A TRAJECTORY OPTIMIZATION PROGRAM FOR SPACECRAFT NAVIGATION

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ABSTRACT/RESUME

The presentation gives an introduction to the calculation program system SPACENAV, a special optimization scheme for three-dimensional multiple-burn manoevre strategies of spacecraft.
The innovative analytical and numerical methods of the program system are introduced.
Finally, some example trajectories are schown. These examples cover the flight planning of an interplanetary solar-electric propulsion Multi-Asteroid-Rendezvous and Flyby Mission.

Keywords: Interplanetary, Solar-electric, Propulsion, Spacecraft, Trajectory, Optimization.

The optimization of spacecraft trajectories is an important aspect of astronautical engineering. Using optimization techniques spacecraft devices are constructed for the cost effective transportation of maximal possible payloads in minimal time.
Generally the payload of a rocket propelled device is rather small in comparison with the overall rocket mass. In these cases only little derivations from optimum in the performance of the system parameters can diminish the payload in a considerable measure. The parameters describing the complicated connections in the propulsion system between jet power, engine thrust, effective exhaust velocity etc.,are to be defined. They are to be interpreted for the spacecraft preliminary design in order to maximize the transportable payload mass. Generally, the problem of the parametrical rocket optimization can not be solved independently of the dynamical optimization of the trajectories, as for each modified spacecraft configuration another trajectory becomes optimal .(Sometimes, in difficult astronautical missions, the trajectories even define the frame conditions for the mission. Optimization techniques are not only applied for enhancing the effectivity of the mission, but at least, for making the mission possible at all.)

Hence, the task is to chose from all possible flight ways through space these ones, in which the particular missions can be performed payload- or time-optimal. A suitable method for the solution of this task is supplied by mathematics with the calculus of variations (Hamilton Lagrange theory). Systems of differential equations can be derived by an application of this difficult comprehensive mathematical theory for the definition of an optimal trajectory. Even if these equations are not complicated in their structur, they can only be solved numerically by a computer, as complete analytical solutions do not exist - at least until now. Rendering more difficult, by the method calculus of variations only the differential equation systems can be constructed, not, however, their belonging initial conditions. The integrals of these differential equations must be adjusted to the demanded initial and end conditions by an iterative search strategy. Therefore the search for the optimal trajectory becomes the solution of a differential equation system; uncompletely defined at two points, but altogether completely defined. This problem is typical for trajectory optimizations and is one of the most difficult problems of numerical mathematics.

Herewith, the solution of the two point boundary value problem becomes especially difficult, because:

- the solutions depend very much on the initial conditions of the differential equations, and small derivations in these initial conditions lead to extremly different trajectories.
- the conditions, which define the switching points for the rocket engines are nearly everywhere valid on the optimal trajectory.
- the equation systems are highly nonlinear.

Proceedings of the Second International Symposium on Spacecraft Flight Dynamics, Darmstadt, FR Germany, 20–23 October 1986 (ESA SP-255, Dec. 1986)

The aim of this study is to give a contribution to the solution of the problem of trajectory optimization in space. For this a completely new developed analytical and numerical method is introduced.

The analytical method is consistent in all points with Lawden's "Primer-Vector" theory (/1/ and /2/). However, in contrary to Lawden's "Primer-Vector" theory, this new method is not based on an inertial carthesian coordinate system, but on a moving coordinate system. In this moving system the three-dimensional motion is regarded as a two-dimensional motion in a moving plane. By this the formulation of the celestial mechanics becomes presentable in a very simple manner. The results become presentable by values with transparent physical interpretations. In some points an important further development beyond the results of Lawden's Primer-Vector theory was possible.(As an example may be mentioned the complete analytical integration of the control laws for the three-dimensional part of the motion).

The special numerical method for the calculation of the trajectories using the new analytical method is innovative, too. In contrary to the often applied hybrid method (/10/ and /13/), which tries to find the switch conditions for the engine by a nonlinear optimization algorithm neglecting the switch function and the transversality conditions, the method developed here is based on the more direct solution of the nonlinear equation system established by calculus of variations. By this the handicap of the hybrid method is avoided.(Possibly the switching function does not vanish at the switching points found by the hybrid calculation).

The new numerical method becomes possible by the fact, that the new analytical method yields the closed analytical solution of motion and control during coast arcs and during impulsive thrust arcs in a comparatively very simple manner. Thus, the calculation of an impulsive thrust program does not need any numerical integrations; first of all the algorithm generates the three-dimensional impulsive transfer trajectory, and afterwards using a logical homotopy method, it diminishes iteratively the thrust level, until finally the desired real thrust trajectory with low thrust level is generated. In this quasi Newton method the nonlinear equation system does not loose its range of convergence.

The trajectories calculated by the computer program SPACENAV (Trajectory Calculation Program for Optimal Spacecraft Navigation) comply with the equations of motion, with the boundary values, with all optimality criterions of the Hamilton Lagrange theory, with the switch conditions and with the transversality conditions. In the calculation program the property of the integration is proved by a comparison of the numerical integrated solution with the closed analytical solved motion of the spacecraft in the coast arcs. Furthermore, the program is independent of initial guesses of path data. The accuracy and the convergence velocity are controlable by the user as a function of convergence security.

Moreover, the program accepts the following frame conditions:

- central body: sun, earth, moon or star
- initial and final orbit: circular,elliptic, parabolic or hyperbolic
- rocket motor: chemical, nuclear or solar electrical
- maneuvre:free transfer, rendezvous or flyby
- final path angle: predescribed or optimized

For example, the program calculates:

- transfer trajectories of solar electric propelled spacecraft to asteroids or comets
- the optimal hyperbolic injection for interplanetary missions
- trajectories of space tugs in earth orbit
- landing and starting on moon
- gravitation turn trajectories at planets with thrust arcs
- midcourse corrections optimally controled
- flight time calculations on conic orbits.

Until now, the program system SPACENAV is soled twice (to the European Space Agency and to a German industry company). ESA has used the software sucessfully for the selection of target objects for an interplanetary electric propulsion multi-asteroid-rendezvous-mission /8/. Now, the program system is prepared to be implemented in a personal computer and to be completed by computer graphics.

The analytical and numerical theory of SPACENAV is described in the report "Ein Flugbahnberechnungsverfahren für optimale Raumfahrzeug Navigation", report RT-TB 85/11 of the Astronautical Department of the Technical University of Munich. A translation of this report is availiable at the Symposium.

Trajectories of electrically propelled spacecraft in an interplanetary multi-asteroid-mission are especially suitable to demonstrate the interactions of boundary conditions and switch function. The most cases of rendezvous and flyby missions occur, and the trajectories are always three-dimensional.

By an application of the principle described in /12/ of the "parallel staging" (that means several spacecraft are launched together by the same launcher into interplanetary space) an enhancement of the mission effectivity (the number of asteroids to investigate) and a mission time reduction is possible. The splitting of the mission up to two spacecraft would not increase the mission costs. For the examples the parallel missions of two probes launched by the European ARIANE 4 launcher are underlied.

The launcher payload mass balance is:

two probes each 900 kg	=	1800 kg
two adapters each 50 kg	=	100 kg
reserve	=	100 kg
ARIANE 4 payload	=	2000 kg

The spacecraft mass balance is:

propellant (mercury)	300 kg
power supply (solar sails, electric,heat control...)	200 kg
thrusters (2 RIT-35,each 50 kg)	100 kg
main structur (incl.tank, heat control...)	100 kg
control devices (attitude, guidance,computer system...)	66 kg
data transmitter(incl. data storage, steerable antenna...)	66 kg
scientific payload & CCD-camera	66 kg
mass of interplanetary probe =	900 kg

The scientific payload mass balance is:
(Voyager V, Giotto G, Polo P)

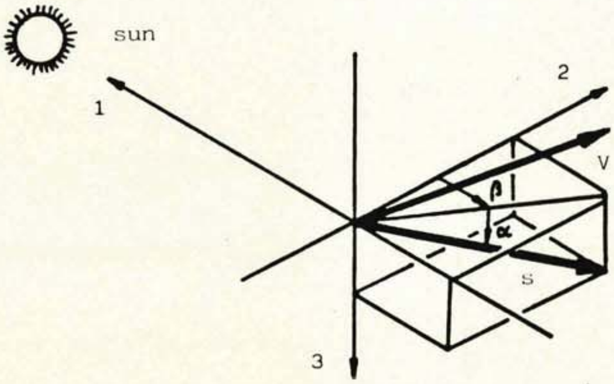
CCD-camera	V	19.0 kg
IR-spectrometer	V	18.6 kg
cosmic radiation detector	V	7.5 kg
particles detector	V	7.5 kg
magnetometer	G	3.6 kg
radar device	P	7.0 kg
ion-detector	V	1.4 kg
electron-detector	V	1.4 kg
scientific payload		66.0 kg

Data of the used RIT-35 thrusters:

name : radiofrequency ion thruster $\emptyset = 35$ cm
design : Prof. Löb from Gießen (Germany)
development : company MBB in Munich (in an ESA-contract)

thrust : 160 mN
exh.velocity : 35240 m/s
mass flow rate:4.5 mg/s
power cons. : 4.0 kW electrical
service life : ca. 2 years
max.throttle : ca. 50 %

Definition of the trust direction:



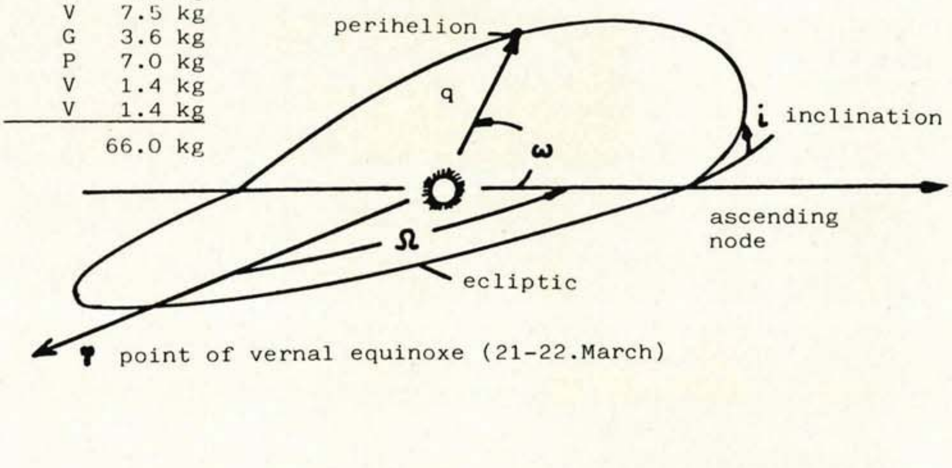
Definition of the thrust magnitude:

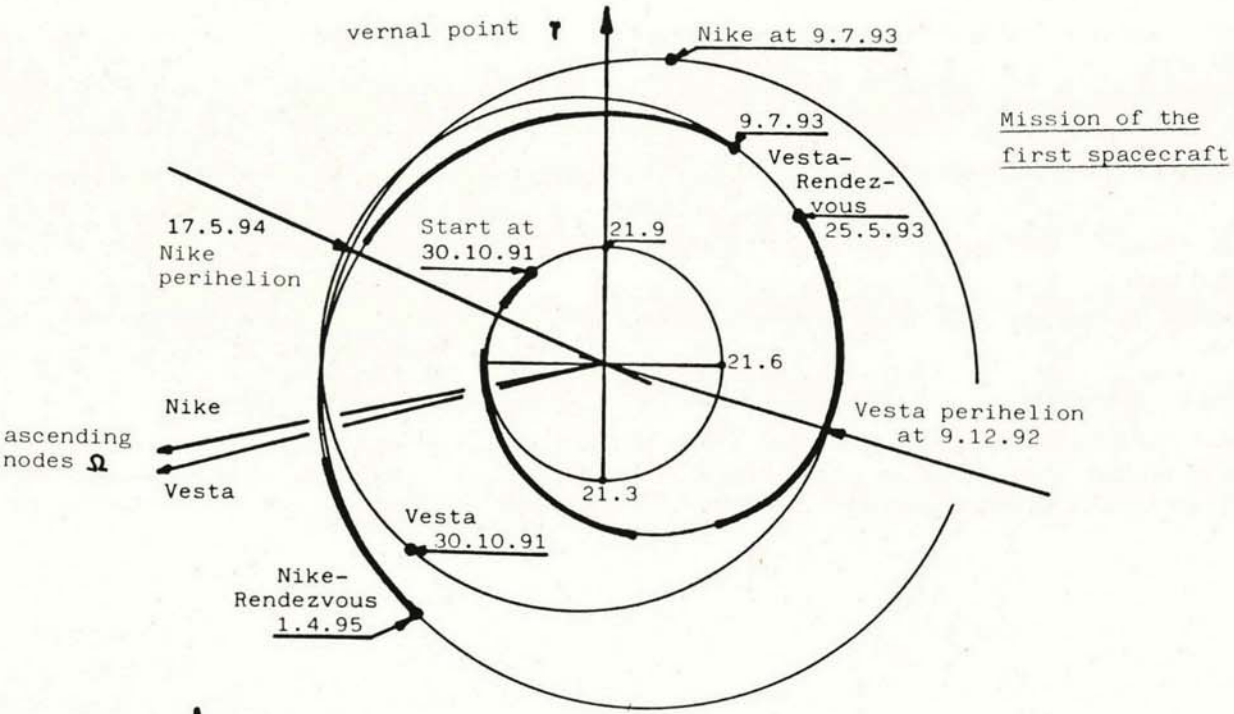
thrust arcs: (thick lines)
sun distance less than 2 astronomical units: thrust = 160 mN
sun distance more than 2 astronomical units:

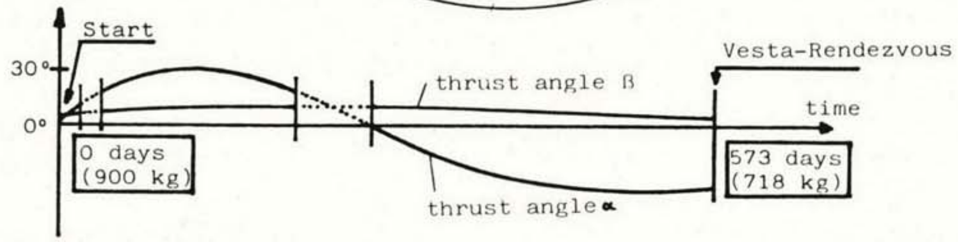
$$\text{thrust} = \left(\frac{\text{sun distance}}{2\ \text{AU}}\right)^{-1.7} * 160\ \text{mN}$$

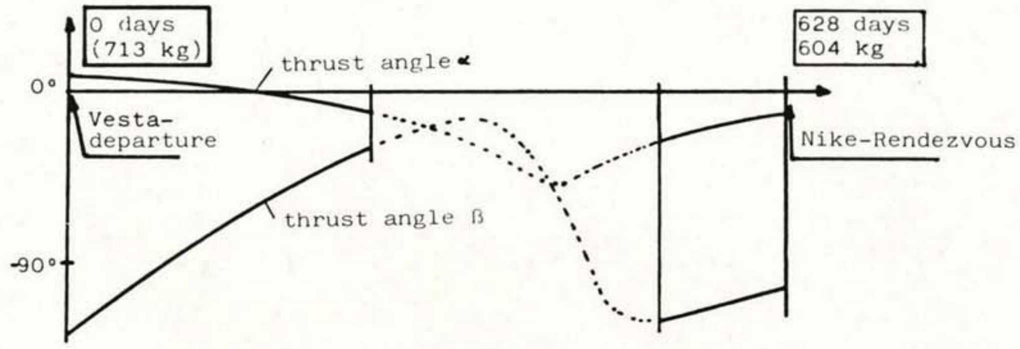
For the mission of the first spacecraft rendezvous maneuvres with the asteroids Vesta and Nike were chosen and for the mission of the second spacecraft rendezvous maneuvres with Parthenope and Amalthea and flyby maneuvres with Vanadis and Glauke were chosen.

Definition of the asteroid orbital data:

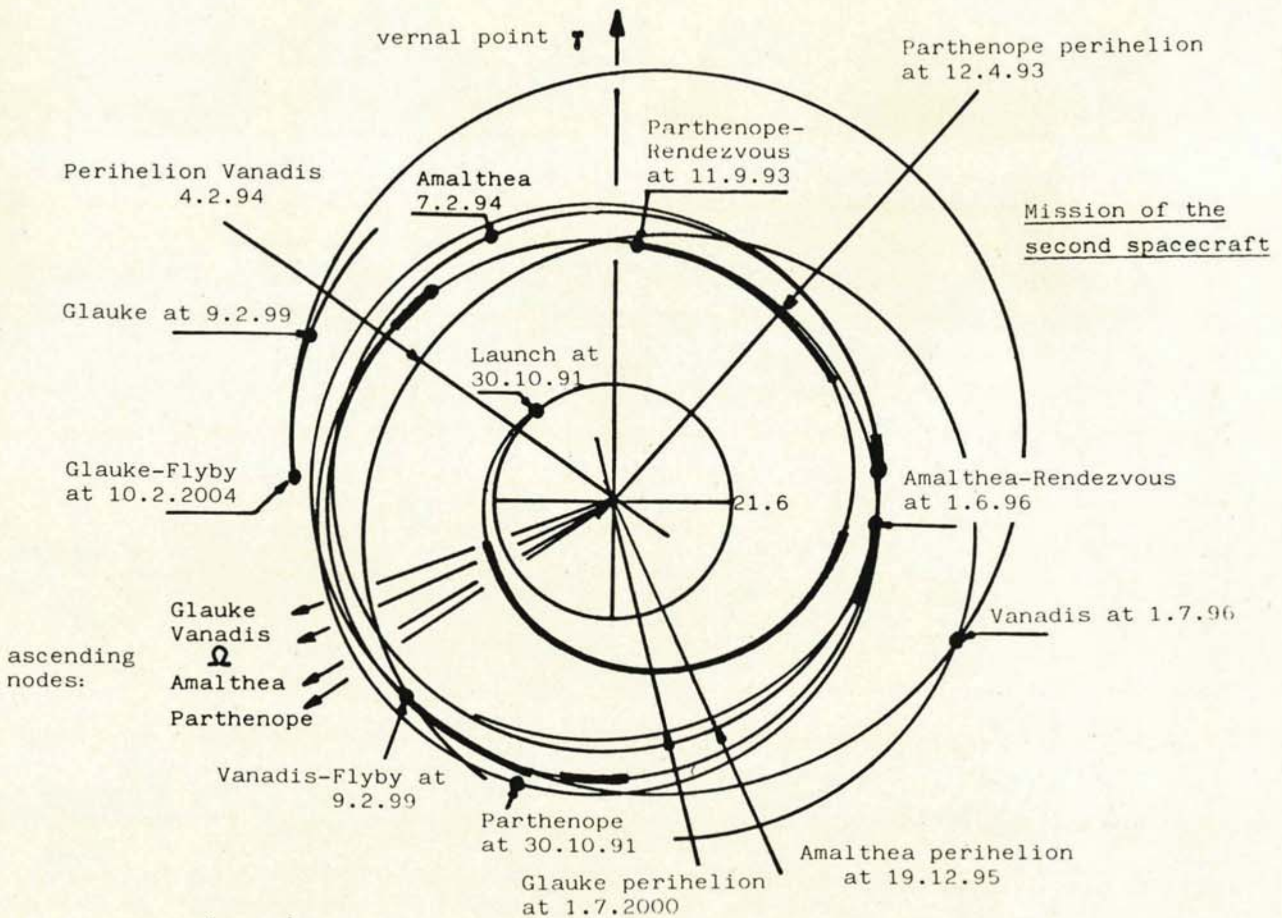


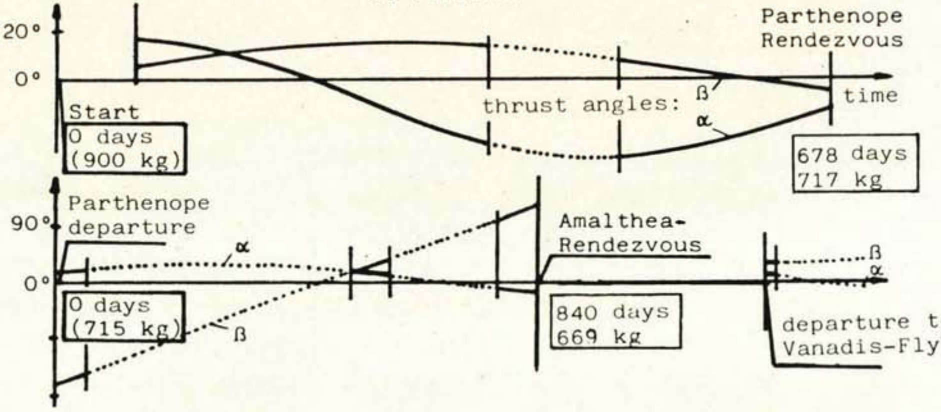






asteroid	4-Vesta	307-Nike
type	V-type	C-type
semiaxis a (AU)	2.3612	2.9072
semiaxis b (AU)	2.352	2.878
perihelion distance q (AU)	2.15	2.49
numerical excentricity e	0.089	0.142
orbital period (years)	3.63	4.95
inclination i	7.144°	6.12°
ascending node Ω	103.4°	100.8°
perihelion angle ω	150.1°	324.0°
perihelion date	9.12.92	17.5.94
estimated radius (km)	288	29





asteroid	11-Parthenope	113-Amalthea	240-Vanadis	288-Glauke
type	S-type	SX-type	C-type	S-type
semiaxis a (AU)	2.4525	2.3760	2.6659	2.7562
semiaxis b (AU)	2.440	2.367	2.261	2.694
perihelion distance q (AU)	2.21	2.17	2.11	2.18
numerical excentricity e	0.100	0.086	0.207	0.211
orbital period (years)	3.84	3.66	4.35	4.58
inclination i	4.62°	5.04°	2.10°	4.33°
ascending node	125.0°	123.1°	114.9°	120.1°
perihelion angle	194.3°	78.8°	299.4°	83.7°
perihelion date	12.4.93	19.12.95	4.2.94	30.12.95
estimated radius (km)	78	24	49	15

REFERENCES:

1) Lawden,D.F.,Analytical Methods of Optimization,Scottish Academic Press,Edinburgh and London,1975.

2) Lawden,D.F.,Optimal Trajectories for Space Navigation,Butterworth,London,1963.

3) Leitmann,G.(ed.),Optimization Techniques with Application to Aerospace Systems, Academic Press,New York and London,1962.

4) Leitmann,G.,Foundations of Optimal Control Theory,McGraw Hill,New York,1966.

5) Ruppe,H.O.,Introduction to Astronautics, Vol.I,Vol.II,Academic Press,New York,1966.

6) Ruppe,H.O.,Die grenzenlose Dimension - Raumfahrt,Bd.1,Bd.2,Econ Verlag,Düsseldorf Wien,1980/81.

7) Hechler,M.,Low Thrust Entry and Exit Spirals around Asteroids, M.A.O.Working Paper No.188,ESOC-Darmstadt,April 1983.

8) Hechler,M.,Serrano-Martinez,J.B.,Low Thrust Asteroids Rendezvous Tours with Vesta,M.A.O.Working Paper No.223,ESOC-Darmstadt,June 1985.

9) Löb,H.W.,Einsatzmöglichkeiten der Ionentriebwerke RIT auf Asteroiden und Kometensonden,Preprint DGLR 83/101, DGLR-Jahrestagung 1983 inMünchen.

10)Dixon,L.C.W.,Hersom,S.E.,Maany,Z., Low Thrust Orbit Optimization for Interplanetary Mission,Contract No.4774/81/D/JS(SC) between ESOC and Hatfield Polytechnic, 1983.

11)Schlingloff,H.,Bahn- und Kostenoptimierung wiederverwendbarer Raumfährenoberstufen, Diss.,Tu-München,1983.

12)Schlingloff,H.,Beiträge zu Missionsstudien über die Erforschung des Asteroidengürtels im interplanetaren Weltraum mittels elektrisch angetriebener Raumsonden,TU-München Raumfahrttechnik,Bericht RT-KB 85/12,1985.

13)Burrows,R.R.,Example Solar Electric Propulsion System Asteroid Tours Using Variational Calculus; AIAA J.Spacecrafts, 325-332,May-June 1985.

14)Zondervan,K.P.,Wood,L.J.,Caughey,T.K., Optimal Low-Thrust,Three-Burn Orbit Transfers with Large Plane Changes,The Journal of Astronautical Sciences,Vol.32,No.3, July-Sept.,pp.407-427,1984.

15)Flury,W.,Finite-Thrust Transfers,ESA-Journal 1984,Vol.8, pp.409-423.

16)Pietrass,A.E.,Trajectory Design for an Ion Drive Asteroids Rendezvous Mission Launched into an Ariane Geostationary Transfer Orbit,AIAA/AAS Astrodynamics Conference,Seattle,Washington,Aug.1984.

17)Pines,S.,Fang,T.C.,A Uniform Closed Solution of the Variational Equations for Optimal Trajectories During Coast, NASA-Contracts NAS 9-4036,Manned Spacecraft Center, Nas 5-9085,Goddard Space Flight Center,Nas 12-114,Electronic research Center.

18)Marec,J.P.,Optimal Space Trajectories, Elsevier Publishing Co.,Amsterdam Oxford New York,1979.

19)Dittberner,W.,Zur dreidimensionalen Bahnsteuerung von Raumsonden mit niedrigem Schub,Diss.,TU-München,1980.

20)Grodzovskii,G.l.,Ivanov,Y.N.,Tokarev, V.V.,Mechanics of Low-Thrust Spaceflight,Israel Program for Scientific Translations,Jerusalem,1969.

21)Pontryagin,L.S.,Boltyanskii,V.G.,Gamkrelidze,R.V.,Mishchenko,E.F.,The Mathematical Theory of Optimal Processes, Interscience,1962.

22)Oberle,H.J.,Numerical Computation of Minimum-Fuel Space-Travel Problems by Multiple Shooting,TU-München,Gruppe Numerische Mathematik,Bericht 7636.

23)Stoer,J.,Bulirsch,R.,Einführung in die Numerische Mathematik,Bd.1,Bd.2, Springer Verlag,Berlin Heidelberg New York,1976/78.