STATE ESTIMATION ALGORITHM CONSIDERING ATTITUDE MOTION FOR APOGEE MOTOR BURNING OF GEOSTATIONARY SATELLITE

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Abstract

This paper presents new algorithm of state estimation with attitude motion for the powered flight phase by Apogee Boost Motor of a geostationary satellite. The measured doppler data oscillate with a spin period for the satellite equipped with an offset antenna, or fluctuate due to the occurence of Apogee Motor Firing error. This paper supplies one of effective techniques for the precise state estimation and also for the specification of the error resouce against the orbit injection error. The estimation results by the simplified filter with attitude motion are presented and discussed. This sort of technique would contribute to the Real Time Estimation, especially which would be essential on the Maneuver Operation of a liquid Apogee Motor satellite.

1 Introduction

The powered flight phase by Apogee Boost Motor (ABM) is one of keys in order to realize a geostationary satellite both on the Satellite System Design, and on the Maneuver Plan/Operation of Station Acquisition. For a solid ABM of short burning time, this phase has been treated and evaluated by the Impulsive Burn technique, which is often adopted on the orbital dynamics but can be applied only for the conditions ; high altitude, short burning time, and small attitude change. In the case of a solid ABM burn of a geostationary satellite, the above conditions are normally satisfied, because of the altitude of about 36000 km, ABM burning time of about 20~60 seconds, and the high spin rate of about 50~100 rpm to maintain the direction of ABM thrust in the inertial space, where the ABM thrust direction used to coincide with the satellite spin-axis.

If any dynamic unbalance exists, C.G. off-set, or ABM mis-alignment, etc, it brings about the attitude coning motion during ABM burn. Then the AMF (Apogee Motor Firing) error occurs of the Drift orbit/attitude injection error by ABM. For large AMF error, the obserbed doppler data have the fluctuations and show the satellite attitude and the spin rate changes through the ABM burning phase. For the satellite with an antenna offset from its body axis, the doppler data oscillate with the spin rate period.
The Impulsive technique cannot estimate the varing states of satellite/ABM with motor burning. Ref.1) treated this powered flight phase by the Continuous Burn technique, where the deterministic and the stochastic estimation method were considered. And the excellent results were obtained. Especially the satellite attitude drifting due to some unbalance was firstly estimated with the reasonably converging covariances. But there were the limitations for the precise specifications of error resources and also for the above oscillating data, because the Dynamic Model of 9 dimensional state vector in Ref.1) was simple of a point mass model without attitude motion.

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Therefore the new algorithm has been expected for an advanced state estimatition considering satellite attitude motion, in order not only to analyse the above phenomena precisely, but also to evaluate the occured AMF error and separate that into the error resources of dynamic unbalance, ABM thrust error, and attitude/orbit determination error. In the Dynamic model construction, the state vector is adopted of 18 dimensions ; position, velocity, acceleration, attitude angle, angular velocity, satellite moment of inertia, C.G.offset, and ABM thrust. The Measurement model is also made up with the motion of an offset antenna laied at any position in the body coordinate.
The estimations by the simplified algorithm of attitude motion are also presented and discussed in this paper.

This simplified method is also applied to the Post Flight analysis for the NASDA launched geostationary satellite with solid ABM. The reasonable results are obtained, and presented in Ref.2). The advanced estimation software by this paper algorithm is under development. The detailed analysis is expected. Moreover this sort of technique will contribute to the Real Time Estimation on the Maneuver Operation of Satellite Geostationing. Even in a case of solid ABM, the Real Time Estimation would be necessary for the prompt Maneuver Decision just after AMF. Especially in a case of liquid ABM of long burning duration, the Operation Decision of engine cut-off would need this Real Time Estimation during AMF for the improvement of maneuver precision and also for the reconstruction of Geostationing Plan.

2 Estimation Algorithm of Attitude Motion

This chapter describes the stochastic estimation of satellite attitude motion. Fig.1 shows the relation between the Earth and the satellite at ABM ignition.
The satellite body coordinate and the Earth centered inertial coordinate are also defined. The satellite is sketched with a doppler antenna offset from its body axis XB, with which the ABM thrust direction coinsides. Fig.2 is the real change of doppler data measured during ABM burn for the satellite of an offset antenna. The data oscillate with a spin period. The spin rate slightly increases through ABM burn. The used Kalman filter algorithm is shown in Ref.1). The constructed Dynamic Model and the Measurement Model are explained as follows.

2 − 1 Dynamic Model

(1) State Vector $\mathbb{X}k$;

The satellite state vector Xk is considered of 18 dimensions and composed of the satellite position $\mathbb{r}k = (X, Y, Z)$, velocity $\mathbb{V}k = (\dot{X}, \dot{Y}, \dot{Z})$, Euler angle (ϕ, θ, ψ), roll/pitch/yaw rate ($\omega_1, \omega_2, \omega_3$), C.G.offset (r_2, r_3), Moment of Inertia (MOI) parameter (ζ_1, ζ_2), and ABM parameter (η_1, η_2) ;

$$\mathbb{X}k = \Big[\underbrace{X, Y, Z}_{\text{position}}, \underbrace{\dot{X}, \dot{Y}, \dot{Z}}_{\text{velocity}}, \underbrace{\phi, \theta, \psi}_{\text{Euler angle}}, \underbrace{\omega_1, \omega_2, \omega_3}_{\text{angle rate}}, \underbrace{r_2, r_3}_{\text{CG offset}}, \underbrace{\zeta_1, \zeta_2}_{\text{MOI p.}}, \underbrace{\eta_1, \eta_2}_{\text{ABM p.}} \Big]_K^T \quad (1)$$

(2) Dynamic Model $\mathbb{F}(\mathbb{X}k, tk)$;

The Dynamic Model $\mathbb{F}$ is defined with the attitude motion during ABM burn.

$$
\left\{
\begin{aligned}
& t_K = t_{K-1} + \Delta t \\
& \mathbb{r}_K = \mathbb{r}_{K-1} + V_{K-1}\cdot\Delta t + \left(\mathbb{a}_{K-1} - \mu \frac{\mathbb{r}_{K-1}}{|\mathbb{r}_{K-1}|^3}\right)\cdot\frac{\Delta t^2}{2} \\
& \mathbb{V}_K = \mathbb{V}_{K-1} + \left(\mathbb{a}_{K-1} - \mu \frac{\mathbb{r}_{K-1}}{|\mathbb{r}_{K-1}|^3}\right)\cdot\Delta t \\
& \mathbb{a}_K = \begin{bmatrix} a_x \\ a_Y \\ a_Z \end{bmatrix}_K = \eta_K\cdot A(t_K)\cdot\begin{bmatrix} \cos\delta\cdot\cos\alpha \\ \cos\delta\cdot\sin\alpha \\ \sin\delta \end{bmatrix}_K = \eta_{1_K} A(t_K)\cdot\begin{bmatrix} u_X \\ u_Y \\ u_Z \end{bmatrix}_K = \eta_{1_K}\cdot A(t_K)\begin{bmatrix} c\alpha_0\cdot c\delta_0\cdot c\psi\cdot c\theta - s\alpha_0 s\psi\cdot c\theta + c\alpha_0\cdot s\delta_0\cdot s\theta \\ s\alpha_0\cdot c\delta_0\cdot c\psi\cdot c\theta + c\alpha_0\cdot s\psi\cdot c\theta + s\alpha_0\cdot s\delta_0\cdot s\theta \\ s\delta_0\cdot c\psi c\theta - c\delta_0\cdot s\theta \end{bmatrix}_K \\
& \phi_K = \phi_{K-1} + \Delta t\left[\omega_1 + (\omega_2\cdot s\phi + \omega_3\cdot c\phi)\tan\theta\right]_{K-1} \\
& \theta_K = \theta_{K-1} + \Delta t\left[\omega_2\cdot c\phi - \omega_3\cdot s\phi\right]_{K-1} \\
& \psi_K = \psi_{K-1} + \Delta t\left[(\omega_2 s\phi + \omega_3\cdot c\phi)/c\theta\right]_{K-1} \\
& [\omega_1]_K = [\omega_1]_{K-1} \\
& [\omega_2]_K = [\omega_2]_{K-1} + \Delta t\cdot\left[-\left\{\frac{\varsigma_1\cdot I_x(t)}{\varsigma_2\cdot I(t)} - 1\right\}\omega_1\cdot\omega_3 - \frac{\eta_2\cdot F(t)}{\varsigma_2\cdot I(t)}\cdot r_3\right]_{K-1} \\
& [\omega_3]_K = [\omega_3]_{K-1} + \Delta t\left[\left\{\frac{\varsigma_1\cdot I_x(t)}{\varsigma_2\cdot I(t)} - 1\right\}\omega_1\omega_2 + \frac{\eta_2\cdot F(t)}{\varsigma_2\cdot I(t)}\cdot r_2\right]_{K-1} \\
& [r_2]_K = [r_2]_{K-1}, \quad [\varsigma_1]_K = [\varsigma_1]_{K-1}, \quad [\eta_1]_K = [\eta_1]_{K-1} \\
& [r_3]_K = [r_3]_{K-1}, \quad [\varsigma_2]_K = [\varsigma_2]_{K-1}, \quad [\eta_2]_K = [\eta_2]_{K-1}.
\end{aligned}
\right\} \quad (2)
$$

where t_k, Δt ; time from ABM ignition, time step of integration
(α_0, δ_0) ; satellite right ascension and declination as nominal AMF direction
(α, δ) ; instantaneous right ascension and declination of satellite
$\eta_{1,2} = 1 + \varepsilon$; ABM parameter of acceleration and thrust ($\varepsilon \ll 1$)
$\zeta_{1,2} = 1 + \varepsilon$; MOI parameter of body axis and cross axis ($\varepsilon \ll 1$)
$A(t_k)$, $F(t_k)$; nominal ABM acceleration and thrust
$I_{xx}(t_k)$, $I(t_k)$; nominal MOI changes of body axis and cross axis

(Assumptions) ; ($\dot{\eta}_1 = \dot{\eta}_2 = \dot{\zeta}_1 = \dot{\zeta}_2 = \dot{r}_2 = \dot{r}_3 = \dot{\omega}_1 = 0$) are introduced from the reasons ;

1) $\dot{\eta}_{1,2} = 0$, $\dot{\zeta}_{1,2} = 0$, $\dot{r}_{2,3} = 0$ ($\varepsilon \ll 1$) ; ABM is normally a high efficient solid motor. The difference would not be large between the predicted nominal acceleration/thrust and the actual on orbit though a small random variation exists. Both MOI and C.G.offset random variations during ABM burn are also assumed to be small from the above same reason.

2) $\dot{\omega}_1 = 0$; This is deduced from the assumptions ; the satellite cross axis MOI $I = I_{yy} = I_{zz}$,and the Product of Inertia $I_{xy} = I_{yz} = I_{zx} = 0$.

A (tk) and F (tk) are selected because of simplicity and formula reduction, although the satellite mass Mk and its flow rate $\dot{M}k$ are related with these. Appendix I is the relations among the Earth centered inertial coordinate, the inertial and instantaneouse satellite body coordinate. Appendix II is the relation between the Euler angle (ϕ, θ, ψ) and (α, δ).

(3) Error Propagation Matrix Φk ;

The Error Propagation Matrix Φk is obtained from the above Dynamic Model. The components of Φk are shown in Appendix IV.

$$
\Phi_K = \left(\frac{\partial \mathbb{F}}{\partial \mathbb{X}}\right)_K =
\left[\begin{array}{cccccc|ccc|ccc|cc|cc|cc}
X_x & X_y & X_z & \Delta t & 0 & 0 & 0 & X_\theta & X_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_{\eta_1} & 0 \\
Y_x & Y_y & Y_z & 0 & \Delta t & 0 & 0 & Y_\theta & Y_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_{\eta_1} & 0 \\
Z_x & Z_y & Z_z & 0 & 0 & \Delta t & 0 & Z_\theta & Z_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{\eta_1} & 0 \\
\dot{X}_x & \dot{X}_y & \dot{X}_z & 1 & 0 & 0 & 0 & \dot{X}_\theta & \dot{X}_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{X}_{\eta_1} & 0 \\
\dot{Y}_x & \dot{Y}_y & \dot{Y}_z & 0 & 1 & 0 & 0 & \dot{Y}_\theta & \dot{Y}_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{Y}_{\eta_1} & 0 \\
\dot{Z}_x & \dot{Z}_y & \dot{Z}_z & 0 & 0 & 1 & 0 & \dot{Z}_\theta & \dot{Z}_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{Z}_{\eta_1} & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & \phi_\phi & \phi_\theta & 0 & \Delta t & \phi_{\omega_2} & \phi_{\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \theta_\phi & 1 & 0 & 0 & \theta_{\omega_2} & \theta_{\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \psi_\phi & \psi_\theta & 1 & 0 & \psi_{\omega_2} & \psi_{\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & [\omega_2]_{\omega_1} & 1 & [\omega_2]_{\omega_3} & 0 & [\omega_2]_{r_3} & [\omega_2]_{\varsigma_1} & [\omega_2]_{\varsigma_2} & 0 & [\omega_2]_{\eta_2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & [\omega_3]_{\omega_1} & [\omega_3]_{\omega_2} & 1 & [\omega_3]_{r_2} & 0 & [\omega_3]_{\varsigma_1} & [\omega_3]_{\varsigma_2} & 0 & [\omega_3]_{\eta_2} \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]_K \quad (3)
$$

(18×18)

2 − 2 Measurement Model $\mathbb{G}$ ($\mathbb{X}$k, t k)

The Measurement Model $\mathbb{G}$ (Xk, t k) is defined for the measurement vector $\mathbb{Y}$k of three sets of doppler data Δ f j of the satellite antenna ;

$$\mathbb{Y}k = (\Delta f1,\ \Delta f2,\ \Delta f3),\quad (j=1,2,3\ ;\ \text{station number}) \tag{4}$$

where

- $\Delta fj = (-f_0/C)(\dot{\rho}_a - \dot{\rho}_{a0})$, ($\dot{\rho}_{a0}$; range rate at ABM ignition)
- slant range of satellite antenna ρ_a and range rate $\dot{\rho}_a$;

$$\dot{\rho}_a = [\ (X+X_a-X_S)(\dot{X}+\dot{X}_a-\dot{X}_S)+(Y+Y_a-Y_S)(\dot{Y}+\dot{Y}_a-\dot{Y}_S)+(Z+Z_a-Z_S)(\dot{Z}+\dot{Z}_a-\dot{Z}_S)\ ] \,/\, \rho_a$$

$$\rho_a = [\ (X+X_a-X_S)^2+(Y+Y_a-Y_S)^2+(Z+Z_a-Z_S)^2\ ]^{1/2}$$

- $\mathbb{R}_a = \begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix} = [C]\cdot[A]\cdot\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}$, $\dot{\mathbb{R}}_a = \begin{bmatrix} \dot{X}_a \\ \dot{Y}_a \\ \dot{Z}_a \end{bmatrix} = [C]\cdot[\dot{A}]\cdot\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}$
- where f_0 = 3000 MHz and C = 299792.5 km/s .

The measurement matrix Hk is defined from $\mathbb{G}$ ($\mathbb{X}$k, t k) , and the components are shown in Appendix V .

$$\underset{(3\times18)}{H_K} = \left(\frac{\partial \mathbb{G}}{\partial \mathbb{X}}\right)_K = \begin{bmatrix} Y1x & Y1y & Y1z & Y1\dot{x} & Y1\dot{y} & Y1\dot{z} & Y_{1\phi} & Y_{1\theta} & Y_{1\psi} & Y_{1\omega_1} & Y_{1\omega_2} & Y_{1\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ Y2x & Y2y & Y2z & Y2\dot{x} & Y2\dot{y} & Y2\dot{z} & Y_{2\phi} & Y_{2\theta} & Y_{2\psi} & Y_{2\omega_1} & Y_{2\omega_2} & Y_{2\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ Y3x & Y3y & Y3z & Y3\dot{x} & Y3\dot{y} & Y3\dot{z} & Y_{3\phi} & Y_{3\theta} & Y_{3\psi} & Y_{3\omega_1} & Y_{3\omega_2} & Y_{3\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_K \tag{5}$$

2 − 3 Simplified Algorithm

The previous complicated algorithm are simplified by the assumption of $\dot{\alpha} = \dot{\delta} = 0$, which means $\dot{\theta} = \dot{\psi} = \dot{\omega}_2 = \dot{\omega}_3 = 0$, as follows ;

(1) Dynamic Model ; the state vector $\mathbb{X}$k becomes of 9 dimensions. **F** and Φk (9×9) are the same as those of Ref.1).

(2) Measurement Model ; the satellite antenna motion is expressed by ω_1 and the offset antenna position. From the above assumption, Hk becomes (3×9) matrix and its components are updated from that in Ref.1).

$$\underset{(3\times9)}{H_K} = \begin{bmatrix} Y_{1x} & Y_{1Y} & Y_{1z} & Y_{1\dot{x}} & Y_{1\dot{Y}} & Y_{1\dot{z}} & 0 & Y_{1\alpha} & Y_{1\delta} \\ Y_{2x} & Y_{2Y} & Y_{2z} & Y_{2\dot{x}} & Y_{2\dot{Y}} & Y_{2\dot{z}} & 0 & Y_{2\alpha} & Y_{2\delta} \\ Y_{3x} & Y_{3Y} & Y_{3z} & Y_{3\dot{x}} & Y_{3\dot{Y}} & Y_{3\dot{z}} & 0 & Y_{3\alpha} & Y_{3\delta} \end{bmatrix}_K$$

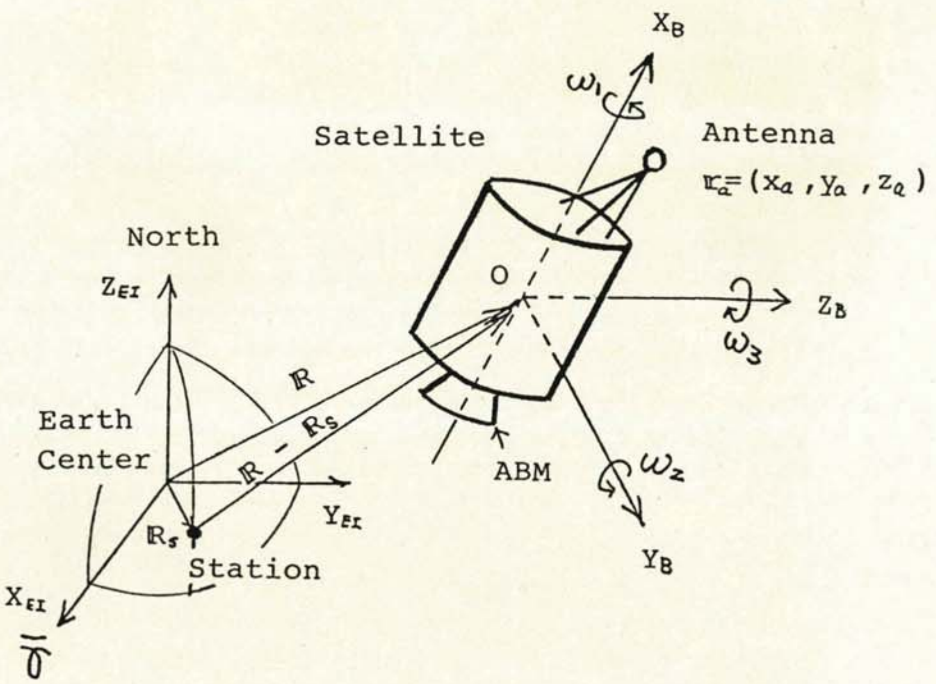

Fig. 1 Satellite and Earth at ABM Ignition

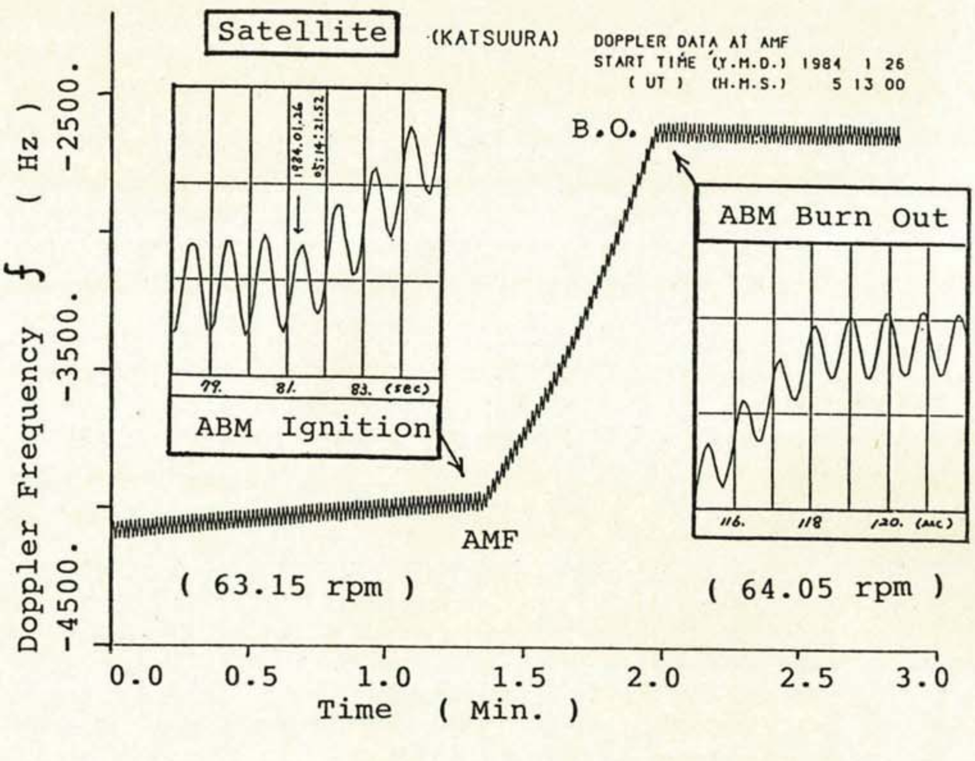

Fig. 2 Real Doppler Data of Offset Antenna Satellite for ABM Burning Phase

3 Results of Estimation

3 − 1 Apri'ori Parameter

The initial states $\mathbb{X}_{K=0}$ and $P_{K=0}$ and the residual matrices Q_K , R_K are apri'ori given. The way to set up these is shown in Ref.1).

3 − 2 Simulation

The simulation was performed in order to verify the filter and to clarify the effect of station geometry. The doppler data of an offset antenna satellite are generated by the normal random numbers, or the uniform random numbers. Two couples of three tracking stations are considered ; the three NASDA stations (Masuda, Katsuura, Okinawa) which are positioned with neighboring geometry, and the three Imaginary stations (P1, P2, P3) which are arranged with an ideal triangle geometry of large distances of one another.

Fig.3 is the results by the Point Mass Model filter without attitude motion. The estimated satellite position, velocity, acceleration, and attitudes are shown as the differences from the nominal states. The covariances are well converged. The neigboring effects of NASDA stations are clarified from the comparison of the covariance convergency with that of the Imarginary stations. But in the attitudes estimation, this filter can not eliminate the data periodic oscillation.

Fig.4 is the results by the Attitude Motion Model filter. The periodic oscillation can be excellently eliminated by this filter. The estimation of attitude α is seen to be well ameliorated. And the covariance covergencies are much more rapid even for the NASDA stations.

From the above results by two filters, the estimation of attitude δ is effected by the neighboring geometry of NASDA stations against the satellite of high altitude. Although there are many ways to smooth the periodic oscilation of data by the curve fitting before estimation, this paper does not adopt those because of the intention to compare the filter closely following the physical phenomena with the filter not so.

Table 5 is the estimated injection orbit and attitude by these filters. The result for the satellite of not offset antenna is also shown, where the doppler data does not periodically oscillate and the Point Mass filter is used. From these comparisons, the introduction of attitude motion is indicated to result the excellent estimation with the covariance converging almost close to that for the not offset antenna satellite.

3 − 3 Real Estimation

For the satellite of an offset antenna, the real estimation results are discussed. Fig.6 shows the real doppler data periodically oscillating and the predicted doppler. The data processing before estimation is performed by the Point Mass Model or by the Attitude Motion Model. For the well estimation by Kalman Filter, it is a key point to fit the prediction to the real data tendency as well as possible. The case of the determined(α, δ)* used as the initial AMF attitude has a better agreement with the real than the case of the nominal (α, δ $)_N$. And the Attitude Motion Model results a excellent fitting. It becomes much better by the division of time span.

Fig.7 is the estimation results by the two filters. As predicted by the simulation, the satellite position, velocity, acceleration, and attitude α are excellently estimated with their covariances by considering attitude motion. But the estimated attitude δ is affected by the neighboring geometry. These results are obtained by the parametric search of $P_{K=0}$, CR and the time span T_{SPAN}. CR is the parameter to control the Measurement Model. The idea of parametric search is described in Ref.1), 2).

Table 8 is the estimated injection orbit and attitude at ABM Burn Out, which have a well agreement with the determined values by the Batch Method. The Attitude Motion filter is shown to be efficient for the improvement of estimation probability, because the covariances converge more rapidly even for NASDA stations than those by the Point Mass Model filter. The results are thought enough to have the technical perspective for the realization of the Real Time Estimation. It will be much more ameriorated by the initial attitude search and the ABM thrust pattern fitting. The Deterministic method in Ref.1).2) will be also efficient by the improvement of adding an iteration logic.

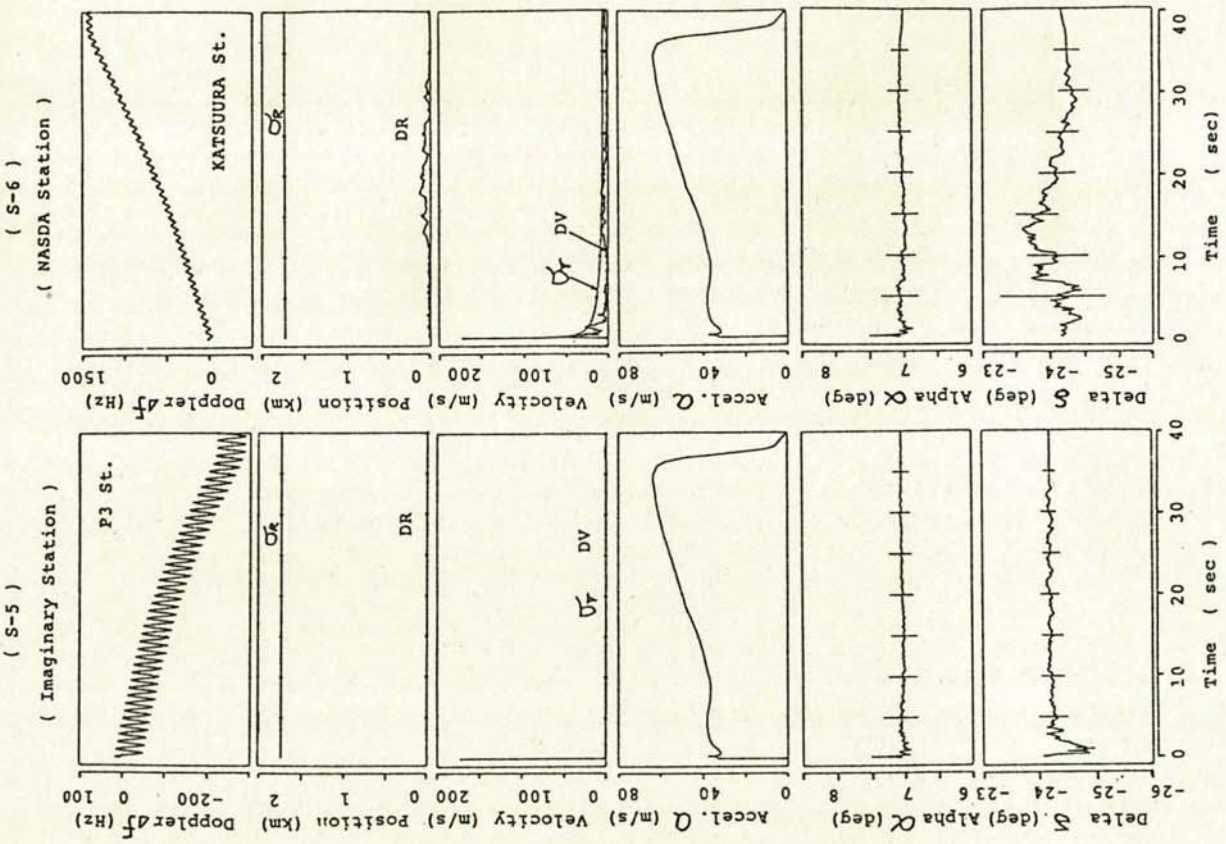

Fig. 4 Simulation (Offset Antenna Data/Attitude Motion Model Filter)

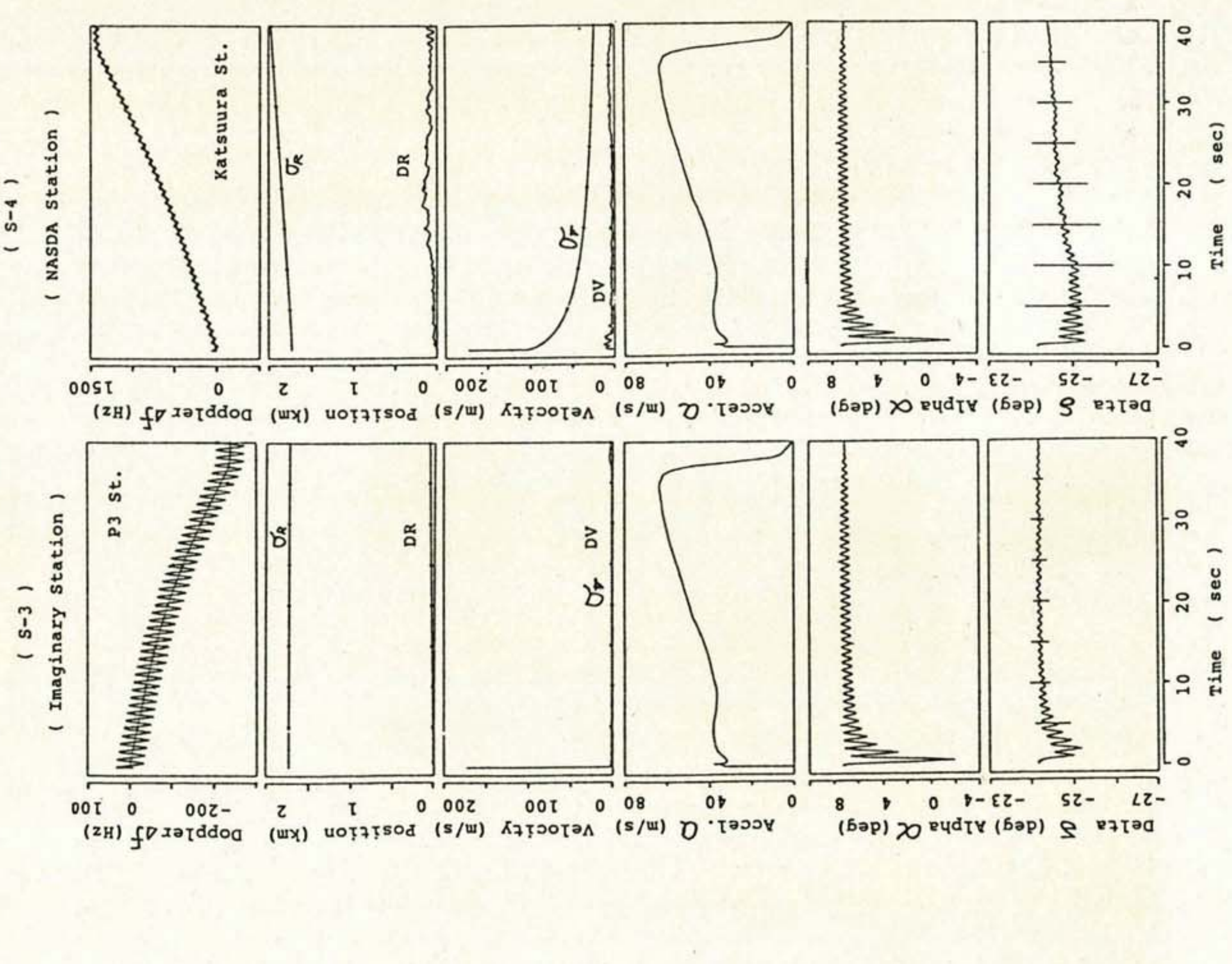

Fig. 3 Simulation (Offset Antenna Data / Point Mass Model Filter)

Table 5 Simulation Results

Orbit/Attitude	Nominal X_N	(S-1) Simulation (Kalman Filter, Normal Random Numbers): Antenna not offset, Point Mass Model: Imagi, St.	(S-2) Antenna not offset, Point Mass Model: NASDA St.	(S-3) Antenna Offset, Point Mass Model: Imagi. St.	(S-4) Antenna Offset, Point Mass Model: NASDA St.	(S-5) Antenna Offset, Attitude Motion Model: Imagi. St.	(S-6) Antenna Offset, Attitude Motion Model: NASDA St.
Drift Orbit							
1) Semi-major Axis a (km)	41753.14	41759.59 (±7.4)	41834.46 (±154.)	41814.00 (±15.)	41725.44 (±628.)	41759.60 (±7.4)	41834.79 (±154.)
2) Eccentricity e	0.016385	0.016236 (±.00016)	0.014506 (±.0036)	0.015033 (±.0004)	0.017084 (±.015)	0.016236 (±.00016)	0.014498 (±.0036)
3) Inclination i (deg)	0.298925	0.296854 (±.0041)	0.273719 (±.019)	0.273180 (±.0099)	0.240074 (±.071)	0.296853 (±.0041)	0.273719 (±.019)
4) Ascending Node Ω (deg)	283.3983	283.4457 (±.28)	284.0229 (± .58)	284.0245 (±.39)	285.0598 (±2.56)	283.4459 (±.28)	284.0227 (± .58)
5) Argument of Perigee ω (deg)	154.0809	153.8529 (±.35)	151.1782 (±4.60)	151.2480 (±.71)	152.5824 (±13.9)	153.8525 (±.35)	151.1677 (±4.60)
6) True Anomaly f (deg)	199.1599	199.3405 (±.20)	201.4380 (±4.60)	201.3665 (±.54)	198.9970 (±13.9)	199.3406 (±.20)	201.4488 (±4.60)
Attitude							
1) Right Ascension α (deg)	7.04	7.04 (±.11)	7.02 (±.11)	7.03 (±.11)	7.06 (±.12)	7.04 (±.11)	7.02 (±.11)
2) Declination δ (deg)	-24.19	-24.19 (±.10)	-24.16 (±.16)	-24.21 (±.11)	-24.39 (±.28)	-24.19 (±.10)	-24.16 (±.16)
Position Diff. ΔR (km)	0.0	0.002	0.016	0.058	0.068	0.002	0.017
Velocity Diff. ΔV (m/s)	0.0	0.267	3.34	2.68	3.37	0.267	3.36
		(σ_D = 1 Hz)	(1 Hz)	(24 Hz)	(24 Hz)	(1 Hz)	(1 Hz)

where EOCH Time is the ABM Burn Out Time.
ΔR, ΔV are the differences of inertial position and velocity between the Nominal and the Simulation.

$$\Delta R = [(X - X_N)^2 + (Y - Y_N)^2 + (Z - Z_N)^2]^{1/2}$$
$$\Delta V = [(\dot{X} - \dot{X}_N)^2 + (\dot{Y} - \dot{Y}_N)^2 + (\dot{Z} - \dot{Z}_N)^2]^{1/2}$$

and (±) means one sigma value in the stochastic Kalman filter estimation.
σ_D is the Standard Deviation of the difference between the real doppler data and the Nominal.

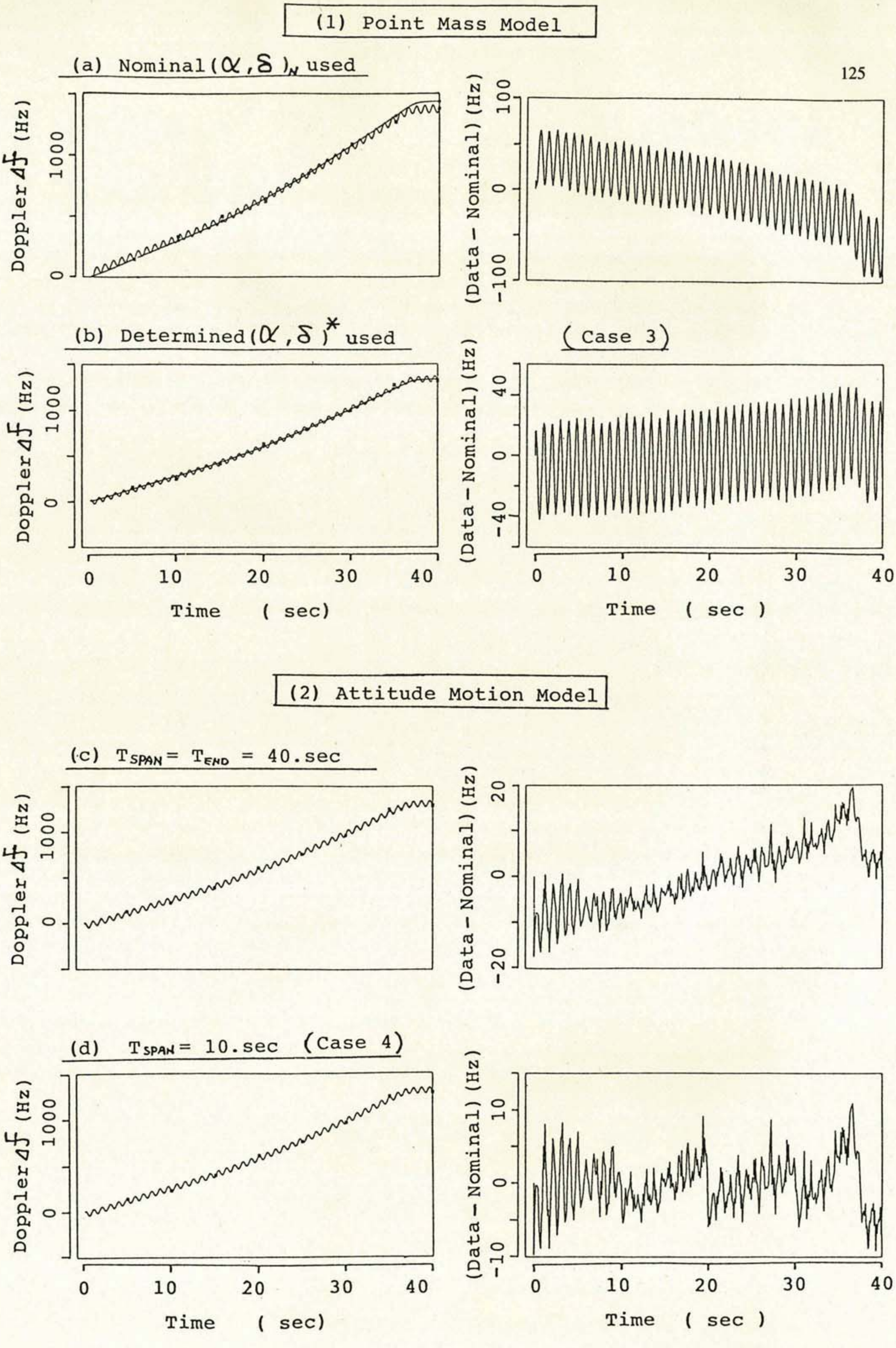

Fig. 6 Doppler Data Processing (KATSUURA St.)

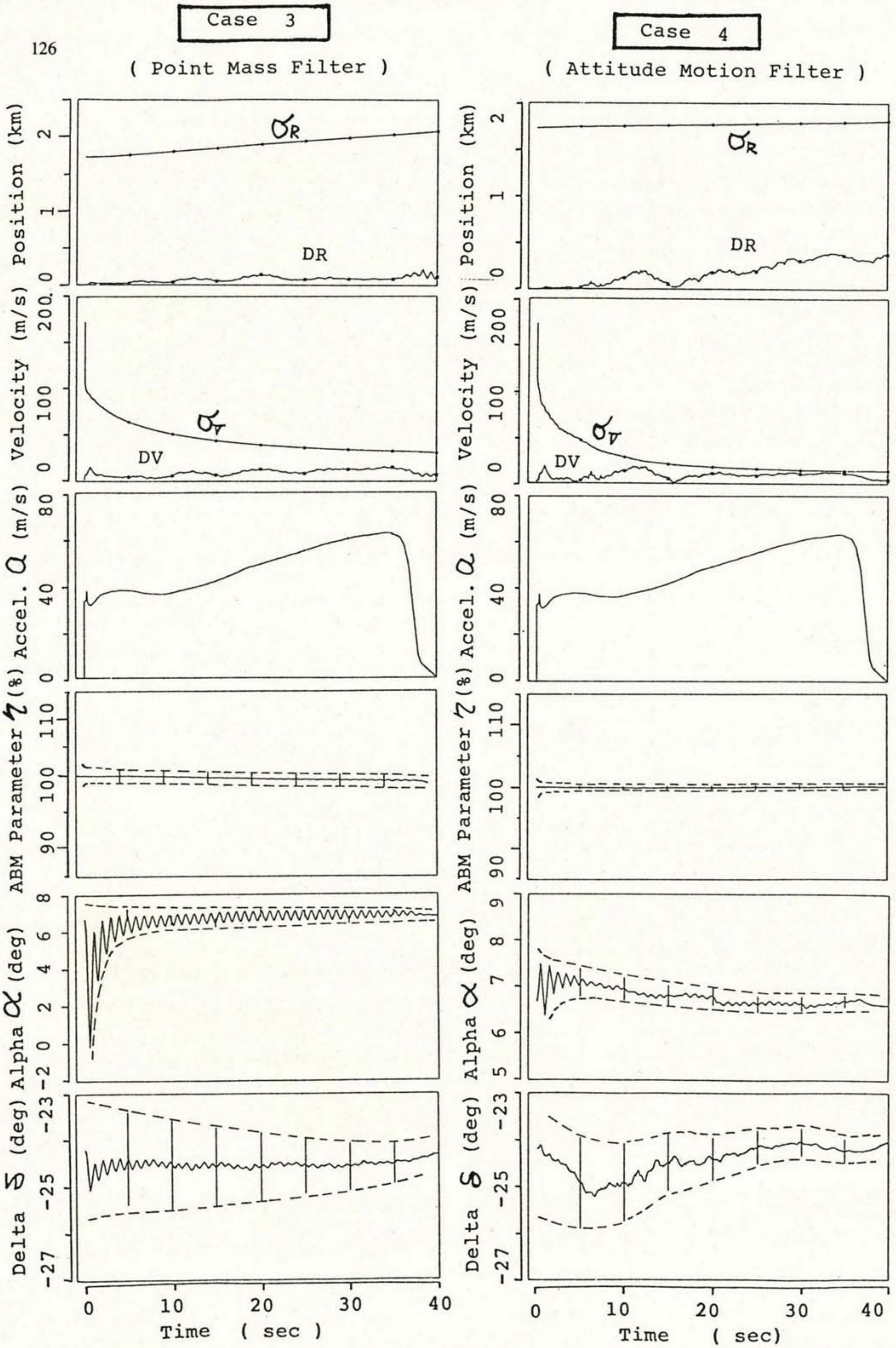

Fig.7 Real Estimation (Case 3,4)

Table 8 Offset Antenna Satellite, Drift Orbit / Attitude Estimation

Orbit/Attitude	Determined Value X^*	Nominal Value X_N	(case 1) Determi.M. $(\alpha, \delta)_N$ used Point Mass Model T_{SPAN}= 9 sec	(case 2) Kalman F. $(\alpha, \delta)_N$ used Point M.M C_I =.0045 C_R = 0.0	$(\alpha, \delta)^*$ Nominal Value X_{NI}	(case 3) Kalman Kilter $(\alpha, \delta)^*$ used Point M.M C_R = 0.16	(case 4) Kalman Kilter $(\alpha, \delta)^*$ used Attitude Motion M. T_{SPAN}=10sec C_R = 1.0
Drift Orbit							
1) Semi-major Axis a (km)	41585.39	41753.15	41258.45	41572.31 (±792)	41750.07	41575.11 (±798)	41647.13 (± 348)
2) Eccentricity e	0.019700	0.016390	0.027793	0.020308 (±.019)	0.015676	0.01997 (±.019)	0.01816 (±.008)
3) Inclination i (deg)	0.351931	0.29887	0.625660	0.31669 (±.086)	0.29368	0.29707 (±.087)	0.27349 (±.047)
4) Ascending Node Ω (deg)	282.5715	283.3117	279.7726	283.0298 (±1.8)	283.5210	283.4551 (±2.0)	284.0351 (± 1.3)
5) Argument of Perigee ω (deg)	167.9021	154.1099	170.8416	161.8599 (±8.2)	165.3289	165.9136 (±4.3)	165.9169 (± 2.1)
6) True Anomaly f (deg)	186.1637	199.2183	186.0281	191.7496 (±8.1)	187.7891	187.2705 (±3.9)	186.6864 (± 1.6)
Attitude							
1) Right Ascension α (deg)	6.69	7.04		6.80 (±.14)	6.69	6.81 (±.14)	6.62 (±.12)
2) Declination δ (deg)	-24.20	-24.19		-24.30 (±.39)	-24.20	-24.31 (±.39)	-24.04 (±.22)
Position Diff. ΔR^* (km)	0.0	2.1	4.3	2.3	2.1	2.1	1.9
Veloity Diff. ΔV^* (m/s)	0.0	12,3	19.3	6.6	7.0	3.2	4.8
				(σ_D =36 Hz)		(24 Hz)	(3 Hz)

where EOCH Time is the ABM Burn Out Time.
ΔR^*, ΔV^* are the differences of inertial position and velocity between the present real estimations from ABM phase doppler data and the past Batch determination from coast phase data after ABM burn out.

$$\Delta R^* = [(X - X^*)^2 + (Y - Y^*)^2 + (Z - Z^*)^2]^{1/2}$$
$$\Delta V^* = [(\dot{X} - \dot{X}^*)^2 + (\dot{Y} - \dot{Y}^*)^2 + (\dot{Z} - \dot{Z}^*)^2]^{1/2}$$

and (±) means one sigma value in the stochastic Kalman filter estimation.
σ_D is the Standard Deviation of the difference between the real doppler data and the Nominal.

4 Conclusions

The conclusions of this paper are as follows ;

(1) One of new attitude motion algorithm of sequential estimations was constructed for the solid ABM burning phase.The estimation results by the simplified filter are reasonable and verify that the technique of attitude motion is efficient for the precise estimation. Therefore the more general analysis by this paper algorithm is expected for the advanced evaluation of ABM phase. That is important on the setting up of AMF error with a high reliability, and also on the reflection upon the Satellite Design Criteria.

(2) The looking again the Determined values,especially the determined attitude, will be necessary for the advanced analyses. By the present time,the covariances of the determined orbit and attitude have not been enoughly evaluated and not obtained. Through the Impulsive evaluations of the occured AMF error,it has been suggested that the attitude determination error would be larger than the considered. This paper verified the above suggestion with a real evidence.

(3) The Real Time Estimation for ABM phase was indicated to be enoughly possible by this paper results. Particularly for the solid ABM satellite, this Real Time Estimation is thought technically to be very easy for the realization because of the short burning time, the possible simplicity of Dynamic Model, and also the the rapid changes of doppler data,etc. And It would be important even for the solid ABM satellite,if the large AMF error is predicted on the narrow visibility of Tracking System,or if the very tight Operation Sequences of Maneuver Decision is planned just after AMF. For the future liquid ABM satellite Operation, this Real Time Estimation would be essential,when the estimation technique would need much more precise because of the long burn duration and the 3 Axis attitude stabilization. Namely the Dynamic Model would be necessary to be constructed with the consideration of the attitude control logic and the perturbation effects of the Earth and the Solar Pressure,etc.

5 Acknowledgements

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References ;

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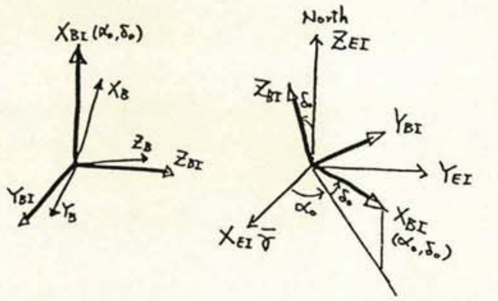
Appendix I Instantaneous Body Coordinate $\mathbb{X}_B$, Inertial Body Co. $\mathbb{X}_{BI}$ and Earth Centered Inertial Co. $\mathbb{X}_{EI}$

$$\mathbb{r}_{EI} = [C]\cdot[A]\cdot\mathbb{r}_B, \quad \mathbb{r}_B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$



$$[C] = \begin{bmatrix} c\alpha_0 & -s\alpha_0 & 0 \\ s\alpha_0 & c\alpha_0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} c\delta_0 & 0 & -s\delta_0 \\ 0 & 1 & 0 \\ s\delta_0 & 0 & c\delta_0 \end{bmatrix} = \begin{bmatrix} c\alpha_0\cdot c\delta_0 & -s\alpha_0 & -c\alpha_0\cdot s\delta_0 \\ s\alpha_0\cdot c\delta_0 & c\alpha_0 & -s\alpha_0\cdot s\delta_0 \\ s\delta_0 & 0 & c\delta_0 \end{bmatrix}$$

where (α_0, δ_0); Nominal AMF Direction

$$[A] = [P]\cdot[T]\cdot[F] = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

$$[\dot{A}] = [\dot{P}]\cdot[T]\cdot[F] + [P]\cdot[\dot{T}]\cdot[F] + [P]\cdot[T]\cdot[\dot{F}] = \dot{\psi}\cdot[P]_\psi\cdot[T]\cdot[F] + \dot{\theta}\cdot[P]\cdot[T]_\theta\cdot[F] + \dot{\phi}\cdot[P]\cdot[T]\cdot[F]_\phi$$

where (ϕ, θ, ψ); Euler angles of transformation from $\mathbb{X}_B$ to $\mathbb{X}_{BI}$

$$\begin{cases} \dot{\phi} = \omega_1 + (\omega_2 s\phi + \omega_3\cdot c\phi)\tan\theta \\ \dot{\theta} = \omega_2\cdot c\phi - \omega_3\cdot s\phi \\ \dot{\psi} = (\omega_2\cdot s\phi + \omega_3\cdot c\phi)\cdot\frac{1}{c\theta} \end{cases}$$

$(\omega_1, \omega_2, \omega_3)$; Body Axis rates

$$\begin{cases} [\dot{P}] = \dot{\psi}\begin{bmatrix} -s\psi & -c\psi & 0 \\ c\psi & -s\psi & 0 \\ 0 & 0 & 0 \end{bmatrix} = \dot{\psi}\cdot[P]_\psi \\ [\dot{T}] = \dot{\theta}\begin{bmatrix} -s\theta & 0 & c\theta \\ 0 & 0 & 0 \\ -c\theta & 0 & -s\theta \end{bmatrix} = \dot{\theta}\cdot[T]_\theta \\ [\dot{F}] = \dot{\phi}\begin{bmatrix} 0 & 0 & 0 \\ 0 & -s\phi & -c\phi \\ 0 & c\phi & -s\phi \end{bmatrix} = \dot{\phi}\cdot[F]_\phi \end{cases} \quad \begin{cases} [P]_{\psi\psi} = \begin{bmatrix} -c\psi & s\psi & 0 \\ -s\psi & -c\psi & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ [T]_{\theta\theta} = \begin{bmatrix} -c\theta & 0 & -s\theta \\ 0 & 0 & 0 \\ s\theta & 0 & -c\theta \end{bmatrix} \\ [F]_{\phi\phi} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -c\phi & s\phi \\ 0 & -s\phi & -c\phi \end{bmatrix} \end{cases}$$

$$\begin{cases} [A]_\phi = \frac{\partial}{\partial\phi}[A] = [P]\cdot[T]\cdot[F]_\phi \\ [A]_\theta = \frac{\partial}{\partial\theta}[A] = [P]\cdot[T]_\theta\cdot[F] \\ [A]_\psi = \frac{\partial}{\partial\theta}[A] = [P]_\psi\cdot[T]\cdot[F] \end{cases}$$

$$\begin{cases} [\dot{P}]_\phi = \dot{\psi}_\phi\cdot[P]_\psi \\ [\dot{P}]_\theta = \dot{\psi}_\theta\cdot[P]_\psi \\ [\dot{P}]_\psi = \dot{\psi}\cdot[P]_{\psi\psi} \\ [\dot{P}]_{\omega_2} = \dot{\psi}_{\omega_2}\cdot[P]_\psi \\ [\dot{P}]_{\omega_3} = \dot{\psi}_{\omega_3}\cdot[P]_\psi \end{cases} \quad \begin{cases} [\dot{T}]_\phi = \dot{\theta}_\phi\cdot[T]_\theta \\ [\dot{T}]_\theta = \dot{\theta}\cdot[T]_{\theta\theta} \\ \\ [\dot{T}]_{\omega_2} = \dot{\theta}_{\omega_2}\cdot[T]_\theta \\ [\dot{T}]_{\omega_3} = \dot{\theta}_{\omega_3}\cdot[T]_\theta \end{cases} \quad \begin{cases} [\dot{F}]_\phi = \dot{\phi}_\phi\cdot[F]_\phi + \dot{\phi}\cdot[F]_{\phi\phi} \\ [\dot{F}]_\theta = \dot{\phi}_\theta\cdot[F]_\phi \\ [\dot{F}]_{\omega_1} = \dot{\phi}_{\omega_1}\cdot[F]_\phi = [F]_\phi \\ [\dot{F}]_{\omega_2} = \dot{\phi}_{\omega_2}\cdot[F]_\phi \\ [\dot{F}]_{\omega_3} = \dot{\phi}_{\omega_3}\cdot[F]_\phi \end{cases}$$

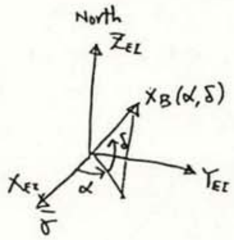
$$\begin{cases} [\dot{A}]_\phi = [\dot{P}]_\phi\cdot[T]\cdot[F] + [\dot{P}]\cdot[T]\cdot[F]_\phi + [P]\cdot[\dot{T}]_\phi\cdot[F] + [P]\cdot[\dot{T}]\cdot[F]_\phi + [P]\cdot[T]\cdot[\dot{F}]_\phi \\ [\dot{A}]_\theta = [\dot{P}]_\theta\cdot[T]\cdot[F] + [\dot{P}]\cdot[T]_\theta\cdot[F] + [P]\cdot[\dot{T}]_\theta\cdot[F] + [P]\cdot[T]_\theta\cdot[\dot{F}] + [P]\cdot[T]\cdot[\dot{F}]_\theta \\ [\dot{A}]_\psi = [\dot{P}]_\psi\cdot[T]\cdot[F] + [P]_\psi\cdot[\dot{T}]\cdot[F] + [P]_\psi\cdot[T]\cdot[\dot{F}] \\ [\dot{A}]_{\omega_1} = [P]\cdot[T]\cdot[\dot{F}]_{\omega_1} \\ [\dot{A}]_{\omega_2} = [\dot{P}]_{\omega_2}\cdot[T]\cdot[F] + [P]\cdot[\dot{T}]_{\omega_2}\cdot[F] + [P]\cdot[T]\cdot[\dot{F}]_{\omega_2} \\ [\dot{A}]_{\omega_3} = [\dot{P}]_{\omega_3}\cdot[T]\cdot[F] + [P]\cdot[\dot{T}]_{\omega_3}\cdot[F] + [P]\cdot[T]\cdot[\dot{F}]_{\omega_3} \end{cases}$$

where

$$\begin{cases} \dot{\phi}_\phi = (\omega_2\cdot c\phi - \omega_3\cdot s\phi)\cdot\tan\theta = \dot{\theta}\cdot\tan\theta \\ \dot{\phi}_\theta = (\omega_2 s\phi + \omega_3 c\phi)\cdot\frac{1}{c\theta^2} = \dot{\psi}/c\theta \\ \dot{\phi}_\psi = 0 \\ \dot{\phi}_{\omega_1} = 1 \\ \dot{\phi}_{\omega_2} = s\phi\cdot\tan\theta \\ \dot{\phi}_{\omega_3} = c\phi\cdot\tan\theta \end{cases} \quad \begin{cases} \dot{\theta}_\phi = -\omega_2\cdot s\phi - \omega_3\cdot c\phi = -\dot{\psi}\cdot c\theta \\ \dot{\theta}_\theta = 0 \\ \dot{\theta}_\psi = 0 \\ \dot{\theta}_{\omega_1} = 0 \\ \dot{\theta}_{\omega_2} = c\phi \\ \dot{\theta}_{\omega_3} = -s\phi \end{cases} \quad \begin{cases} \dot{\psi}_\phi = (\omega_2\cdot c\phi - \omega_3\cdot s\phi)\frac{1}{c\theta} = \dot{\theta}/c\theta \\ \dot{\psi}_\theta = (\omega_2\cdot s\phi + \omega_3\cdot c\phi)\cdot(\frac{s\theta}{c\theta^2}) = \dot{\psi}\cdot\tan\theta \\ \dot{\psi}_\psi = 0 \\ \dot{\psi}_{\omega_1} = 0 \\ \dot{\psi}_{\omega_2} = s\phi/c\theta \\ \dot{\psi}_{\omega_3} = c\phi/c\theta \end{cases}$$

Appendix II Relation between (α, δ) and (ϕ, θ, ψ)

$$\begin{bmatrix} u_x \\ u_Y \\ u_Z \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{EI} = [C]\cdot[A]\cdot\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} c\alpha_0\cdot c\delta_0\cdot c\psi\cdot c\theta - s\alpha_0\cdot s\psi\cdot c\theta + c\alpha_0\cdot s\delta_0\cdot s\theta \\ s\alpha_0\cdot c\delta_0\cdot c\psi\cdot c\theta + c\alpha_0\cdot s\psi\cdot c\theta + s\alpha_0\cdot s\delta_0\cdot s\theta \\ s\delta_0\cdot c\psi\cdot c\theta \qquad - c\delta_0\cdot s\theta \end{bmatrix}$$



$$\alpha = \tan^{-1}\left[\frac{U_Y}{U_X}\right]_{EI} = \tan^{-1}\left[\frac{s\alpha_0\cdot c\delta_0\cdot c\psi\cdot c\theta + c\alpha_0\cdot s\psi\cdot c\theta + s\alpha_0\cdot s\delta_0\cdot s\theta}{c\alpha_0\cdot c\delta_0\cdot c\psi\cdot c\theta - s\alpha_0\cdot s\psi\cdot c\theta + c\alpha_0\cdot s\delta_0\cdot s\theta}\right]$$

$$\delta = \tan^{-1}\left[\frac{U_Z}{\sqrt{U_X^2+U_Y^2}}\right]_{EI} = \tan^{-1}\left[\frac{s\delta_0\cdot c\psi\cdot c\theta - c\delta_0\cdot s\theta}{[(c\delta_0 c\psi\cdot c\theta)^2 + (s\psi\cdot c\theta)^2 + (s\delta_0\cdot s\theta)^2 + 2(c\delta_0\cdot s\delta_0\cdot c\psi\cdot c\theta\cdot s\theta)]^{1/2}}\right]$$

$$\begin{cases} \begin{bmatrix} (U_X)_\theta \\ (U_Y)_\theta \\ (U_Z)_\theta \end{bmatrix} = \dfrac{\partial}{\partial\theta}\begin{bmatrix} U_X \\ U_Y \\ U_Z \end{bmatrix} = [C]\cdot[A]_\theta\cdot\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} (U_X)_\psi \\ (U_Y)_\psi \\ (U_Z)_\psi \end{bmatrix} = \dfrac{\partial}{\partial\psi}\begin{bmatrix} U_X \\ U_Y \\ U_Z \end{bmatrix} = [C]\cdot[A]_\psi\cdot\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{cases}$$

Appendix III Antenna Position, Velocity, and Partial Derivatives

(Position)

$$\mathbb{R}_a = \begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix} = [C]\cdot[A]\cdot \mathbb{r}_a, \quad \mathbb{r}_a = \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}$$

$$\begin{cases} (\mathbb{R}_a)_\phi = \begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix}_\phi = [C]\cdot[A]_\phi\cdot \mathbb{r}_a \\ (\mathbb{R}_a)_\theta = \begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix}_\theta = [C]\cdot[A]_\theta\cdot \mathbb{r}_a \\ (\mathbb{R}_a)_\psi = \begin{bmatrix} X_a \\ Y_a \\ Z_a \end{bmatrix}_\psi = [C]\cdot[A]_\psi\cdot \mathbb{r}_a \\ (\mathbb{R}_a)_{\omega_1} = (\mathbb{R}_a)_{\omega_2} = (\mathbb{R}_a)_{\omega_3} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{cases}$$

(Velocity)

$$\dot{\mathbb{R}}_a = \begin{bmatrix} \dot{X}_a \\ \dot{Y}_a \\ \dot{Z}_a \end{bmatrix} = [C]\cdot[\dot{A}]\cdot \mathbb{r}_a$$

$$\begin{cases} (\dot{\mathbb{R}}_a)_\phi = \begin{bmatrix} \dot{X}_a \\ \dot{Y}_a \\ \dot{Z}_a \end{bmatrix}_\phi = [C]\cdot[\dot{A}]_\phi\cdot \mathbb{r}_a \\ (\dot{\mathbb{R}}_a)_\theta = \begin{bmatrix} \dot{X}_a \\ \dot{Y}_a \\ \dot{Z}_a \end{bmatrix}_\theta = [C]\cdot[\dot{A}]_\theta\cdot \mathbb{r}_a \\ (\dot{\mathbb{R}}_a)_\psi = \begin{bmatrix} \dot{X}_a \\ \dot{Y}_a \\ \dot{Z}_a \end{bmatrix}_\psi = [C]\cdot[\dot{A}]_\psi\cdot \mathbb{r}_a \\ (\dot{\mathbb{R}}_a)_{\omega_1} = \begin{bmatrix} \dot{X}_a \\ \dot{Y}_a \\ \dot{Z}_a \end{bmatrix}_{\omega_1} = [C]\cdot[\dot{A}]_{\omega_1}\cdot \mathbb{r}_a \\ (\dot{\mathbb{R}}_a)_{\omega_2} = \begin{bmatrix} \dot{X}_a \\ \dot{Y}_a \\ \dot{Z}_a \end{bmatrix}_{\omega_2} = [C]\cdot[\dot{A}]_{\omega_2}\cdot \mathbb{r}_a \\ (\dot{\mathbb{R}}_a)_{\omega_3} = \begin{bmatrix} \dot{X}_a \\ \dot{Y}_a \\ \dot{Z}_a \end{bmatrix}_{\omega_3} = [C]\cdot[\dot{A}]_{\omega_3}\cdot \mathbb{r}_a \end{cases}$$

Appendix IV Error Propagation Matrix Φ_K

$$
\Phi_K = \left(\frac{\partial \mathbb{F}}{\partial \mathbb{X}}\right)_K = \left[\begin{array}{cccccc|ccc|ccc|cc|cc|cc}
X_x & X_y & X_z & \Delta t & 0 & 0 & 0 & X_\theta & X_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_{\eta_1} & 0 \\
Y_x & Y_y & Y_z & 0 & \Delta t & 0 & 0 & Y_\theta & Y_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Y_{\eta_1} & 0 \\
Z_x & Z_y & Z_z & 0 & 0 & \Delta t & 0 & Z_\theta & Z_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z_{\eta_1} & 0 \\
\dot{X}_x & \dot{X}_y & \dot{X}_z & 1 & 0 & 0 & 0 & \dot{X}_\theta & \dot{X}_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{X}_{\eta_1} & 0 \\
\dot{Y}_x & \dot{Y}_y & \dot{Y}_z & 0 & 1 & 0 & 0 & \dot{Y}_\theta & \dot{Y}_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{Y}_{\eta_1} & 0 \\
\dot{Z}_x & \dot{Z}_y & \dot{Z}_z & 0 & 0 & 1 & 0 & \dot{Z}_\theta & \dot{Z}_\psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{Z}_{\eta_1} & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & \phi_\phi & \phi_\theta & 0 & \Delta t & \phi_{\omega_2} & \phi_{\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \theta_\phi & 1 & 0 & 0 & \theta_{\omega_2} & \theta_{\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \psi_\phi & \psi_\theta & 1 & 0 & \psi_{\omega_2} & \psi_{\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & [\omega_2]_{\omega_1} & 1 & [\omega_2]_{\omega_3} & 0 & [\omega_2]_{r_3} & [\omega_2]_{\zeta_1} & [\omega_2]_{\zeta_2} & 0 & [\omega_2]_{\eta_2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & [\omega_3]_{\omega_1} & [\omega_3]_{\omega_2} & 1 & [\omega_3]_{r_2} & 0 & [\omega_3]_{\zeta_1} & [\omega_3]_{\zeta_2} & 0 & [\omega_3]_{\eta_2} \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]_K
$$

(18×18)

where

$$\left(\mu 1 = \mu / r^3 , \quad \mu 2 = 3\mu / r^5 \right)$$

$$
\left\{\begin{array}{lllll}
X_x & = & \frac{\partial X_K}{\partial X_{K-1}} & = & 1 - (\Delta t^2/2)\ (\mu 1 - \mu 2\ X^2) \\
Y_x & = & \frac{\partial Y_K}{\partial X_{K-1}} & = & (\Delta t^2/2)\ \mu 2\ XY \\
Z_x & = & \frac{\partial Z_K}{\partial X_{K-1}} & = & (\Delta t^2/2)\ \mu 2\ X\dot{Z} \\
\dot{X}_x & = & \frac{\partial \dot{X}_K}{\partial X_{K-1}} & = & -\ \Delta t\ (\mu 1 - \mu 2\ X^2) \\
\dot{Y}_x & = & \frac{\partial \dot{Y}_K}{\partial X_{K-1}} & = & \Delta t\ \mu 2\ XY \\
\dot{Z}_x & = & \frac{\partial \dot{Z}_K}{\partial X_{K-1}} & = & \Delta t\ \mu 2\ XZ
\end{array}\right.
$$

$$
\left\{\begin{array}{lllll}
X_y & = & \frac{\partial X_K}{\partial Y_{K-1}} & = & (\Delta t^2/2)\ \mu 2\ XY \\
Y_y & = & \frac{\partial Y_K}{\partial Y_{K-1}} & = & 1 - (\Delta t^2/2)\ (\mu 1 - \mu 2\ Y^2) \\
Z_y & = & \frac{\partial Z_K}{\partial Y_{K-1}} & = & (\Delta t^2/2)\ \mu 2\ YZ \\
\dot{X}_y & = & \frac{\partial \dot{X}_K}{\partial Y_{K-1}} & = & \Delta t\ \mu 2\ XY \\
\dot{Y}_y & = & \frac{\partial \dot{Y}_K}{\partial Y_{K-1}} & = & -\Delta t\ (\mu 1 - \mu 2\ Y^2) \\
\dot{Z}_y & = & \frac{\partial \dot{Z}_K}{\partial Y_{K-1}} & = & \Delta t\ \mu 2\ YZ
\end{array}\right.
$$

$$
\left\{\begin{array}{lllll}
X_z & = & \frac{\partial X_K}{\partial Z_{K-1}} & = & (\Delta t^2/2)\ \mu 2\ XZ \\
Y_z & = & \frac{\partial Y_K}{\partial Z_{K-1}} & = & (\Delta t^2/2)\ \mu 2\ YZ \\
Z_z & = & \frac{\partial Z_K}{\partial Z_{K-1}} & = & 1 - (\Delta t^2/2)\ (\mu 1 - \mu 2\ Z^2) \\
\dot{X}_z & = & \frac{\partial \dot{X}_K}{\partial Z_{K-1}} & = & \Delta t\ \mu 2\ XZ \\
\dot{Y}_z & = & \frac{\partial \dot{Y}_K}{\partial Z_{K-1}} & = & \Delta t\ \mu 2\ YZ \\
\dot{Z}_z & = & \frac{\partial \dot{Z}_K}{\partial Z_{K-1}} & = & -\Delta t\ (\mu 1 - \mu 2\ Z^2)
\end{array}\right.
$$

$$\begin{cases} \phi_\phi = 1 + \Delta t [\omega_2 \cdot c\phi - \omega_3 \cdot s\phi] \cdot \tan\theta \\ \theta_\phi = -\Delta t [\omega_2 \cdot s\phi + \omega_3 \cdot c\phi] \\ \psi_\phi = \Delta t [\omega_2 \cdot c\phi - \omega_3 \, s\phi] / c\theta \end{cases}$$

$$\begin{cases} X_\theta = \frac{\Delta t^2}{2} \cdot \eta_1 \cdot A(t) \cdot [-c\alpha_0 \cdot c\delta_0 \cdot c\psi \cdot s\theta + s\alpha_0 \cdot s\psi \cdot s\theta + c\alpha_0 \cdot s\delta_0 \cdot c\theta] \\ Y_\theta = \frac{\Delta t^2}{2} \cdot \eta_1 \cdot A(t) \cdot [-s\alpha_0 \cdot c\delta_0 \cdot c\psi \cdot s\theta - c\alpha_0 \cdot s\psi \cdot s\theta + s\alpha_0 \cdot s\delta_0 \cdot c\theta] \\ Z_\theta = \frac{\Delta t^2}{2} \cdot \eta_1 \cdot A(t) \cdot [-s\delta_0 \cdot c\psi \cdot s\theta - c\delta_0 \cdot c\theta] \\ \dot{X}_\theta = \Delta t \cdot \eta_1 \cdot A(t) \cdot [-c\alpha_0 \cdot c\delta_0 \cdot c\psi \cdot s\theta + s\alpha_0 \cdot s\psi \cdot s\theta + c\alpha_0 \cdot s\delta_0 \cdot c\theta] \\ \dot{Y}_\theta = \Delta t \cdot \eta_1 \cdot A(t) \cdot [-s\alpha_0 \cdot c\delta_0 \cdot c\psi \cdot s\theta - c\alpha_0 \cdot s\psi \cdot s\theta + s\alpha_0 \cdot s\delta_0 \, c\theta] \\ \dot{Z}_\theta = \Delta t \cdot \eta_1 \cdot A(t) \cdot [-s\delta_0 \cdot c\psi \cdot s\theta - c\delta_0 \cdot c\theta] \\ \phi_\theta = \Delta t \cdot [\omega_2 \cdot s\phi + \omega_3 \cdot c\phi] / c\theta^2 \\ \psi_\theta = \Delta t \cdot [\omega_2 \cdot s\phi + \omega_3 \cdot c\phi] \cdot s\theta / c\theta^2 \end{cases}$$

$$\begin{cases} X_\psi = \frac{\Delta t^2}{2} \cdot \eta_1 \cdot A(t) \cdot [-c\alpha_0 \cdot c\delta_0 \cdot s\psi \cdot c\theta - s\alpha_0 \cdot c\psi \cdot c\theta] \\ Y_\psi = \frac{\Delta t^2}{2} \cdot \eta_1 \cdot A(t) \cdot [-s\alpha_0 \cdot c\delta_0 \cdot s\psi \cdot c\theta + c\alpha_0 \cdot c\psi \cdot c\theta] \\ Z_\psi = \frac{\Delta t^2}{2} \cdot \eta_1 \cdot A(t) \cdot [-s\delta_0 \cdot s\psi \cdot c\theta] \\ \dot{X}_\psi = \Delta t \cdot \eta_1 \cdot A(t) \cdot [-c\alpha_0 \cdot c\delta_0 \cdot s\psi \cdot c\theta - s\alpha_0 \cdot c\psi \cdot c\theta] \\ \dot{Y}_\psi = \Delta t \cdot \eta_1 \cdot A(t) \cdot [-s\alpha_0 \cdot c\delta_0 \cdot s\psi \cdot c\theta + c\alpha_0 \cdot c\psi \cdot c\theta] \\ \dot{Z}_\psi = \Delta t \cdot \eta_1 \cdot A(t) \cdot [-s\delta_0 \cdot s\psi \cdot c\theta] \end{cases}$$

$$\begin{cases} [\omega_2]_{\omega_1} = \Delta t \left[-\left\{ \frac{S_1 \cdot I_X(t)}{S_2 \cdot I(t)} - 1 \right\} \omega_3 \right] \\ [\omega_3]_{\omega_1} = \Delta t \cdot \left[\left\{ \frac{S_1 \cdot I_X(t)}{S_2 \cdot I(t)} - 1 \right\} \omega_2 \right] \end{cases}$$

$$\begin{cases} \phi_{\omega_2} = \Delta t \cdot s\phi \cdot \tan\theta \\ \theta_{\omega_2} = \Delta t \cdot c\phi \\ \psi_{\omega_2} = \Delta t \cdot s\phi / c\theta \\ [\omega_3]_{\omega_2} = \Delta t \cdot \left[\left\{ \frac{S_1 \cdot I_X(t)}{S_2 \cdot I(t)} - 1 \right\} \omega_1 \right] \end{cases}$$

$$\begin{cases} \phi_{\omega_3} = \Delta t \cdot c\phi \cdot \tan\theta \\ \theta_{\omega_3} = -\Delta t \cdot s\phi \\ \psi_{\omega_3} = \Delta t \cdot c\phi / c\theta \\ [\omega_2]_{\omega_3} = \Delta t \cdot \left[-\left\{ \frac{S_1 \cdot I_X(t)}{S_2 \cdot I(t)} - 1 \right\} \omega_1 \right] \end{cases}$$

$$\begin{cases} [\omega_3]_{r_2} = \Delta t \cdot \left[\frac{\eta_2 \cdot F(t)}{S_2 \cdot I(t)} \right] \\ [\omega_2]_{r_3} = \Delta t \cdot \left[-\frac{\eta_2 \cdot F(t)}{S_2 \cdot I(t)} \right] \end{cases}$$

$$\begin{cases} [\omega_2]_{S_1} = \Delta t \left[-\left\{ \frac{I_X(t)}{S_2 \cdot I(t)} \right\} \omega_1 \omega_3 \right] \\ [\omega_3]_{S_1} = \Delta t \cdot \left[\left\{ \frac{I_X(t)}{S_2 \cdot I(t)} \right\} \omega_1 \omega_2 \right] \end{cases}$$

$$\begin{cases} [\omega_2]_{S_2} = \Delta t \cdot \left[-\left\{ \frac{S_1 \cdot I_X(t)}{I(t)} \right\} \omega_1 \omega_3 - \frac{\eta_2 \cdot F(t)}{I(t)} \cdot r_3 \right] \cdot \left(-\frac{1}{S_2^2} \right) \\ [\omega_3]_{S_2} = \Delta t \left[\left\{ \frac{S_1 \cdot I_X(t)}{I(t)} \right\} \cdot \omega_1 \omega_2 + \frac{\eta_2 \cdot F(t)}{I(t)} \cdot r_2 \right] \cdot \left(-\frac{1}{S_2^2} \right) \end{cases}$$

$$\begin{cases} X_{\eta_1} = \frac{\Delta t^2}{2} \cdot A(t) \cdot u_X \\ Y_{\eta_1} = \frac{\Delta t^2}{2} \cdot A(t) \cdot u_Y \\ Z_{\eta_1} = \frac{\Delta t^2}{2} \cdot A(t) \cdot u_Z \\ \dot{X}_{\eta_1} = \Delta t \cdot A(t) \cdot u_X \\ \dot{Y}_{\eta_1} = \Delta t \cdot A(t) \cdot u_Y \\ \dot{Z}_{\eta_1} = \Delta t \cdot A(t) \cdot u_Z \end{cases}$$

$$\begin{cases} [\omega_2]_{\eta_2} = \Delta t \cdot \left[-\frac{F(t)}{S_2 \cdot I(t)} \cdot r_3 \right] \\ [\omega_3]_{\eta_2} = \Delta t \cdot \left[\frac{F(t)}{S_2 \cdot I(t)} \cdot r_2 \right] \end{cases}$$

Appendix V Measurement Matrix H_K

$$H_K = \left(\frac{\partial G}{\partial X}\right)_k = \begin{bmatrix} Y1x & Y1y & Y1z & Y1\dot{x} & Y1\dot{y} & Y1\dot{z} & Y_{1\phi} & Y_{1\theta} & Y_{1\psi} & Y_{1\omega_1} & Y_{1\omega_2} & Y_{1\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ Y2x & Y2y & Y2z & Y2\dot{x} & Y2\dot{y} & Y2\dot{z} & Y_{2\phi} & Y_{2\theta} & Y_{2\psi} & Y_{2\omega_1} & Y_{2\omega_2} & Y_{2\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ Y3x & Y3y & Y3z & Y3\dot{x} & Y3\dot{y} & Y3\dot{z} & Y_{3\phi} & Y_{3\theta} & Y_{3\psi} & Y_{3\omega_1} & Y_{3\omega_2} & Y_{3\omega_3} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_K$$

(3×18)

where

$$\left(\begin{matrix} C_0 = \left(-\frac{f_0}{c}\right), & C_1 = (X + X_a - X_s), & C_2 = (Y + Y_a - Y_s), & C_3 = (Z + Z_a - Z_s), \\ C_4 = (\dot{X} + \dot{X}_a - \dot{X}_s), & C_5 = (\dot{Y} + \dot{Y}_a - \dot{Y}_s), & C_6 = (\dot{Z} + \dot{Z}_a - \dot{Z}_s) & \end{matrix}\right)$$

($s = 1, 2, 3$; Station Number)

$$\begin{cases} Y_{sX} \equiv \frac{\partial \Delta f_s}{\partial X} = C_0 \left[\left(C_4 - C_1 \dot{\rho}_a / \rho_a \right) / \rho_a \right]_s \\ Y_{sY} \equiv \frac{\partial \Delta f_s}{\partial Y} = C_0 \left[\left(C_5 - C_2 \dot{\rho}_a / \rho_a \right) / \rho_a \right]_s \\ Y_{sZ} \equiv \frac{\partial \Delta f_s}{\partial Z} = C_0 \left[\left(C_6 - C_3 \dot{\rho}_a / \rho_a \right) / \rho_a \right]_s \\ Y_{s\dot{X}} \equiv \frac{\partial \Delta f_s}{\partial \dot{X}} = C_0 \left[C_1 / \rho_a \right]_s \\ Y_{s\dot{Y}} \equiv \frac{\partial \Delta f_s}{\partial \dot{Y}} = C_0 \left[C_2 / \rho_a \right]_s \\ Y_{s\dot{Z}} \equiv \frac{\partial \Delta f_s}{\partial \dot{Z}} = C_0 \left[C_3 / \rho_a \right]_s \end{cases}$$

$$\begin{cases} Y_{s\phi} \equiv \frac{\partial \Delta f_s}{\partial \phi} = C_0 \left[C_1 (\dot{X}_a)_\phi + C_2 (\dot{Y}_a)_\phi + C_3 (\dot{Z}_a)_\phi + C_4 (X_a)_\phi + C_5 (Y_a)_\phi + C_6 (Z_a)_\phi - \left\{ C_1 (X_a)_\phi + C_2 (Y_a)_\phi + C_3 (Z_a)_\phi \right\} \cdot \dot{\rho}_a / \rho_a \right] / \rho_a \Big|_s \\ Y_{s\theta} \equiv \frac{\partial \Delta f_s}{\partial \theta} = C_0 \left[C_1 (\dot{X}_a)_\theta + C_2 (\dot{Y}_a)_\theta + C_3 (\dot{Z}_a)_\theta + C_4 (X_a)_\theta + C_5 (Y_a)_\theta + C_6 (Z_a)_\theta - \left\{ C_1 (X_a)_\theta + C_2 (Y_a)_\theta + C_3 (Z_a)_\theta \right\} \cdot \dot{\rho}_a / \rho_a \right] / \rho_a \Big|_s \\ Y_{s\psi} \equiv \frac{\partial \Delta f_s}{\partial \psi} = C_0 \left[C_1 (\dot{X}_a)_\psi + C_2 (\dot{Y}_a)_\psi + C_3 (\dot{Z}_a)_\psi + C_4 (X_a)_\psi + C_5 (Y_a)_\psi + C_6 (Z_a)_\psi - \left\{ C_1 (X_a)_\psi + C_2 (Y_a)_\psi + C_3 (Z_a)_\psi \right\} \cdot \dot{\rho}_a / \rho_a \right] / \rho_a \Big|_s \\ Y_{s\omega_1} \equiv \frac{\partial \Delta f_s}{\partial \omega_1} = C_0 \left[C_1 (\dot{X}_a)_{\omega_1} + C_2 (\dot{Y}_a)_{\omega_1} + C_3 (\dot{Z}_a)_{\omega_1} \right] / \rho_a \Big|_s \\ Y_{s\omega_2} \equiv \frac{\partial \Delta f_s}{\partial \omega_2} = C_0 \left[C_1 (\dot{X}_a)_{\omega_2} + C_2 (\dot{Y}_a)_{\omega_2} + C_3 (\dot{Z}_a)_{\omega_2} \right] / \rho_a \Big|_s \\ Y_{s\omega_3} \equiv \frac{\partial \Delta f_s}{\partial \omega_3} = C_0 \left[C_1 (\dot{X}_a)_{\omega_3} + C_2 (\dot{Y}_a)_{\omega_3} + C_3 (\dot{Z}_a)_{\omega_3} \right] / \rho_a \Big|_s \end{cases}$$

where $(X_a)_\phi$, $(\dot{X}_a)_\phi$, etc, are in Appendix III.