it is therefore proposed that the inertial angular rate of the line-of-sight (LOS) vector from spacecraft to comet should be measured e.g. three mutually perpendicular gyros mounted on a steerable platform carrying an imaging system which points at the nucleus. The three measured angular rates constitute a vector, the magnitude of which is denoted $\hat{e}$ and its direction is the normal to the orbit plane of motion with respect to the comet.

The latter vector is with respect to spacecraft axes and it must be possible to relate these to inertial axes by having two reference directions available in the spacecraft. It is assumed (a) that the Sun is always visible as the first reference direction and (b) stars or the Earth provide a second reference direction in the spacecraft. In order to maintain communications it is postulated that the encounter would be chosen so that the Earth and Sun are not very close to coincidence, hence the Earth as a second directional reference if stars are not visible. Therefore the relationship between spacecraft and inertial axes would be known in the spacecraft.

Distance from the spaceraft to the surface of the nucleus is expected to be available either by a laser system up to about 1000 km or by radar up to 5000 km. The accuracy of both would be around 0.1 per cent but, more importantly, the distance to the 'centre' of the nucleus would be uncertain due to the unknown shape and size of the nucleus early in the mission. The distance at which radar range becomes available is significant because (as mentioned in Section 3.2.1) there may be large uncertainties in the distance along the LOS. If such a large error is detected too late by radar then there might be insufficient throttling capability in the low thrust motor to prevent under or overshoot. In order to avoid the latter, simple considerations show that range should be available at a distance given by

LOS position error $\div$ throttling ratio (3)

This may be too large, in which case autonomous procedures for under and overshoot must be included; they are not difficult (Appendix 7.19, Ref 1).

In approaching from say 2000 to 200 km the angle subtended by the nucleus would increase between the orders of magnitude 0.2 to 2 degrees. The imaging system must point at the 'centre' of an irregularly shaped body (complicated by gas emissions) of such angular sizes. The 'centre' is obviously ill defined and some allowance must be made in simulations for the corruption of angular rate and range measurements. A crude model of such effects was therefore included (Section 4.3.1, Ref 1); it was a periodic function of ($\dot{\theta} - W_S$), where W_S is the spin rate of the nucleus, and it allowed for angular effects as a function of distance from a nucleus of a given mean radius. The resulting errors in angular rates also give rise to errors in the calculated normal to the orbit plane although, with illustrative parameters, such errors have been estimated (Appendix 7.6.1, Ref 1) to be only a few degrees at distances of 200 to 2000 km from the nucleus.

For a nominal acceleration (70 per cent of the maximum of 4.2×10^{-5} m/s^2) and a typical initial miss-distance of 1000 km, we obtain as follows for the angular rate of the LOS vector:

10^4 km, (8.0 days to go) 0.060 deg/hour

10^5 km, (25.3 days to go) 0.0019 deg/hour

A typical threshold for a high quality gyro (such as the Ferranti 125) is 0.002 deg/hour from which it is estimated that the use of angular rates could not usefully start before about 20000 km (11.3 days to go). Range is an important measurement but it is not expected to be available until within 1000 to 5000 km of the nucleus.

Given the approximately straight line nature of the approach trajectory the in-plane motion can be characterized by three state equations for a non-unique choice of state variables as follows. If v is speed, h angular momentum and r distance from nucleus to spacecraft, then

$$\dot{v} = f_t \qquad (4)$$

$$\dot{h} = h f_t / v - (r f_n / v)(v^2 - h^2/r^2) \qquad (5)$$

$$\dot{r} = -(v^2 - h^2/r^2)^{1/2} \qquad (6)$$

where (f_t, f_h) are thruster accelerations along and perpendicular to the velocity vector in the orbit plane.

The measurements are

$$y_1 = \dot{\theta} = h/r^2, \quad y_2 = r \qquad (7)$$

An extended Kalman Filter based on these state and measurement equations was implemented, although the simulated measurements incorporated noise and the corruptions arising typically from sighting on an irregularly shaped nucleus.

Table 5 shows the results from one example starting at 2000 km, including the measurement of range from that distance.

TIME days	r km	ESTIMATION ERROR in V, m/s	ESTIMATION ERROR in h, km^2/s
0.2	1486	-1.1	0.010
1.0	798	0.37	0.095
2.0	317	0.19	0.074
2.4	233	0.17	0.053
2.6	212	0.17	0.048

TABLE 5: EXAMPLE OF APPROACH STATE ESTIMATION

The last row of Table 5 corresponds to estimation errors just before the coast arc near the nucleus, which in this case is a parabola with a perigee nominally at 100 km. In the case of a parabola it can be shown (Section 4.3.2, Ref 1) that errors in (v, h, r) and gravitational factor at the start of this coast can be related to errors in the radius r_1 of perigee by

$$\Delta r_1 / r_1 = 2\Delta h/h - \Delta v/v - \tfrac{1}{2}\Delta r/r - \tfrac{1}{2}\Delta\mu/\mu \qquad (8)$$

The nominal values of v and h at 212 km are 1.25 m/s and 0.18 km^2/s, hence the last errors of Table 5 correspond to a perigee error of 40 per cent. This is of course probably less than the dispersion due to uncertainty in the gravitational factor of the nucleus.

3.2.3 Autonomous Approach Guidance

Let $\bar{v}(r)$, $\bar{h}(r)$ define a nominal desired approach trajectory as a function of distance r from the nucleus, and assuming a nominal value $\bar{f}_t$ of constant retro tangential thrusting e.g. 70 per cent of the maximum. From equations (4, 5, 6) with f_n equal to zero, it is possible to derive an analytical solution (Appendix 7.4, Ref 1) for $\bar{v}(r)$, $\bar{h}(r)$ and this can easily be implemented on board. By taking a first-order expansion of equations (4, 5, 6) about this nominal trajectory we can also derive a feedback guidance law (Appendix 7.20, Ref 1) to ensure that, if the states are estimated perfectly, then the deviations Δv and Δh converge exponentially to zero, i.e.

$$\Delta\dot{v} = -\Delta v / T_1 \qquad (9)$$

$$\Delta\dot{h} = -\Delta h / T_2 \qquad (10)$$

That feedback law is as follows, with over bar symbols omitted for brevity.

$$\Delta f_t = -(1/T_1 - v p^2 f_t)\Delta v - (h p^3 f_t / r^2)\Delta h \qquad (11)$$

$$\Delta f_n = -\left[(h/r)(p/T_1 - v p^3 f_t) + h p f_t (1 + p^2 v^2)/rv\right]\Delta v + \left[(v/r)(p/T_2 - h p^3 f_t / r^2) + (p f_t / r)(1 + p^2 h^2/r^2)\right]\Delta h \qquad (12)$$

$$p = (v^2 - h^2/r^2)^{-1/2} \qquad (13)$$

It is important to note that this feedback law, which would be implemented onboard, is a function of distance to the target and not time. The suitability and convenience of this feedback law has been confirmed for both onboard autonomous guidance from 20000 km or for the last phase of the ground-based navigation from one million km.

An example of simulated approach navigation is taken as that already used for the state estimation from 2000 km in Table 5. In order to implement guidance the terms Δv and Δh of equations (11) and (12) are formed as

$$\Delta v = \hat{v} - \bar{v}(\hat{r}) \qquad (14)$$

$$\Delta h = \hat{h} - \bar{h}(\hat{r}) \qquad (15)$$

where $\hat{v}$, $\hat{h}$ and $\hat{r}$ come from the state estimator. Table 6 shows the total errors in v and h arising from state estimation and guidance.

TIME days	r km	NAVIGATION ERROR in V, m/s	NAVIGATION ERROR in h, km^2/s
0.2	1486	1.26	-1.49
1.0	798	0.65	-0.034
2.0	317	0.27	0.10
2.4	233	0.23	0.077
2.6	212	0.22	0.069

TABLE 6: AUTONOMOUS APPROACH NAVIGATION

The last row of Table 6 corresponds to the end of the retro-thrusting approach phase just before the start of a coast. By using equation (8) we can again relate these errors to dispersion in the coast orbit around the nucleus. The final navigation errors are only a little greater than the estimation errors of Table 5 and they correspond to a 59 per cent change in the perigee of the coast parabola. It is however again asserted that, by reference to equaton (8), the dispersion due to uncertainties in the gravitational factor of the nucleus would probably be much greater.

A feedback law for out-of plane guidance is also possible (Appendix 7.5, Ref 1) to provide limited correction of the orbit plane during the approach phase.

4. MANOEUVRES IN THE VICINITY OF THE COMET

4.1 MANOEUVRES AND ORBITS ABOUT THE NUCLEUS

For manoeuvres and orbiting within a radius less than 200 km the continued use of solar electric low-thrust propulsion is considered extremely unlikely for the following reasons:

(a) damage to the arrays from cometary debris over a long period and uncertain solar intensity.

(b) large slew manoeuvres might be needed to point the low-thrust vector.

(c) a requirement for huge batteries or a separate RTG power source when the sun is eclipsed.

(d) any manoeuvres are tiny and would require only a very small amount of chemical propellant, e.g. 0.23 per cent of spacecraft mass for several typical increments of velocity.

Orbital adjustment and maintenance by means of small conventional thrusters is therefore considered more realistic. However the thrusting strategies developed could be realised either by continuous thrusting with electric propulsion or by impulsive thrusting using chemical propellant.

It is helpful to appreciate the strength and uncertainty in the gravitational field of the nucleus. For a mean specific gravity ρ and radius r_o the gravitational factor is (with $c = 2.794\times10^{-7}$)

$$\mu = c\rho r_c^3 \quad km^3/s^2 \qquad (16)$$

The acceleration in a circular orbit of N radii is

$$a = c\rho r_c / N^2 \qquad (17)$$

and the perturbing acceleration due to the Sun is approximately

$$\Delta a = \mu_s N r_c / R^3 \qquad (18)$$

where R is distance to the Sun (with gravitational factor μ_s). Thus a dimensionless ratio, to which solar perturbations are proportional, is

$$\Delta a / a = \mu_s N^3 / c\rho R^3 \qquad (19)$$

Some ilustrative values for this ratio and periods are given in Table 7. By way of comparison, the ratio (19) is 7.5 x 10^{-6} for a geostationary satellite and 5.6 x 10^{-3} for the Moon about the Earth. In other words, despite the weak gravitational field of a comet, relatively stable orbits about the nucleus are meaningful.

r_o km	N	r km	$\Delta a/a$	Period, hours
10	2	20	1.4×10^{-4}	9.33
10	4	40	1.1×10^{-6}	26.4
10	10	100	1.8×10^{-5}	104
1	2	2	1.4×10^{-7}	9.33
1	4	4	1.1×10^{-6}	26.4
1	100	100	1.8×10^{-2}	3300

TABLE 7: CIRCULAR ORBITS ABOUT THE NUCLEUS OF A COMET

Although the theory of orbits perturbed by non-spherical bodies is well developed, the useful results refer to cases where the departures from a spherically symmetrical inverse square - law field are small. The nuclei of comets are expected to be very irregularly shaped and therefore they have been represented in these computer studies as a collection of N point masses distributed approximately within a given nuclear radius r_o. The assemblage can be distributed at will, but it is arranged within the program that the gravitational factor is given by eqn (16), ρ and r_o being inputs.

The following distribution of mass was used to illustrate departure from a point mass; the angles being R.A. and Dec. in comet - centred ecliptic axes

$$\left.\begin{array}{l} 1)\ 16.7\%\ \text{at}\ r_o,\ (0, -90)\ \text{deg.} \\ 2)\ 33.3\%\ \text{at}\ r_o,\ (120, 0)\ \text{deg.} \\ 3)\ 50.0\ \text{at}\ r_o,\ (240, 90)\ \text{deg.} \end{array}\right\} \quad (20)$$

Period of rotation about Z axis = 20 hours, nuclear radius 4 km.

By way of an example the following changes occurred in the orbital elements when the orbit was computed over 100 hours, the period being nominally 24 hours. The two cases are for positive and negative spin.

Semi-major axis: (15.0-18.3), (14.0-15.4)
eccentricity : (0.10-0.32), (0.10-0.22)
inclination : (42.4-45.2), (41.1-45.2)
line of nodes : 0.41, 0.53 deg/hr

4.2 DETERMINATION OF THE GRAVITATIONAL FACTOR

Due to the large uncertainties in the gravitational factor of the nucleus, significant dispersions will occur in coasting or thrusting trajectories within about 200 km of the nucleus. A coast phase starting at about this distance is therefore proposed to permit a rapid autonomous (onboard) first estimate. The assumed measurements are angular rate, already described for the approach navigation, and distance to the nucleus by a spacecraft-borne radar, some such measurement being essential at this stage. However these measurements would be seriously corrupted by the irregular shape of the nucleus and consequently the crude model already mentioned in Section 3.2.2 was included in simulations to represent such effects.

After one or two abortive tests with Kalman and simpler sequential estimators it was apparent that a very stable process was needed because of the corrupted measurements. A simple linear least-squares fit on a batch of data was therefore adapted, based on the energy equation in the field of the comet, viz.

$$v^2 = \text{constant} + \mu/r \tag{21}$$

The left-hand side of (21) was approximated from the measurements $(\dot\theta_n, r_n)$ by

$$v_n^2 = \left[q_n^2 + (r_n + r_{n-1})^2/4\right](\dot\theta_n + \dot\theta_{n-1})^2/4 \tag{22}$$

$$q_n = 2(r_n - r_{n-1})/\left[(t_n - t_{n-1})(\dot\theta_n + \dot\theta_{n-1})\right] \tag{23}$$

The estimation of gravitational factor was tested for a coast phase of 200 km to 100 km, with the nuclear specific gravity equal to unity but for three nuclear radii of 1, 4 and 10 km (Section 4.3.2, Ref 1). Even with solar perturbations and the conglomerate nuclear model (20), the gravitational factor was determined within 2 per cent for the 4 km cases. The estimates were not so good for 1 and 10 km radius. The errors were -21 and 36 per cent respectively, and it was ascertained that (a) the former arose because the field of the 1 km nucleus is relatively weak at 100-200 km and , (b) the latter arose from the seriously corrupted measurements at only 10-20 radii distance from the 10 km nucleus, i.e. of an irregular shape. These estimates could be improved by continuing nearer the nucleus in case (a) and prolonging the coast interval to take more measurements in case (b), if necessary by making a correction to avoid impact.

4.3 AUTONOMOUS ORBIT DETERMINATION NEAR THE NUCLEUS

Determination of a necessary subset of the orbital elements near the nucleus depends on the measurements $\dot\theta$ and r as for the approach and coast phases. The chosen elements in conventional notation were (p, e, θ), the latter being the angle from perigee. An extended Kalman Filter was tried but, even without the corrupting measurement errors, the convergence could be assured only with very good first estimates. A simpler more robust recursive estimator was therefore formulated. It is based on the following standard equations, assuming that μ is known.

$$p = (r^2\dot\theta)^2/\mu \tag{24}$$

$$r = p/(1+e\cos\theta) \tag{25}$$

$$q = dr/d\theta = (er^2/p)\sin\theta \tag{26}$$

The derivation of the estimator is detailed in the original study (Section 4.3.3, Ref 1); it is sufficient to say here that it converged well on an eccentric orbit (e.g. e = 0.3, a = 10 radii), despite the corrupted measurements. It was however unreliable for less eccentric orbits e.g. e equal to 0.1. Big changes in the estimated value of θ occurred if the estimated e became small; of course θ is not defined as e tends to zero.

Estimation of a minimum set of in-plane orbital elements must therefore exclude θ for nearly circular orbits. By the way, the use of nonlinear least-squared batch fitting was not considered at this stage, because the determination was intended to provide continual sequential updating of the orbit as an input to an algorithm for orbital maintenance.

4.4 ORBIT ADJUSTMENT AND MAINTENANCE

The manoeuvres required to make transitions between orbits and to circularize them are conventional and need not be detailed here. Orbits close to an irregularly shaped nucleus may however need frequent corrections to maintain the elements within limits as illustrated in Section 4.1, hence the use of the following feedback law to regulate semi-major axis a and eccentricity e.

Assume that the orbit is an ellipse with eccentricity less than 0.7; transition to or from hyperbolic orbits would be handled by discrete manoeuvres. Thrusting acceleration only in the orbit plane perpendicular to the radius vector is proposed (denoted f_θ), and continuous thrusting is to be approximated by impulsive thrusting at regular intervals. In conventional notation it can be shown that

$$da/dt = (2a^2/h)(1+e\cos\theta)f_\theta \tag{27}$$

$$de/dt = (p/h)\left[\cos\theta + (e+\cos\theta)/(1+e\cos\theta)\right]f_\theta \tag{28}$$

It is shown (Section 4.4.2, Ref. 1) that the guidance law

$$f_\theta = w_c + w_1\cos\theta \tag{29}$$

where

$$w_c = (h/p)\left[(p/2a^2)(1-e^2/8)\Delta a/T_5 - \tfrac{1}{2}e\Delta e/T_6\right]/(1-3e^2/8) \tag{30}$$

$$w_1 = (h/p)\left[\Delta e/T_6 - (pe/4a^2)\Delta a/T_5\right]/(1-3e^2/8) \tag{31}$$

will provide the approximate exponential convergence desired, and

$$\Delta a = \bar a - a, \quad \Delta e = \bar e - e \tag{32}$$

$(\bar a, \bar e)$ being the desired values. Thus

$$(da/dt)_{av} = (\bar a - a)/T_5 \tag{33}$$

$$(de/dt)_{av} = (\bar e - e)/T_6 \tag{34}$$

where 'av' denotes averaging per orbit. The implementation of this algorithm requires estimated values for (a, e) to be inserted in (32), in order to generate the thrusting law (29). A corresponding feedback law for the two out-of-plane elements has also been derived (Appendix 7.7, Ref. 1).

The stability and convergence of the above algorithm was confirmed first by a simulation using perfect estimates of a and e. Gradual convergence to desired values was obtained for an eccentric orbit (e = 0.3), but with a nearly circular orbit (e = 0.1), only the major axis was controllable.

In the presence of estimation errors arising from typical corrupted measurement due to an irregularly shaped nucleus, simultaneous adjustment of neither major axis nor eccentricity was achieved, even in the case of the eccentric orbit for which the estimation appeared to be satisfactory.

More work is needed on the orbit determination and control around the nucleus but it is suggested that,

(a) a nonlinear batch fit to equations 24-26 should be employed to estimate (p, e, θ) and,

(b) infrequent conventional impulse-type corrections are applied only when necessary, thus providing relatively long intervals to estimate mean values of the in-plane elements (p, e, θ) between corrections.

5. CONCLUSIONS

5.1 CRUISE PHASE NAVIGATION

5.1.1 Ground Based Orbit Determination

- Provided good first estimates are available (if necessary from batch-fitting) satisfactory convergence has been demonstrated using tracking from only one station at a time, in the N and S hemispheres.

- Orbit determination on the coast arc (at about 1 AU from the Earth) has been demonstrated to be compatible with ESOC/GIOTTO experience, e.g. 130 km in position and 0.06 m/s in velocity.

- Orbit determination in the thrusting phases is seriously degraded if uncertainties in the magnitude and direction of the thrust vector are not measured or estimated (as part of the orbit determination).

- Constant or slowly varying uncertainties in the thrust vector have been successfully included and estimated in the state vector as part of the orbit determination, both near the Earth and when approaching the Comet with the thrust vector nearly perpendicular to the line-of-sight.

- Despite the above, it is recommended that the new type of solid state accelerometer (to be tested as part of the ESA Technology Demonstration Programme) should be mounted along the spacecraft axes if it can yield an accuracy of 10^{-8} g.

5.1.2 Guidance

- Effective guidance about a nominal (but not necessarily optimal) trajectory by means of a feedback algorithm has been demonstrated in which rendezvous occurs at the nominal mission time. Adjustment of the coast arc and final time is not sufficient; instead small positive and negative changes of thrust magnitude (throttling) are employed, together with adjustments to the pointing angles.

- This guidance law can handle errors of orbit determination and coarse initial speed offsets using (separately) propellant less than 0.4 per cent of the initial mass of the spacecraft. In view of its operational simplicity it is therefore recommended.

- A provisional specification for pointing the thrust vector with respect to spacecraft axes (in the presence of a shifting centre of mass) would be about 1 degree. Similar constant or slowly varying errors in pointing the spacecraft could be tolerated because the total pointing error of the thrust vector can be calibrated as part of the orbit determination.

- It is suggested that power for the motor is regulated to about ±1 per cent. Allowing for uncertainties in thrust per unit power and spacecraft mass, this would then imply several per cent uncertainty in thruster acceleration. Provided that this error is slowly varying (e.g. 1 per cent per month) then in-flight calibration is possible.

5.2 NAVIGATION APPROACHING AND NEAR THE COMET

5.2.1 Approach from a Million Kilometres

- Ground-based orbit determination (augmented by spacecraft camera data) can usefully continue and overlap with an autonomous system until within a few thousand km of the comet. However the major source of error (important for a rendezvous) is then likely to be the positional uncertainty of the comet resolved along the approach velocity vector e.g. 1000 km. Much earlier use of camera data should be helpful but the combined spacecraft plus comet state vector estimation has not been included in this study.

- Simplified and satisfactory estimation (from 20000 km) of three approach state variables has been simulated for an autonomous system based on angular rate of the line-of-sight vector and range by radar.

- The availability of radar range from at least 2000 km is virtually essential for comet rendezvous. There is a trade-off between radar range and the use of autonomous procedures for handling under and overshoot.

- A feedback law for approach guidance has been demonstrated; it is indexed by distance-to-go and can be used in the latter part of the ground-based phase as well as the autonomous phase. The specification on pointing the thrust vector would not be demanding because of the feedback loop; in any case errors of several degrees may arise in determining the normal to the orbital plane.

5.2.2 Near the Nucleus of the Comet

- Despite the weak gravitational field of a comet, the orbits about a nucleus should be perfectly stable up to at least 50 nuclear radii. The evolution of orbital parameters for close orbits about such irregular shapes will however depend on the nuclear spin period and orientation of the axis.
- A crude representation of the uncertainty in locating the 'centre' of a nucleus in an imaging and ranging system has been included in the simulations, but it underlines the need for a closer evaluation of these practical aspects. The corruption of such meansurements near the nucleus is likely to be serious.
- A robust procedure (using the above corrupted data) for an early autonomous determination of the gravitational factor has been simulated. It is intended for a coast phase between about 200 and 100 km in order to permit early correction for the large a priori uncertainty in this parameter. This first estimate may be no more accurate than 20 per cent but subsequent refinement can occur, if necessary by ground-based Doppler measurements.
- Recursive estimation (not a Kalman Filter) of major axis, eccentricity and angle-from-perigee has been tested using corrupted measurements of angular rate and range, but least-squares batch fitting is suggested as a more stable robust procedure.
- Manoeuvres near the nucleus would probably not employ a low thrust system, since tiny amounts of cold gas or mono-propellants would suffice. A feedback law for the gradual adjustment of major axis and eccentricity near the nucleus has been formulated but it was not found to be satisfactory except for well determined eccentric orbits. It is suggested that manoeuvres are restricted to infrequent conventional impluse-type corrections.

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