GRAVITY-ASSIST ORBITS

M Belló Mora

G M V S.A.
Madrid, Spain

M Hechler

European Space Operations
Centre (E S O C)

ABSTRACT

This paper expands on the calculation of three different types of gravity assist orbits. The common approach is the formulation of the problem as constrained parameter optimisation and the application of powerful optimisation routines to their solution.
The different examples of gravity assist orbits are: a lunar swingby to rotate the orbit plane of CLUSTER after an ARIANE 4 double launch with SOHO; the GIOTTO Post Halley Earth flyby to comet Grigg-Skjellerup and an energy raising ΔVEGA (ΔV Earth Gravity Assist) orbit for the Comet Nucleus Sample Return Mission.
The proper choice of variables and constraints on the optimisation, and the availability of good initial solutions are necessary for convergence of the method. The latter are obtained by heuristic considerations of celestial mechanics or by solving approximate problems (e.g. patched conics, Lambert problem).

Keywords: Gravity Assist, Flyby, ΔVEGA, Constrained Optimisation

1. INTRODUCTION

Gravity assisted trajectories are powerful means of improving the payload capability of some spacecraft missions. A gravity assist may add energy with respect to the central body by passing the spacecraft through a gravitational field of a planet or moon, it may change orbital parameters to values which otherwise could not easily be reached.

This technique has been successfully employed on the Mariner Venus-Mercury 1973 mission and the Pioneer 10 and 11 missions, the Jupiter and Saturn swingbys of Voyager are well known, Vega has used a gravity assist at Venus, lunar gravity assists have been used by ICE on its way to comet Giacobini-Zinner, Ulysses is planned to use a Jupiter gravity assist and Galileo will utilise a Delta V-Earth Gravity Assist (ΔVEGA).

This paper expands on the methods and results of three different mission analysis studies using gravity assist orbits:

- a lunar swingby to inject CLUSTER in a polar orbit after an ARIANE 4 double launch with SOHO;

- the GIOTTO Post-Halley mission using an Earth flyby;

- the Comet Nucleus Sample Return mission (CNSR) with a ΔV-Earth gravity assist (ΔVEGA).

The common approach in the three gravity assist studies is the formulation of the problem as a constrained parameter optimisation and the use of the Bigg's Recursive Quadratic Programming Algorithm contained in the optimisation routines of the Numerical Optimisation Centre (NOC) Hatfield. The missions for which a gravity assist has been planned or considered are at a different stage of the scientific program of ESA.

<u>CLUSTER</u> is a four spacecraft mission for the three dimensional study of plasma turbulence and small-scale structure in the magnetosphere, phase A has been completed. When descoping the SOHO/ CLUSTER mission after Phase A a dual launch into a 200x15000 km parking orbit followed by perigee manoeuvres and lunar flybys of CLUSTER has been found to increase the overall payload capability by over 500 kg. The required final orbit of CLUSTER has an inclination of 90 degrees and passes through the polar cusp, which is neither compatible with the SOHO orbit requirements nor the ARIANE capabilities. The gravity assist is mainly employed to rotate the orbit plane.

As the proposed <u>GIOTTO mission continuation</u> to the comet Schwassmann-Wachmann 3 had turned out to be not attractive because of the doubtful knowledge on the orbit of that comet, the search for GIOTTO Post-Halley mission opportunities had been reduced to the analysis of the possible targets using an Earth Gravity Assist.

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The Comet Nucleus Sample Return (CNSR) mission is one of the cornerstones in the scientific program of ESA. The mission objective is to return a sample of the comet nucleus to the laboratories on Earth in an unaltered state. Although the use of a ΔVEGA (ΔV Earth Gravity Assist) adds two years to the mission duration, it has been found mandatory because the launch energy can be significantly reduced and much higher spacecraft masses can be brought to the comet and returned to Earth. For CNSR the only alternative to gravity assists techniques within the capabilities of launchers available before the end of the century would be the use of electric propulsion.

2. THE PRINCIPLE OF A GRAVITY ASSIST

The phrases 'gravity assist', 'swingby' or 'planet flyby' are defined as a significant trajectory perturbation due to a close approach (less than 25 planet radii) of a celestial body. The gravitational attraction changes the spacecraft velocity, and usually its energy with respect to the central body (see ref. 25).

The foundations for the study of gravity assists have been laid by some of the early investigators: Kondratyuk (1), Tsander (2), Firsoff (3) and Lawden (4). Following them, the analysis of swingbys to achieve inner and outer planet trajectories was done by Minovitch (5,7), Flandro (6) and Niehoff (8,9). The use of gravity assists from Jupiter's natural satellites in order to achieve Jupiter orbit capture was analysed by Longman (10,11) and Uphoff (12); gravity assist orbit control after capture into the Jupiter system has been addressed by general authors including Minovitch (13), Beckman and Smith (14) and Uphoff, Roberts and Friedman (15).

The principle of gravity assist can be explained by means of a vector diagram (Fig. 1). The spacecraft is in orbit about the primary body and has an encounter with the secondary body, which can either be a planet with $\underline{V}_p$ representing the planet's heliocentric velocity, or a satellite with $\underline{V}_p$ representing the planetocentric velocity vector. $\underline{V}_1$ and $\underline{V}_2$ are the spacecraft velocity vectors with respect to the primary before and after the flyby of the secondary body. $\underline{V}_{\infty 1}$ and $\underline{V}_{\infty 2}$ are the spacecraft velocity vectors with respect to the secondary before and after the flyby. During the swingby the velocity vector is rotated as illustrated in the Figure. Its magnitude does not change since energy is conserved with respect to the secondary body.

However, the velocity rotation causes a change in the magnitude and/or direction of the velocity vector with respect to the primary. This results in a ΔV having been added to the spacecraft's velocity; the change in spacecraft energy with respect to the primary is provided through an exchange of energy with the secondary body. Since the secondary body is many times more massive than the spacecraft, the velocity change of the secondary is totally insignificant.

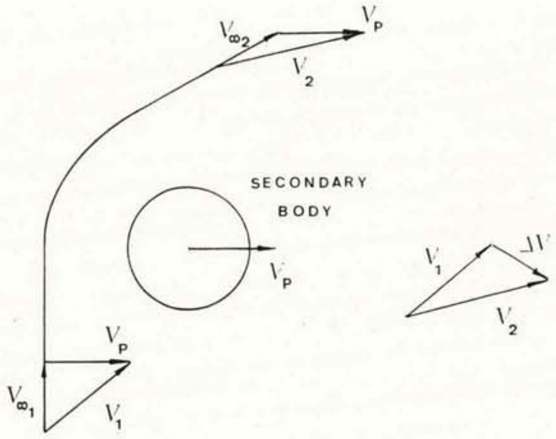


Fig. 1: Flyby principle; $\underline{V}_{\infty 1}$, $\underline{V}_{\infty 2}$ relative velocity at arrival and departure respectively. $\underline{V}_1$, $\underline{V}_2$ absolute velocity before and after the flyby.

The ΔVEGA (ΔV-Earth Gravity Assist) trajectory mode (ref. 18) is a flight technique which utilises the gravity field of the Earth in a swingby mode to reduce the energy requirements for missions to the outer planets or to minor bodies. Several investigators analysed the use of the ΔVEGA technique, including Hollenbeck (16), Bender (17), Hendricks, Satin and Tindle (18).

The basic technique is explained in Fig. 2. The S/C is launched from the Earth (E_2) with a low energy C_3 into a heliocentric trajectory of a period near 2 or 3 years. A manoeuvre $\Delta \underline{V}_a$ is performed near aphelion, targeted to an Earth swingby (E_2) either before (E_2-) or after (E_2+) the integer number of years.

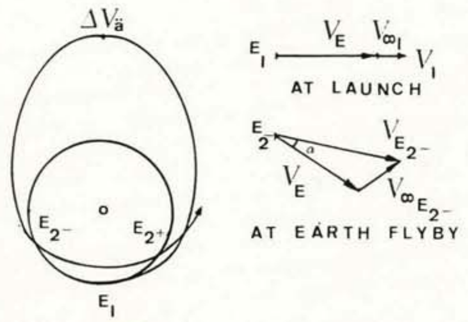


Fig. 2: ΔVEGA trajectory mode; E_2 launch, $\Delta \underline{V}a$ Earth targetting manoeuvre, E_2^- flyby ΔVEGA class I, E_2^+ flyby ΔVEGA class II

The vector diagram (Fig. 2) shows how the flyby velocity at Earth is increased: $\underline{V}_E$ is the Earth heliocentric velocity; $\underline{V}_1$ and $\underline{V}_{E2}^-$ are the spacecraft heliocentric velocity vectors at launch and swingby respectively, and $\underline{V}_{\infty L}$ and $\underline{V}_{\infty E2^-}$ are the velocities relative to the Earth. The spacecraft velocity magnitudes are nearly the same at launch ($\underline{V}_L$) and at swingby ($\underline{V}_{E2^-}$) ($\Delta\underline{V}a$ is a relatively small manoeuvre), but the change of orientation with respect to the Earth velocity produces a higher Earth return speed $\underline{V}_{\infty E2^-}$ at flyby. The ΔVa in deep space depends strongly on the flyby velocity at Earth. Fig. 3 shows the increase of return speed as a function of ΔVa;

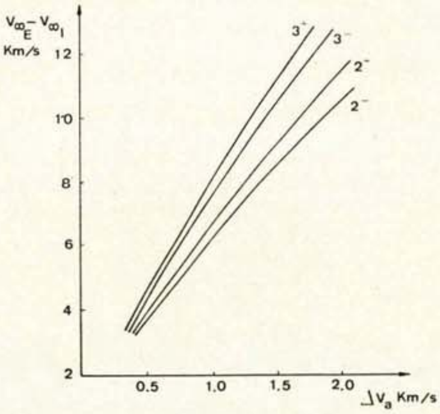


Fig. 3: Increment of hyperbolic excess velocity as a function of the Earth targetting manoeuvre size.

a typical value of 800 m/s, with a two years ΔVEGA, yields a velocity of 10 km/s at flyby for a departure velocity of 5 km/s, so the solar centric gain is almost 5 km/s which as additional velocity increase at launch perigee would be over 3 km/s. This saving accounts for the usefulness of a ΔVEGA to increase the launch masses of missions to outer planets, asteroids or comets at the cost of an additional two years mission duration.

3. CALCULATION OF GRAVITY ASSIST TRAJECTORIES

3.1 The Optimisation Problem

For any gravity assist orbit the flyby conditions, the flyby date and, depending on the problem, other orbital parameters have to be chosen such that the required final orbit conditions and other constraints are satisfied. This usually has to be done to optimise the useful mass of a spacecraft or to minimise a combination of propellant consumption and launch energy.
Apparently calculation of gravity assist orbits are typical parameter optimisation problems.

3.2 Mathematical Formulation

We define a parameter optimisation problem in the usual way as the calculation of the minimum value of a function F(x) of n variables, $x \in E^n$, subject to equality and inequality constraints:

$$g_i(x) = 0 \quad i = 1, \ldots\ldots q \qquad (1)$$

$$g_i(x) \geqslant 0 \quad i = q+1, \ldots., m \qquad (2)$$

The problem formulation, namely the selection of the adequate cost function, variables and con straints, is essential for the convergence of the optimisation process. In our applications the cost function F(x) is typically the usefull mass (with a negative sign to maximise) or the sum of the moduli of the velocity increments ΔV (addition of all the manoeuvres) possibly with weighting factors and special moduli considering the decomposition on the spacecraft.

The n variables of optimisation can be position vectors, velocity vectors at some times, dates, orbital elements, etc.
The equality and inequality constraints primarily are the initial and final conditions to be satisfied or conditions at midcourse events like a minimum flyby radius, etc. On top of that a set of 'technical constraints' may be introduced to accelerate the convergence or avoid divergence during the optimisation process.

The optimisation method requires an 'initial guess' of the chosen variables. There is no systematic way to generate this initial set of values; it has been done in a heuristic way for each problem. E.g. approximate solutions have been generated (like patched conics or Lambert problem solutions), scans on some parameters have been done or the most common values found in literature have been taken. In the description of the three different examples a short summary of this 'tricky' part of the solution finding will be given.

3.3 Solution Technique

Once the problem has been expressed into above standard form, powerful general purpose parameter optimisation programs can be employed. The pro gram used in the present study is OPRQP, developed by the Numerical Optimisation Centre (NOC) at Hatfield. If contains the Bigg's Recur sive Quadratic Programming Algorithm as well as other subroutines required for the solution of non-linear programming problems. The program OPRQP has been successfully used at ESOC/MAS for variety of other optimisation problems of different nature. For details of the method see refs. 23 and 24.

3.4 Selection of Optimisation Parameters and Constraints

Constraints and cost Functional may be quite different in different gravity assist orbit optimisation problems. Two typical sets of optimisation variables can be identified.

3.4.1 Selection of Optimisation Parameters for a single swingby. The optimum trajectory from a departure orbit to a final orbit using the gravitational field of a secondary celestial body (Fig. 4) has to be calculated.

The optimisation variables selected are the following:

t_p : Time of pericentre passage on the flyby hyperbola;
V_∞ : Arrival hyperbolic velocity vector osculating at pericentre;
P : Bidimensional impact vector osculating at pericentre (vector from the centre of the

flyby central body to the intersection of the arrival asymptote with the plane perpendicular to the incoming asymptotic velocity).

The total number of variables for the swingby phase is 6.
Once these variables are defined, the state vector at pericentre of the flyby hyperbola can be easily obtained; integrating backwards from the pericentre to the initial state, and forwards to the arrival conditions the cost function and constraints can be evaluated.

An additional inequality constraint has to be considered for the swingby phase: the pericentre radius can not be lower than the radius of the flyby central body plus a critical height.

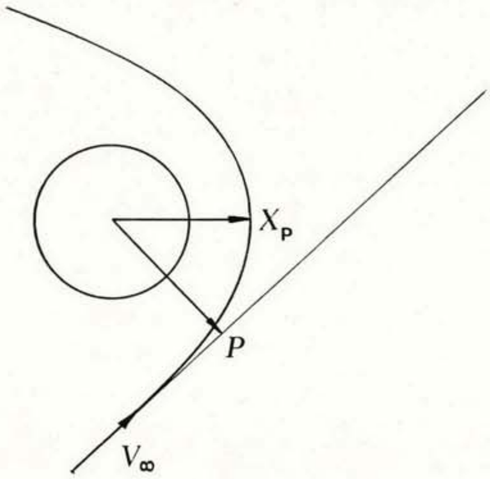


Fig. 4 : Flyby hyperbola; $\underline{V}_\infty$ arrival velocity, $\underline{p}$ impact vector, $\underline{X}_p$ pericentre position vector.

3.4.2 Selection of Optimisation Parameters and Constraints for a ΔVEGA Trajectory Mode

The ΔVEGA Earth Gravity Assist problem is defined in a similar way as the previous basic swingby problem. The optimisation variables selected are the following:

- t_0 : Earth departure date
- t_e : Earth targetting manoeuvre date
- t_1 : Date of perigee passage on the flyby hyperbola
- $V_{\infty 0}$: Hyperbolic excess velocity vector at launch
- P : Bidimensional impact vector at Earth flyby.

The total number of variables for the ΔVEGA case is 8.

Fig. 5 shows the geometry in the terminology of above variables; once these parameters are defined, the heliocentric state after Earth flyby can be easily obtained, and propagating the orbit, the midcourse and final states are computed. From the above all constraints and the cost function can be evaluated.

In addition to the perigee radius condition for a single swingby (section 3), imposing of 'technical constraints' is useful during the convergence process. They are lower and upper limits for t_0 and t_1, and an additional constraint which forces the Earth targetting manoeuvre date to be between the launch and the Earth flyby data.

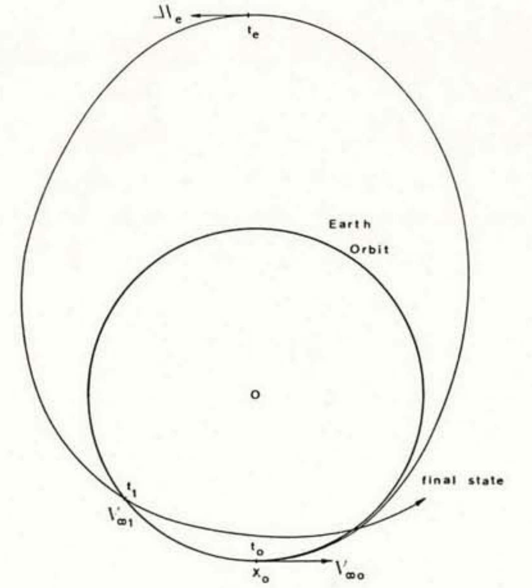


Fig.5: ΔVEGA geometry; $\underline{X}_0$ initial state at launch (t_0), $\underline{V}_{\infty 0}$ hyperbolic excess velocity at launch, $\Delta\underline{V}_e$ Earth targetting manoeuvre, at t_e, $\underline{V}_{\infty 1}$ Earth arrival velocity at flyby (t_1).

4. CLUSTER TRANSFER ORBIT

The first application is the generation of the CLUSTER initial orbit transfer trajectories for a double launch with SOHO on ARIANE4. These are lunar flyby orbits starting from a 200 x 15000 km parking orbit and finally achieving the polar operational orbit of the CLUSTER project.

The specific constraints imposed by this project are the following:

I. Equality constraints:

1. The perigee radius at the parking orbit is fixed (H_p = 200km).
2. The fact that the optimum burn out point of ARIANE should be close to perigee approximately can be formulated as follows:

$$\omega_{north} = \text{arc sin} \frac{\sin \lambda_K}{\sin i} + \theta \qquad (3)$$

$$\omega_{south} = -\omega_{north} + \pi + \theta \qquad (4)$$

where:

λ_K = Kourou latitude
i = Launch inclination
θ = 40.5 deg.

3. The final orbit inclination is fixed ($i_c = 90°$).
4. The final perigee radius is fixed (R_c=4R :R = Earth eq. radius).
5. The argument of perigee of the final orbit is fixed ($\omega_c = 336°$).

II. Inequality constraints:

1. The inclination at launch is limited ($i_{max} = 15°$)

2. The fact that the CLUSTER orbit should pass through the cusp at the begin of the mission adds an upper and lower limit to the longitude of the post flyby perigee relative to the sun:

$$5° \leq \lambda \leq 30°$$

where: $\lambda = \lambda_p - \lambda_s$

λ_p = longitude of the final perigee

λ_s = longitude of the Sun at perigee passage.

There is one degree of freedom (number of variables = 6, number of equality constraints = 5) in this problem, which allows the minimisation of the cost function F (F = sum of perigee ΔV at parking and at final CLUSTER operational orbit).

The initial guess required by the optimisation method is obtained from the analysis of a flyby target map which satisfies initial and final conditions for patched conics.

Two different Lunar Transfer Orbits (LTO) have been found for December/January and July/August 1993 respectively (ref. 20). In the July/August case we will have a post-apogee lunar flyby near the descending node of the lunar orbit (with respect to the Earth equator). In the December/January case the flyby occurs near the ascending node of the lunar orbit, and before the apogee of the LTO, hence, about 2 days after injection into LTO. The time difference from perigee injection into LTO to the lunar flyby is considerably longer (19 days) in July/August.

Fig. 6 shows the SOHO/CLUSTER dual launch transfer schematic.

5. GIOTTO POST-HALLEY MISSION

The second example is the GIOTTO Post Halley Mission, starting from a given initial heliocentric state vector x_0 at epoch t_0 (immediately after encounter with the comet Halley). A manoeuvre ΔV at time t_1 had to be calculated such that a flyby with the Earth gives the adequate transfer orbit in order to reach a comet or an asteroid previously selected.

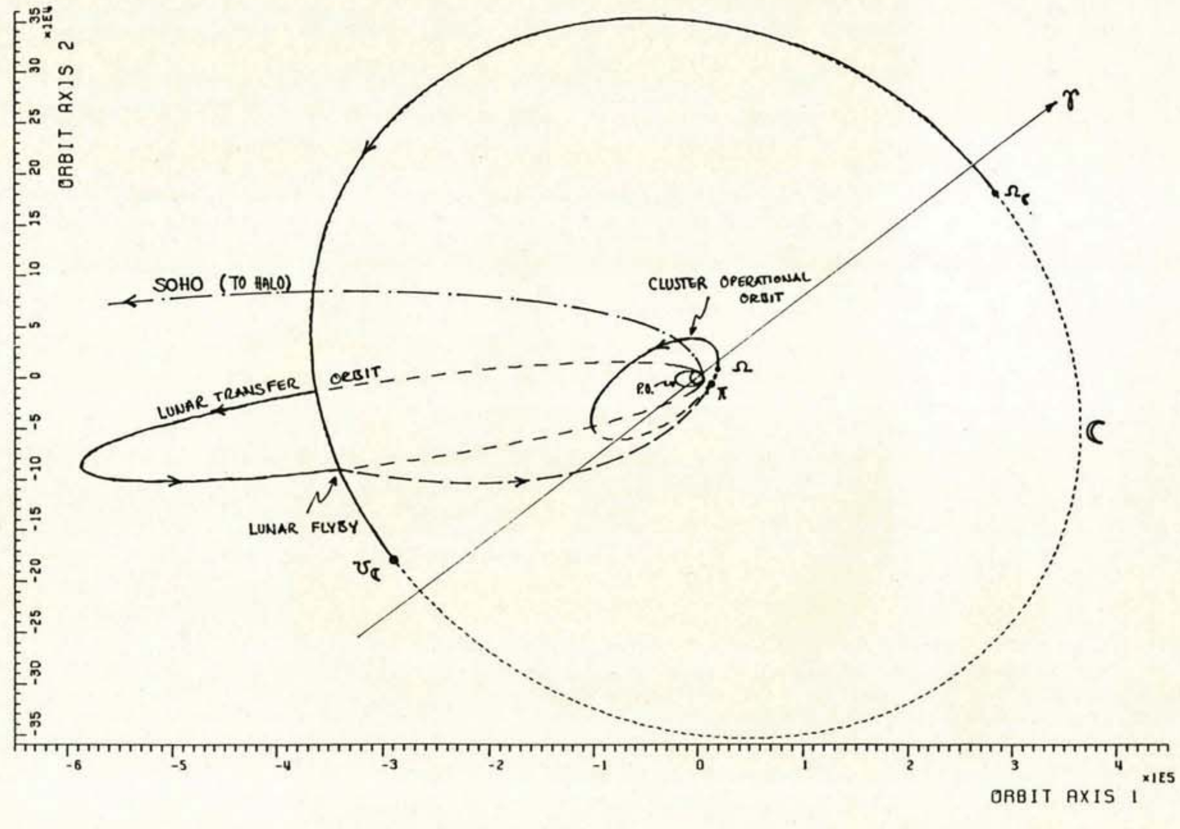


Fig. 6 : SOHO/CLUSTER Dual Launch

In addition to the optimisation parameters described in section 3., two dates are taken as new variables:

t_1 = Earth targetting manoeuvre date
t_3 = Comet or asteroid encounter date.

The total number of optimisation variables becomes 8.

The particular constraints for this problem are the following:

I. Equality constraints:

1. At the manoeuvre date t_1, the position vector $\underline{x_1}$ obtained integrating forwards from the initial state $\underline{x_0}$, has to be equal to the position vector $\underline{x_{1p}}$ obtained integrating backwards from the flyby perigee.

2. At comet or asteroid arrival date t_3, the position vector $\underline{x_3}$ obtained integrating forwards from the flyby perigee, and the celestial body position $\underline{x_c}$ obtained from the catalogue have to be equal.

II. Inequality constraints:

Upper and lower limits are imposed on all dates.

There are two degrees of freedom (number of variables = 8, equality constraints = 6).

The functional is defined as follows:

$$F = \Delta V_{ax} + K \; \Delta V_{rad}$$

where:

ΔV_{ax} = Axial component of the manoeuvre at t_1

ΔV_{rad} = Radial component of the manoeuvre at t_1

K = Efficiency ratio axial/radial manoeuvre.

To calculate the possible spacecraft axis directions the GIOTTO antenna properties are taken into account. Communication during the manoeuvre is assumed.

The initial guess and the selection of candidate comets and asteroids were done using patched conic orbit generation; the 'window' for the Earth targetting manoeuvre is very narrow, and a typical manoeuvre date t_1, a flyby date t_2, and an hyperbolic arrival velocity at the Earth V_∞ can be selected. Scanning over the arrival data t_3 for all comets and asteroids of the catalogue, and solving the corresponding Lambert problem produces a list of candidate missions and the corresponding initial guesses for the optimisation method (Table 1).

Table 1 gives all the comets, numbered and unnumbered asteroids which possibly could be reached within five years after an Earth flyby on 2nd July 1990: among the comets, the mission to the periodic comet Grigg-Skjellerup was assumed to be the more attractive one because of the earlier arrival date.

FLYBY TIME :14793.0 MJD (02/07/90) MAX DEF = 112.06 DEG

COMET OR ASTEROID	DELT-V(KM/S)	N.REV	ENCOUNTER DATE	DEF(DEG)
3813 GRIGG-SKJELLERUP	2.026	1	15549. (07/92)	75.776
3883 NEUJMIN 2	2.405	1	15540. (07/92)	135.906
3993 TUTTLE-GIACO-KRES	2.593	3	16638. (07/95)	125.780
4175 DU TOIT-HARTLEY	2.889	1	15555. (08/92)	137.161
433 EROS	1.380	3	16230. (06/94)	156.625
719 ALBERT	3.018	3	16370. (10/94)	98.724
1011 LAODAMIA	3.641	2	16294. (08/94)	126.110
1221 AMOR	1.427	2	15878. (06/93)	179.162
1566 ICARUS	3.541	3	16142. (03/94)	108.362
1620 GEOGRAPHOS	3.675	0	15144. (06/91)	86.115
1620 GEOGRAPHOS	1.891	1	15300. (11/91)	126.010
1620 GEOGRAPHOS	2.447	2	15810. (04/93)	133.858
1620 GEOGRAPHOS	2.080	3	16182. (04/94)	103.501
1862 APOLLO	1.430	0	15082. (04/91)	67.706
1862 APOLLO	2.716	1	15724. (01/93)	126.888
1862 APOLLO	1.691	2	15734. (01/93)	70.385
1862 APOLLO	1.457	3	16382. (11/94)	104.000
1863 ANTINOUS	3.125	3	16718. (10/95)	130.161
1865 CERBERUS	3.398	0	14938. (11/90)	87.560
1865 CERBERUS	3.267	1	15374. (01/93)	100.970
1865 CERBERUS	3.627	2	15734. (01/93)	102.603
1865 CERBERUS	3.432	3	16578. (05/95)	128.532
1915 QUETZALCOAT	1.891	2	15790. (03/93)	137.328
1917 CUYO	1.088	2	15678. (12/92)	89.624
1943 ANTEROS	3.513	1	15406. (03/92)	110.344
1943 ANTEROS	2.220	2	16034. (11/93)	128.936
1943 ANTEROS	2.692	3	16658. (08/95)	132.304
1951 LICK	3.494	0	15140. (06/91)	104.075
1980 TEZCATLIPOC	2.065	2	15878. (06/93)	114.587
1981 MIDAS	0.354	1	15408. (03/92)	121.226
1981 MIDAS	3.258	2	15502. (06/92)	66.451
1981 MIDAS	1.536	3	16270. (07/94)	86.275
2061 ANZA	3.294	3	16350. (10/94)	117.886
2063 BACCHUS	3.571	0	15282. (11/91)	129.327
2063 BACCHUS	2.587	1	15686. (12/92)	138.377
2063 BACCHUS	2.240	2	16090. (01/94)	141.966
2063 BACCHUS	2.057	3	16498. (03/95)	138.16
2100 RA-SHALOM	2.796	2	15770. (03/93)	101.615
2100 RA-SHALOM	2.868	3	16318. (09/94)	108.124
2101 ADONIS	1.809	1	15530. (07/92)	131.757
2101 ADONIS	2.051	3	16470. (02/95)	129.652
2135 ARISTAEUS	2.012	0	15132. (06/91)	105.163
2135 ARISTAEUS	0.847	2	15866. (06/93)	118.372
2135 ARISTAEUS	3.316	3	16614. (06/95)	107.490
2201 OLJATO	2.577	1	15736. (01/93)	141.982
2201 OLJATO	3.219	3	16446. (01/95)	130.022
2202 PELE	3.018	2	15902. (07/93)	123.293
2329 ORTHOS	2.713	1	15214. (08/91)	126.859
2340 HATHOR	1.502	2	15758. (02/93)	138.184
2340 HATHOR	0.228	3	16022. (11/93)	105.303
2608 SENECA	2.319	2	15990. (10/93)	83.416
4263 72RB	2.636	3	16354. (10/94)	126.642
4315 77VA	1.240	2	15754. (02/93)	105.264
4316 78CA	1.299	0	15082. (04/91)	122.92
4316 78CA	0.562	1	15240. (09/91)	125.253
4316 78CA	1.310	2	15678. (12/92)	155.687
4316 78CA	1.558	3	16114. (02/94)	158.548
4392 79VA	1.651	2	15570. (08/92)	44.200
4396 80AA	3.533	3	16734. (10/95)	107.234
4411 80PA	1.169	3	16154. (03/94)	105.193
4430 80WF	1.715	3	16158. (03/94)	109.856
4444 81ET3	1.269	1	15264. (10/91)	106.043
4444 81ET3	1.430	3	16122. (02/94)	120.247
4430 80WF	3.188	0	14942. (11/90)	166.731
4516 82HR	2.549	0	15210. (08/91)	126.312
4516 82HR	2.622	1	15696. (12/92)	124.151
4516 82HR	1.372	2	15658. (11/92)	56.604
4516 82HR	0.430	3	16166. (04/94)	77.951
4538 82XB	3.618	3	16670. (08/95)	129.467
4608 83RD	1.960	2	15622. (10/92)	53.492
4627 1983 RD	1.879	2	15626. (10/92)	53.345
4689 1982 TA	2.554	2	15822. (04/93)	107.789
4689 1982 TA	3.693	3	15918. (08/93)	38.86
4714 1982 FT	3.459	0	15194. (07/93)	145.637
4714 1982 FT	3.621	3	16918. (04/96)	132.886
4719 1983 TF2	1.680	1	15333. (12/91)	84.572
4719 1983 TF2	1.567	2	15926. (08/93)	82.695

TABLE 1 : GIOTTO Post Halley Gravity Assist Flyby Opportunities

Fig. 7 shows the projection on the ecliptic plane of the trajectory. The size of the Earth targetting manoeuvre is 108.8 m/s, the perigee radius of the flyby hyperbola is 28691 km and the inclination 44.34°; the arrival date to Grigg-Skjellerup is 1992/07/14, with a relative velocity of 14 km/s.

The manoeuvre was executed in the last week of March 1986 very similar to the above proposed by mission analysis.

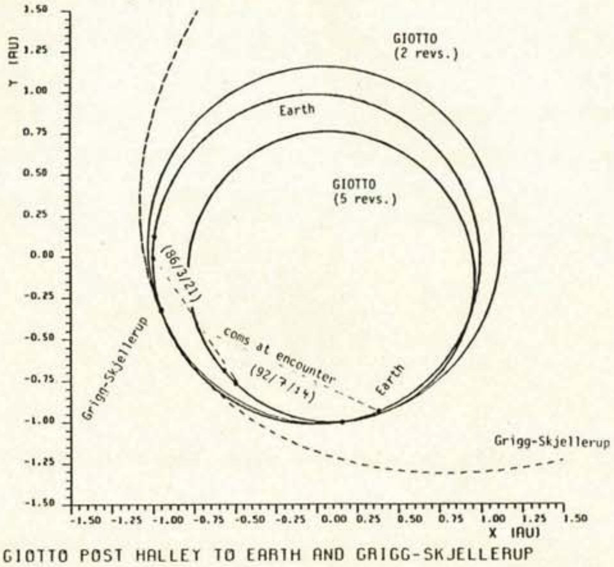


Fig. 7 : Ecliptic projection of GIOTTO transfer to Grigg-Skjellerup

6. COMET NUCLEUS SAMPLE RETURN (CNSR)

The CNSR is the most complicated of the examples given. The spacecraft is injected into a two-year ΔVEGA trajectory mode. If required a further manoeuvre $\Delta\underline{V}_2$ changes the orbital plane at t_2; the comet rendez-vous condition is generated by a manoeuvre $\Delta\underline{V}_a$ at comet arrival t_a. The Earth return starts at t_d ($\geqslant t_a$ + stay time) with a manoeuvre $\Delta\underline{V}_d$ to target to Earth flyby; even tually another manoeuvre $\Delta\underline{V}_4$ at t_4 is required. Aerocapture is assumed at Earth return t_5 (Fig. 8).

In addition to the 8 parameters described in section 2.2.5 for a ΔVEGA trajectory, the following optimization variables have been taken into account:

- t_2 Deep space manoeuvre date on transfer
- t_a Comet arrival date
- t_d Comet departure date
- t_4 Deep space manoeuvre date on transfer
- t_r Earth return date
- $V_{\infty c}$ Relative velocity at comet departure
- δ_c Celestial latitude of the relative velocity vector at departure
- λ_c Celestial longitude of the relative velocity vector at departure.

The following particular inequality constraints have been considered:

- minimum stay time on the comet (100 days)
- lower and upper limits for the comet arrival t_a and the Earth return t_r date
- the midcourse manoeuvre data t_2 and t_4 are forced between the flyby and the comet arrival data and between the comet departure and Earth arrival data respectively.
- For ARIANE 5 launcher:
  - Limit on the hyperbolic excess velocity at launch (V_{min} = 4.5 km/s, V_{max} = 6.5 km/s)
  - Limit on the asymptote declination at launch ($\delta_{min} = -15°$, $\delta_{max} = 5°$)
- For Shuttle/Centaur G' launcher:
  - Limit on the asymptote declination at launch ($\delta_{min} = -28.5°$, $\delta_{max} = 28.5°$).

The cost function is the final mass M_f at Earth return which is computed using a fit of the launch mass (ref. 22) as function of the asymptote declination and escape energy, and the rocket equation for the deep space manoeuvre including proper tankage factors depending on the staging.

The initial guess is obtained by dividing the global problem in three different parts:

1. ΔVEGA trajectory
2. Three impulses Earth-comet rendez-vous trajectory
3. Three impulses comet-Earth flyby trajectory.

The second part is executed as first step; the optimum three impulses transfer is obtained for Keplerian orbits. The second step is to fit a Δ VEGA trajectory using a typical value for the launch energy C_3. Finally, after a minimum stay time on the comet, a three impulses transfer comet-Earth completes the initial guess.

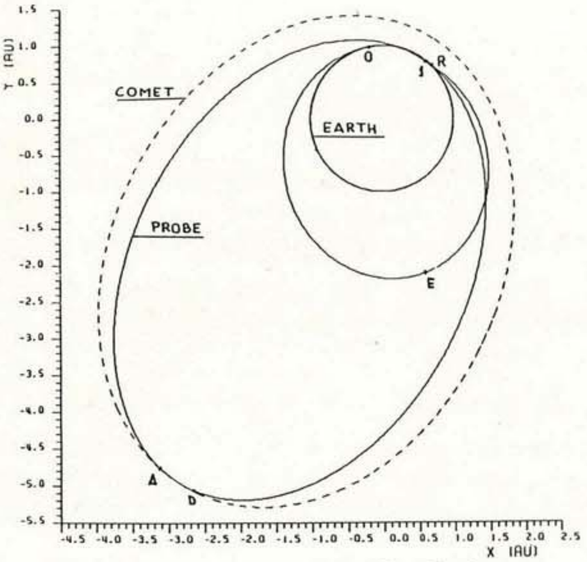
A previous selection of candidate comets has been done considering the launch date (1995-2001) and the global mission duration (not exceeding 8 years). For all comets in the list of candidates a mission has been obtained for the two classes of ΔVEGA, and for two different launchers: ARIANE 5 and Shuttle-Centaur G'.

Table 2 summarised the best CNSR missions for each year, and Fig. 8 shows the ecliptic projection of a typical rendez-vous round trip to the comet Churyumov-Gerasimenko.

SELECTION OF CANDIDATE MISSIONS

YEAR	COMET	FINAL MASS (SHUTTLE) KG	FINAL MASS (ARIANE 5) KG	DURATION (YEARS)
1995	HANEDA CAMPOS	1252	415	8
1996	TRITTON	951	401	8
	NEUJMIN 2	1625	775	7
1997	HOWELL	763	322	7
1998	TEMPEL 1	1018	475	7
	HONDA-MRKOS-PAJDU	1120	457	8
1999	SCHWA-WACH 3	1568	643	7
	FINLAY	2128	651	9
2000	WIRTANEN	1539	570	7
	CHURYUMOV-GER.	1928	999	8
2001	WILD - 2	936	377	8
	WIRTANEN	1559	767	7
	DU TOIT-HARTLEY	1429	693	7

Table 2 : CNSR Mission Opportunities



OVEGA TRANSFER TO CHURYUMOV-GER (ARN 5)

Fig. 8 : Ecliptic projection of Churyumov-Gerasimenko CNSR mission; 0 launch, E Earth targetting manoeuvre, 1 flyby, A comet arrival, D comet departure, R Earth return.

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