TO BUS AND BACK

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ABSTRACT

The Numerical Optimisation Centre, whilst under contract to the Mission Analysis Office at ESOC, has developed a software capability for the computation of optimal satellite trajectories.

Following delivery of the software to ESOC we decided, as part of the second author's doctoral programme, to use the software to investigate the problem of rendezvousing with Comet Bus and subsequently returning to earth. This investigation was carried out for a number of different power levels. On a number of these the optimisation algorithm produced a solution in which the sign of the switching function was incorrect over all or part of the trajectory. Subsequent investigation has indicated that this implies that an incorrect pattern of thrust and coast arcs had been specified, and the method we have adopted for using this information to determine the correct pattern will be described.

1. INTRODUCTION

In this paper we study the optimisation of a hypothetical mission to the comet Bus. It is assumed that the satellite will be launched from an Ariane 5 launch vehicle on 28 December 1997 with launch mass 4388.2 kg and launch velocity 2.0 km/sec. The objective is to rendezvous with the comet, deliver 200 kg of equipment and stay 90 days in the comet's orbit before leaving to rendezvous with the earth.

It is assumed that the power unit is a solar array unit generating P_o = 34.546 kw, specific impulse 3383 sec, efficiency 61% and a solar constant $k = 1.6$. It is further assumed that a degraded power level of 32 kw is available for the return mission.

The approach adopted is based on the indirect, neighbouring extremal, philosophy in which the thrust angles are determined by Pontryagin's Maximum principle [1] enabling the optimal control problem to be converted into a standard nonlinear programming problem.

It is very well-known that such a nonlinear programming problem is very sensitive to the initial values of the adjoint variables if these are used as optimisation variables so, to avoid this difficulty, an adjoint-control variable transformation [2] is introduced which enables the initial values of the thrust angles and their derivatives to be used as optimisation variables. As it is known that most optimal trajectories consist of both thrust and coast phases the starting time of these phases are also used as optimisation variables [3].

Experience demonstrated that recursive quadratic programming codes frequently terminate at feasible points at which the reduced gradient is nonzero [4] and a special purpose code OPQFQC designed to cope with highly nonlinear constraints had to be written to reliably solve the optimisation problem [5], [6].

The OPQFQC code [5] uses a quadrtic approximation to both the objective function and the constraints at each iteration. This algorithm successfully solves the problem given an assumed combination of coast and thrust arcs. This paper is concerned with the problem of determining the correct combination when faced with a new type of mission. The return mission from Bus was the first mission of that type attempted.

2. THE FORWARD MISSION

In the forward mission the mission profile used was initially C/T/C/T with the length of all the four arcs as optimisation variables. Seven runs were carried out and a summary of the results is given in table (1). In this table the final mass in orbit, the total mission time, the length of the arcs, the summation of the absolute values of the constraint violations at the final point |C| and the free gradient are given for each run. The optimisation algorithm converged in all cases. In run 1 the second coast arc was limited to be not greater than 200 days. This limit was increased to 220, 240, 260, 280, 300 and 400 days for runs 2 to 7 respectively. Table (1) shows that for run 1 to run 6 the upper limit on this coast arc was active and this is shown by the underline of the second coast arc in the table. The table also shows that the final mass in orbit increases as the limit of the coast arc increases as would be expected. The total mission time also increases as the upper limit on the second coast arc increases.

The switching functions for these seven runs are shown in figure (1). Theoretically at the ends of optimal (unconstrained) coast or thrust arcs the switching function should pass through zero. It

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should be negative during thrust arcs and positive during coast arcs. This figure indicates that the switching function does not pass through zero at the start and the end of the coast arc when its upper limit is active. This result was also expected. In run 1 to run 4 the slope of the switching function at the start and end of the second coast arc suggests that the switching function is correct. For run 5 the slope of the switching function at the start of the second coast arc seems to be incorrect. The switching function of run 6 indicates that although the optimisation code converges to a solution of the nonlinear programming problem, this solution may not be the optimal solution for the original control problem. For run 7 where the upper limit on the second coast arc was not active the switching function passes through zero at the correct switching times but has the wrong sign through the whole mission. This indicates again that although the optimisation code OPQFQC converges to a solution which satisfies the necessary conditions for the nonlinear programming problem, this may not be the optimal solution of the original optimal control problem. These results indicate that the mission profile used may not be the best and introduction of more arcs may improve the final mass in orbit and correct the switching function.

3. THE FORWARD MISSION WITH MORE ARCS

The switching function of run 7 suggested that it may be better to introduce more arcs rather than using the C/T/C/T strategy. Some more runs to investigate this possibility were carried out. In the first run the mission profile was changed to C/T/C/T/C/T. Using a starting point near the result of run 7 and dividing the first thrust arc into the three arcs of 300 days thrust, 20 days coast and 262 days thrust to obtain an initial point OPQFQC converges in 20 iterations to a solution where the final mass in orbit is 2 kg better than run 7 and a total mission time 8 days longer than run 7. The result of this new run (run 8) is given in table (2). The corresponding switching function is the solid curve in figure (2). This switching function shows that:

1) The switching function does not pass through zero at the end of the first coast arc. This is because the first coast arc was now only 2 days (the lower limit for this arc).

2) The switching function passes through zero as expected at all the other switching times.

3) The sign of the switching function is correct for all arcs other than the last thrust arc.

This result was promising and indicated that errors in sign of the switching function can be used to improve the result, so we decided to add another coast arc by replacing the last thrust arc by the three arcs 330 days thrust, 30 days coast and 220 days thrust. Using the output of run 8 with the new arcs as a starting point to the new run (run 9) the result is given in table (2). From this table we can see that two arcs were moved to their lower limit of 0.1 day and the result suggests that only 6 arcs are needed. It is also noticed that the final mass in orbit is about 9 kg better than that of run 7, however the total mission time increased by about a year. The switching function for this run was correct and very close to the curve marked (run 10) in figure (2). One more run was then tested in which we split the last thrust arc of run 7 into the three arcs, 300 days thrust, 40 days coast and 240 days thrust. Using a starting point close to the output of run 7 and running OPQFQC for the new 6 arcs case (run 10), OPQFQC converges to the solution given in table (2). It is clear that this solution is very close to that of run 9. The switching function for run 10 is also shown in figure (2). This switching time is correct at all times. This investigation shows that the nonlinear programming problem formulated in [3] can have a solution which is not the optimal solution for the control problem if the thrusting strategy is incorrect. It also demonstrated that by studying the switching function we can get an idea about the changes in the mission profile that may produce an optimal solution for the original control problem.

Run	Final mass (kg)	Total time (day)	Arcs length (day)	|C|	Free gradient
run 1	3258.2	1375.2	23.9/600.6/$\underline{200.0}$/550.7	3.69 10^{-9}	9.16 10^{-7}
run 2	3265.5	1386.7	22.0/592.9/$\underline{220.0}$/551.8	3.57 10^{-9}	1.60 10^{-7}
run 3	3271.2	1400.0	19.6/585.6/$\underline{240.0}$/554.8	1.22 10^{-8}	1.35 10^{-7}
run 4	3275.2	1415.0	16.6/578.9/$\underline{260.0}$/559.5	6.31 10^{-8}	2.19 10^{-6}
run 5	3277.5	1431.8	13.1/572.9/$\underline{280.0}$/565.8	5.14 10^{-8}	2.48 10^{-6}
run 6	3278.6	1450.2	9.3/567.0/$\underline{300.0}$/573.6	3.85 10^{-8}	3.54 10^{-7}
run 7	3278.8	1469.4	5.4/562.3/319.5/582.6	1.49 10^{-7}	5.87 10^{-7}

Table (1). The results for the forward mission

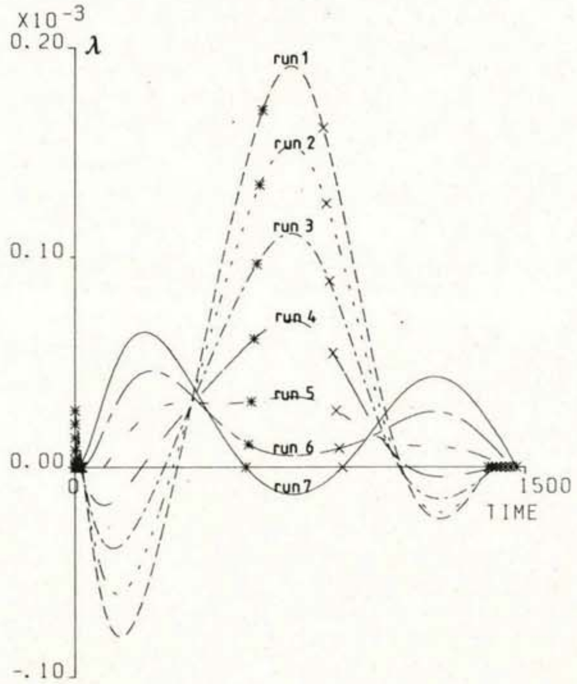

Figure 1. The switching function for the forward missions in table (1)

4. THE CONTINUATION METHOD

Any optimisation code runs more efficiently if it can be started from a reasonable estimate of the solution. In trajectory optimisation the problem becomes more difficult as the power available decreases. We have found it useful to approach the solution of difficult low power problems from easier high power solutions by a continuation strategy.

In the limit of impulsive flow Lambert's method leads to a simple optimisation problem involving simply the length of flight and angle of entry into the required orbit. This can be computed readily [4]. This result can be used to obtain the solution for a high powered (1000 ks say) vehicle by putting the finite thrusts in the same direction and equivalent magnitude to the Lambert solution. The usual optimisation variables [3] can then be computed that lead to this solution and an optimisation carried out.

The power level can then be reduced steadily using the solution at one level as the starting point of the next.

If the final problem is a rendezvous problem and the initial problem is the simpler same orbit problem then a switch can be made at the appropriate power level when the optional same orbit trajectory is approximately a rendezvous. This approach was used to solve the return mission described in the next section.

5. THE BACKWARD MISSION

In the previous sections we showed that the forward mission could be performed with more than one set of control arcs. The final mass in orbit was in the range 3258.2 - 3287.6 kg. The arrival date at comet Bus was in the range October 2001 - 26 January 2003. It was then decided to select the result of run 7 in section 3 which gives a final mass in orbit of 3278.8 kg and an arrival date of 6 January 2002 as a basis for the backward mission.

For the backward mission the initial mass is assumed to be 3078 kg and the launch date is 6 April 2002. This means that the mass delivered at the comet Bus is 200.8 kg and that the satellite stayed 90 days moving with Bus before starting the backward mission. The power available to the backward mission is assumed to be P_O = 32 kw compared with 34.546 kw for the forward mission.

The Lambert/continuation approach described in the last section was implemented in this mission. The Lambert two impulse solution of this mission was as follows:

the propellant used = 1047.9 kg,
the final mass in orbit = 2030.1 kg
δv_1 = 5.53 km/sec, δv_2 = 10.12 km/sec,
t_f = 736.1 day and the earth was about 125° in advance of the satellite. We then proceeded by assuming a power unit of 1000 kw instead of infinite power and found an estimate for the initial value of the optimisation variables needed to solve the corresponding same orbit optimal trajectory problem. The results are given as run 1 in table (3). In this table $\theta_e - \theta_s$ is the angle between the earth and the satellite at the end of the mission. Using the continuation approach the results for power levels 500, 400 and 300 kw are shown in table (3). The switching function for run 4 (P_O = 300 kw) is

Run	Final mass (kg)	Total time (day)	Arcs length (day)	|C|	Free grad.
run 7	3278.8	1469.4	5/562/320/583	$1.5\ 10^{-7}$	$5.9\ 10^{-7}$
run 8	3280.8	1477.8	$\underline{2}$/384/34./186/292/580	$1.4\ 10^{-8}$	$3.7\ 10^{-7}$
run 9	3287.6	1854.9	$\underline{2}$/561/$\underline{0.1}$/$\underline{0.1}$/358/438/310/186	$3.6\ 10^{-6}$	$4.5\ 10^{-6}$
run 10	3287.6	1854.9	$\underline{2}$/561/358/438/310/186	$8.0\ 10^{-9}$	$1.1\ 10^{-6}$

Table (2). The results for the forward mission using different configurations

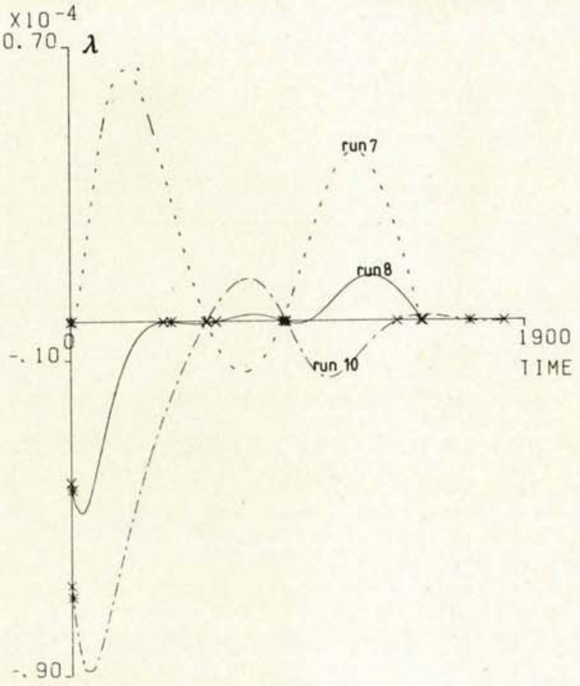

Figure 2. The switching function for the forward missions in table (2)

Run	1	2	3	4
power P_0 (kw)	1000.0	500.0	400.0	300.0
fuel used (kg)	842.8	840.1	838.7	836.4
mass in orbit (kg)	2235.2	2237.9	2239.3	2241.6
thrust arc (day)	31.7	63.6	79.7	106.7
coast arc (day)	995.5	983.3	975.7	961.7
thrust arc (day)	7.2	14.5	18.2	24.3
total time (day)	1034.4	1061.4	1073.6	1092.8
|C|	$6.1\ 10^{-6}$	$5.1\ 10^{-7}$	$8.8\ 10^{-7}$	$5.7\ 10^{-8}$
free gradient	$3.8\ 10^{-6}$	$2.9\ 10^{-6}$	$2.8\ 10^{-6}$	$1.1\ 10^{-7}$
$\theta_e - \theta_s$ (deg)	41.3	18.7	8.8	-8.1

Table (3). The results for the same orbit mission from comet Bus to the earth for different power level

Run	1	2	3	4
power P_0 (kw)	300.0	200.0	150.0	100.0
fuel used (kg)	836.5	833.6	834.0	851.3
mass in orbit (kg)	2241.5	2244.4	2244.0	2226.7
thrust arc (day)	106.9	163.8	224.6	371.1
coast arc (day)	954.8	891.8	824.3	663.2
thrust arc (day)	24.3	36.7	49.5	77.1
total time (day)	1086.0	1092.3	1098.5	1111.4
$\|C\|$	$3.1 \; 10^{-9}$	$1.2 \; 10^{-7}$	$1.4 \; 10^{-8}$	$8.9 \; 10^{-10}$
free gradient	$4.8 \; 10^{-8}$	$3.9 \; 10^{-6}$	$5.2 \; 10^{-7}$	$6.0 \; 10^{-9}$
$\theta_e = \theta_s$ (deg)	184.5	190.7	196.8	209.4

Table (4a). The results for the rendezvous mission Bus-earth for different power level

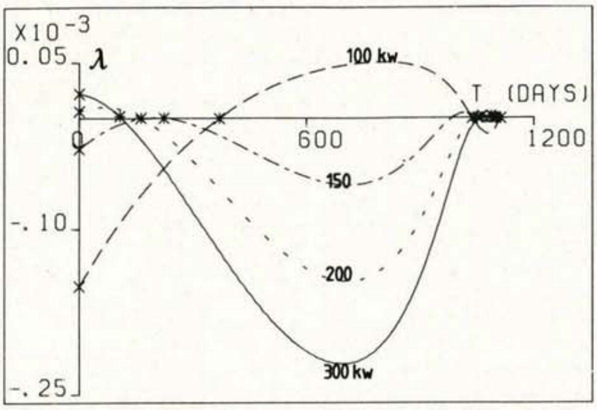

Figure 3. The switching function for the rendezvous missions from comet Bus to the earth in table (4a)

Run	5	6	7	8
power P_0 (kw)	75.0	50.0	40.0	38.0
fuel used (kg)	854.0	857.9	892.5	928.4
mass in orbit (kg)	2224.0	2220.1	2185.5	2149.6
thrust arc (day)	439.9	483.9	389.0	433.8
coast arc (day)	942.9	775.5	573.6	403.0
thrust arc (day)	125.4	287.5	623.3	760.0
total time (day)	1508.3	1546.9	1586.0	1596.7
$\|C\|$	$8.8 \; 10^{-9}$	$5.2 \; 10^{-8}$	$2.7 \; 10^{-8}$	$1.3 \; 10^{-9}$
free gradient	$8.0 \; 10^{-8}$	$5.6 \; 10^{-6}$	$1.1 \; 10^{-6}$	$6.0 \; 10^{-8}$
$\theta_e = \theta_s$ (deg)	240.1	277.1	314.4	324.3

Table (4b). The results for the rendezvous mission Bus-earth for different power level

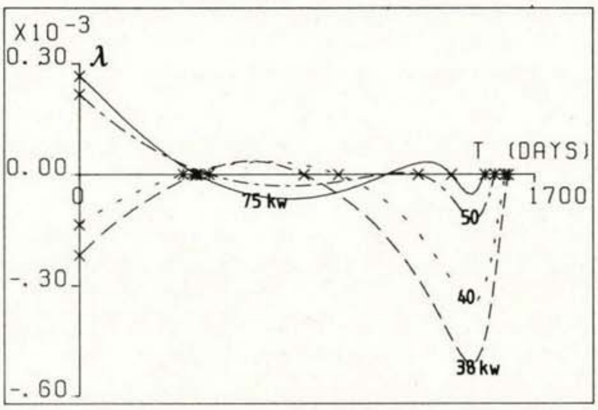

Figure 4. The switching function for the rendezvous missions from comet Bus to the earth in table (4b)

Run	9	10	11
power P_0 (kw)	36.0	34.0	32.0
fuel used (kg)	1002.8	1207.0	1231.9
mass in orbit (kg)	2075.2	1871.0	1846.1
thrust arc (day)	1602.9	1698.9	1744.9
$\|C\|$	$7.0 \; 10^{-9}$	$2.0 \; 10^{-9}$	$2.1 \; 10^{-10}$
free gradient	$6.2 \; 10^{-6}$	$7.3 \; 10^{-7}$	$8.9 \; 10^{-7}$
$\theta_e = \theta_s$ (deg)	330.6	65.4	112.3

Table (4c). The results for the rendezvous mission Bus-earth for different power level

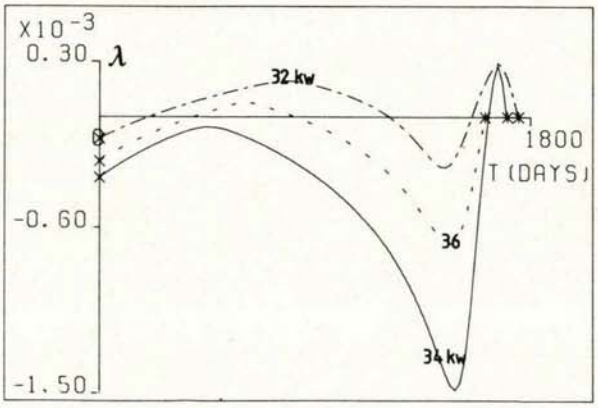

Figure 5. The switching function for the rendezvous missions from comet Bus to the earth in table (4c)

shown in figure (3). The switching function for runs 1, 2 and 3 are very close to that of run 4. This switching function indicates that the solution obtained may not be optimal for the original trajectory problem and that the use of less power may lead to a better result. The results in table (3) confirm this since the final mass in orbit improves slightly with reducing the power level. From this table we can see that the mission time also increases as P_o decreases and as a result of this the angle between the earth and the satellite changes. It is also clear that since at P_o = 400 kw the earth was 8.8° in advance of the satellite and at P_o = 300 kw the satellite was 8.1° in advance of the earth we expect that a rendezvous mission will be possible at a power level between 300 and 400 kw.

It was therefore decided to switch to rendezvous mission at P_o = 300 kw. The initial value used for this rendezvous mission was the final value of the same orbit mission with the same power level [run 4 table (3)]. The results for this rendezvous mission are given in table (4a). In this table the last row is the position of the earth and hence the satellite at the end of the mission. The main difference between the same orbit mission and the rendezvous mission at P_o = 300 kw is a reduction of the final mass in orbit of about 0.1 kg. The continuation approach was then used for the rendezvous mission and the results for P_o = 200, 150 and 100 kw are given in table (4a). The corresponding switching functions are shown in figure (3). It can be seen from this figure that the switching function changes rapidly as the power is reduced. For P_o = 300 and P_o = 200 kw the sign of the switching function was wrong during the whole trajectory while for P_o = 150 it was correct for the two thrust arcs and part of the coast arc. At P_o = 100 kw the switching function was correct all the time. The final mass in orbit improved by reducing the power level from 300 to 200 kw which agrees with the incorrect sign of the switching function, but then it starts to be reduced when the power is reduced from 200 to 150 then 100 kw. It is then noticeable that the total mission time increases by about 25 days when the power is reduced from 300 to 100 kw. As expected the thrust time increases as the power is reduced but the main change in the arcs is the reduction of the coast arc from 955 days at P_o = 300 kw to 663 days at P_o = 100 kw. The results for the continuation for P = 75, 50, 40 and 38 kw are given in table (4b) and the corresponding switching functions are shown in figure (4). By reducing the power from 100 to 74 kw the final mass in orbit was reduced from 2226.7 to 2224.0 kg, a reduction of only 2.7 kg but the total mission time jumps from 1111.4 to 1508.3 days, an increase of more than one year and significantly the switching function now has the wrong sign during the first thrust arc and part of the coast arc. By reducing the P_o to 50 kw the switching function improved a little and the final mass in orbit reduced by 4 kg. Reducing P_o to 40 then 38 kw the switching function becomes correct but the final mass in orbit starts to drop quickly.

Run	1	2	3	4
fuel used (kg)	1231.9	1195.7	927.3	926.3
mass in orbit (kg)	1846.1	1882.3	2150.7	2151.8
thrust arc (day)	1744.9	834.9	927.1	924.5
coast arc (day)	---	226.9	102.5	113.5
thrust arc (day)	---	663.2	484.4	467.5
coast arc (day)	---	---	296.7	123.4
thrust arc (day)	---	---	47.3	5.7
coast arc (day)	---	---	---	175.4
thrust arc (day)	---	---	---	48.6
total time (day)	1744.9	1765.1	1857.9	1858.6
$\lvert C \rvert$	$2.1\ 10^{-10}$	$5.0\ 10^{-8}$	$1.1\ 10^{-7}$	$5.7\ 10^{-8}$
free gradient	$8.9\ 10^{-7}$	$7.0\ 10^{-7}$	$7.9\ 10^{-6}$	$3.4\ 10^{-6}$
$\theta_e = \theta_s$ (deg)	112.3	132.9	225.0	225.6

Table (5). The results for the rendezvous mission Bus-earth for P_o = 32 kw

Reducing P_o to 36 kw the coast arc moves to its lower limit of 0.1 day and the only possible answer found to this problem was to use continuous thrusting. The results for the continuation using continuous thrusting for P_o = 36, 34 and 32 kw are given in table (4c) and the corresponding switching functions are shown in figure (5). As expected the thrusting time increased as P_o decreased and the main remark was the very fast drop in the final mass in orbit. The switching function curves for these power levels indicate that continuous thrusting is not the best mission profile although it was the only solution found for P_o = 36 kw.

We may conclude that using the Lambert/continuation approach helps us to find a solution for the non-

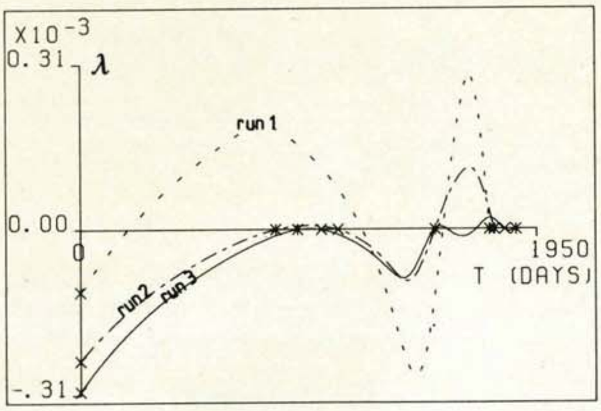

Figure 6. The switching function for the rendezvous missions Bus-earth for P_o = 32 kw in table (5)

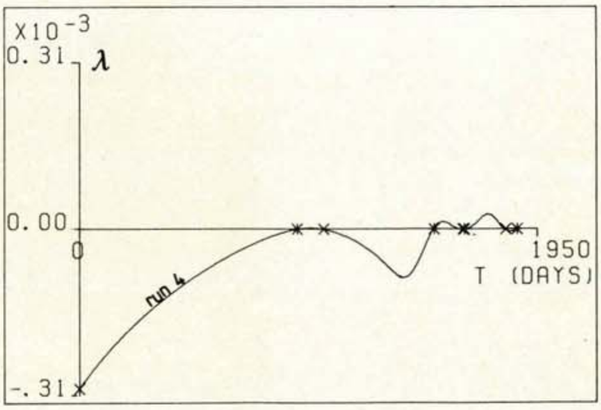

Figure 7. The switching function for the rendezvous mission Bus-earth for P_o = 32 kw in table (5)

linear programming problem formulated from the original optimal control problem but the switching function suggests that continuous thrusting is not the best mission profile.

To investigate this conclusion further we decided to examine the effect of changing the mission profile on the switching function and the final mass in orbit at $P_o = 32$ kw. The switching function using continuous thrusting suggests that adding a coast arc after about 750 days may improve the performance. A run using T/C/T strategy was then carried out and the initial value of the optimisation variables for this run were the final values obtained for the continuous thrust case [run 11 table (4c)] other than the thrust arc which was replaced by the three arcs 750 days thrust, 40 days coast and 960 days thrust. The results of this run are given in table (5) in the column run 2. In this table run 1 is the continuous thrust case reported before as run 11 in table (4c). The switching function for both T and T/C/T configurations are shown in figure (6). The final mass in orbit improved by about 36 kg while the total mission time increased by about 20 days by changing the mission profile from continuous thrusting to T/C/T. However, while the switching function sign for the T/C/T strategy was correct for the first thrust arc and the coast arc it was still partially wrong for the second thrust arc. A run using T/C/T/C/T strategy was then carried out with the initial value of the optimisation variables the same as the final values obtained in run 2 and replacing the last thrust arc by the three arcs 480 days of thrust, 20 days of coast and 165 days of thrust. The results of this run were very good and they are reported as run 3 table (5). The final mass in orbit is about 268 kg better than the result for the T/C/T strategy and more than 305 kg better than the continuous thrust case. The total mission time increased by 92 days compared with the T/C/T mission and by 113 days compared with the continuous thrust case. The switching function for this T/C/T/C/T mission is shown in figure 6 run 3. It is clear that this switching function has the correct sign for all the arcs other than the last coast arc and that an introduction of an additional thrust arc at the middle of this coast arc may still improve the performance. A final run using the final value of the optimisation variables of run 3 as an initial estimate and replacing the last coast arc with the three arcs 120 days coast, 20 days thrust and 160 days coast was then carried out. The results of this run are reported as run 4 in table (5) and the corresponding switching function is shown in figure 7. The switching function is correct at all times but the improvement in the final mass in orbit was only 1 kg compared with the T/C/T/C/T strategy with one day increase in the total mission time. We note that occasionally a change in switching strategy can lead to a dramatic improvement but on the other occasions the improvement is negligible.

6. A FINAL COMMENT

While discussing the results of the return mission we noted that the switching function curves of the high powered results used in the continuation strategy indicated that these were not optimal. Out of curiosity we applied a T/C/T/C/T strategy at a power level of 300 kw, this gave a correct switching function and a mass in orbit of 2249.8. This small change of 8 kg is sufficient to give a higher mass in orbit than any lower powered system and to remove the anomaly of better performance apparently being associated with low power. For completeness the projection of the combined mission is shown in figure 8.

7. ACKNOWLEDGEMENTS

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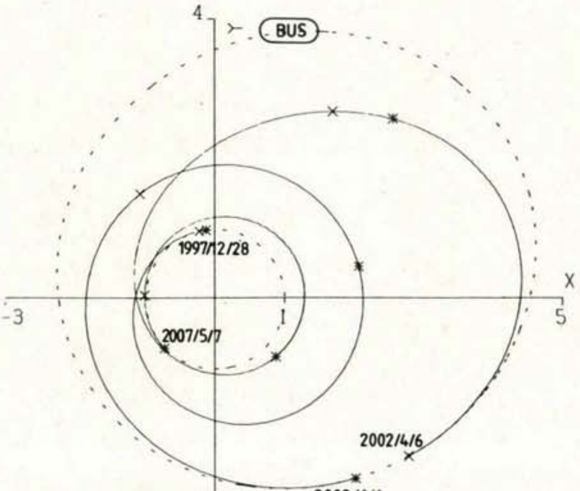

Figure 8. The optimal trajectory for the combined mission