NEAR-OPTIMAL STRATEGIES FOR SUB-DECIMETER SATELLITE TRACKING WITH GPS

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ABSTRACT

Decimeter tracking of low earth orbiters can be achieved with a variety of differential GPS techniques. A precisely known global network of GPS ground receivers and a receiver aboard the user satellite are needed, and all techniques simultaneously estimate the user and GPS satellite orbits. Three basic strategies include a purely geometric, a fully dynamic, and a hybrid strategy. The last combines dynamic GPS solutions with a geometric user solution. Two powerful extensions of the hybrid strategy show the most promise. The first uses an optimized synthesis of dynamics and geometry in the user solution, while the second uses a novel gravity adjustment method to exploit data from repeat ground tracks. These techniques promise to deliver sub-decimeter accuracy down to the lowest satellite altitudes.

Keywords: *GPS, differential GPS, decimeter tracking, low earth orbiter, dynamic & geometric tracking, gravity adjustment*

1. INTRODUCTION

Tracking requirements for earth sensing satellites are becoming increasingly stringent, reaching the decimeter level for several missions proposed for the 1990's. NASA's Ocean Topography Experiment (TOPEX, Ref 1), set for a 1991 launch, has a goal of 15 cm altitude accuracy, but would benefit from an accuracy comparable to the 2.5 cm precision of its altimeter. A number of similar missions, including the Navy Remote Ocean Sensing System (NROSS, Ref 2), the European Space Agency's Earth Remote Sensing-1 (ERS-1, Ref 3), and a series of altimetry experiments planned for NASA's Earth Observing System (EOS, Ref 4), are also seeking decimeter altitude accuracy.

TOPEX will carry an experimental tracking system based on the U.S. Defense Department's Global Positioning System (GPS, Ref 5). The basic technique, called differential GPS, makes use of a high performance GPS receiver aboard the orbiter and a small network of precisely located receivers around the globe. All receivers continuously observe the GPS satellites, making dual frequency measurements of pseudorange and L-band carrier phase (Fig 1). Orbiter and ground measurements are then analyzed to recover the orbiter and GPS satellite states in the reference frame defined by the ground network (Refs 6-10). For the TOPEX demonstration, the reference network will include NASA's three Deep Space Network (DSN) tracking stations in California, Spain, and Australia, and at least three complementary sites operated by the Defense Mapping Agency (DMA). A

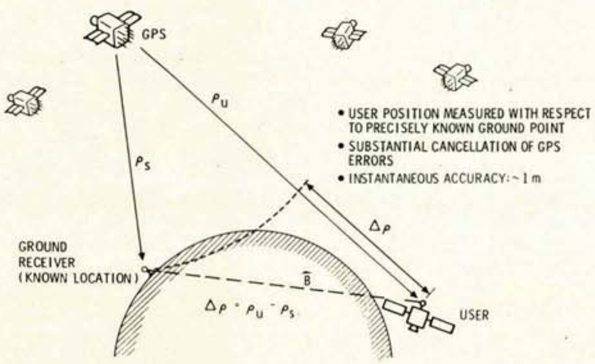

Fig 1. Differential pseudorange observations of four GPS satellites provide the user position and time offset with respect to the reference point.

map of the sites used in error analysis is given in Fig 2. At TOPEX launch, ground site relative positions will be known to about 5 cm and their geocentric positions to better than 20 cm.

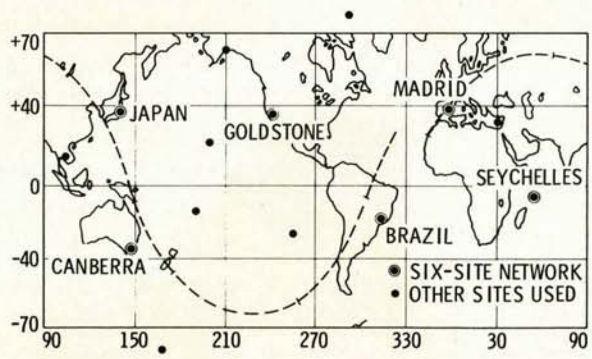

Fig 2. GPS ground receiver sites used in error studies.

Our purpose is to investigate how best to apply differential GPS carrier and pseudorange data to determine the user orbit. A first principle (Refs 6-10) is that any decimeter strategy must perform a joint solution for the user and GPS satellite states. Otherwise, user position accuracy is limited at the meter level by the a priori error in GPS orbits. Considerable data strength is therefore needed to achieve decimeter accuracy.

2. THREE TRACKING STRATEGIES

Here we present three fundamental strategies: a purely geometric strategy, a fully dynamic strategy, and a hybrid strategy in which GPS states are determined dynamically and the user state

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geometrically. First, a word on data combining will be helpful. All of our strategies can use either the undifferenced, singly differenced, or doubly differenced GPS data types. These correspond respectively to the cases in which (a) all clock behavior is modeled over time; (b) GPS clocks are eliminated and only receiver clock behavior is modeled over time; and (c) all clocks are eliminated (or solved for) at each time step. In general, the less differencing the greater the solution strength. When receiver oscillators are unstable, however, it can be advantageous to eliminate their effects. Analysis presented here therefore assumes doubly differenced data. Results will invariably improve if clocks are sufficiently stable to permit less differencing.

2.1 The Geometric Strategy

This is the differential analog of classical point positioning in which a user makes pseudorange measurements to four or more GPS satellites, obtaining a quick geometric solution for position and time offset from GPS time. The conventional user is dependent upon a priori knowledge of GPS satellite positions and time offsets, which are expected to be in error by about 5 m. This limits final position error to 10-15 m. In the simple differential approach (Fig 1) the user and a reference receiver make pseudorange measurements to a common set of at least four satellites permitting a geometric solution for the baseline and the time offset between receivers. GPS orbits are left unadjusted, but cancellation of common GPS clock and orbit errors improves user position accuracy to a meter or two.

In the general approach a network of reference receivers and the user observe the GPS satellites. With sufficient measurements, the user and all GPS positions can be determined geometrically with respect to the reference network. To illustrate, consider a set of four GPS satellites and assume the use of doubly differenced measurements. Including the user there are 5 satellites or 15 position components to estimate, implying a need for at least 15 independent doubly differenced measurements. Since each user-to-ground baseline yields three double differences, five baselines and thus five reference receivers are needed, all viewing the same four satellites. GPS orbits can now be estimated and performance will be limited primarily by measurement precision and observing geometry.

Pseudorange measurements are typically precise to 0.5-1.0 m over one second. This limits instantaneous position accuracy to a meter or worse, depending upon observing geometry. Geometric strategies can be markedly improved by introducing GPS carrier phase, continuously counted, and subtracting it from pseudorange. This removes receiver and GPS dynamics, permitting pseudorange to be averaged over time, a procedure known as "smoothing pseudorange against the carrier." More generally, position change obtained from carrier phase measurements can be subtracted from successive position solutions obtained with pseudorange. This permits the averaging of position solutions over arbitrarily long time periods despite frequent switching of GPS satellites. By this means the general strategy can in principle approach decimeter accuracy for any low orbiter. In reality, however, an impractically large receiver network is needed to maintain strong determination of all parameters.

2.2 The Dynamic Strategy

Here we switch to a classical dynamic formulation in which the satellite state parameters at a single epoch are estimated using an extended arc of data (Refs 6-8). Observations at different times are related to the epoch state parameters by integrating the equations of motion, a procedure requiring accurate analytical models of the observing system and the forces acting on the satellites. The further in time an observation is from the solution time, the greater the expected error from dynamic mismodeling. In other words, the effect of force model errors increases with increasing arc length. In the notation of least squares, if we have an observation vector **z**, we can write the regression equation

$$\mathbf{z} = \mathbf{A}\mathbf{x} + \mathbf{n}, \tag{1}$$

where **x** is the vector of parameters to be estimated, **A** is the matrix of partial derivatives of the observations with respect to the estimated parameters, and **n** is the vector of data noise accompanying the observations. **A** thus contains the information relating observations at one time to the satellite states at epoch. The solution $\hat{\mathbf{x}}$ is obtained with the familiar expression

$$\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\mathbf{z}, \tag{2}$$

where **W** is the inverse of the data noise covariance matrix.

Compared with the general geometric strategy, this approach vastly reduces the number of estimated parameters, thus increasing data strength and permitting far fewer reference receivers. And by introducing dynamic constraints, it permits solutions that are impossible geometrically, such as satellite positions from carrier phase alone. The dynamic technique can therefore be used with carrier phase, pseudorange, or the two together. All this is gained in exchange for a dependence upon dynamic models, with a consequent vulnerability to modeling errors.

This vulnerability is illustrated in two TOPEX studies. Figure 3 is from a covariance study of the altitude error for a single TOPEX orbit using carrier phase data. Key assumptions are

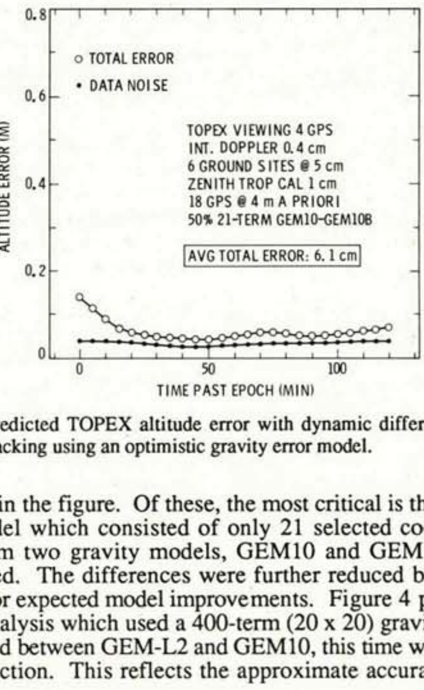

Fig 3. Predicted TOPEX altitude error with dynamic differential GPS tracking using an optimistic gravity error model.

indicated in the figure. Of these, the most critical is the gravity error model which consisted of only 21 selected coefficients taken from two gravity models, GEM10 and GEM10B, and differenced. The differences were further reduced by 50% to account for expected model improvements. Figure 4 presents a similar analysis which used a 400-term (20 x 20) gravity model differenced between GEM-L2 and GEM10, this time without the 50% reduction. This reflects the approximate accuracy of the

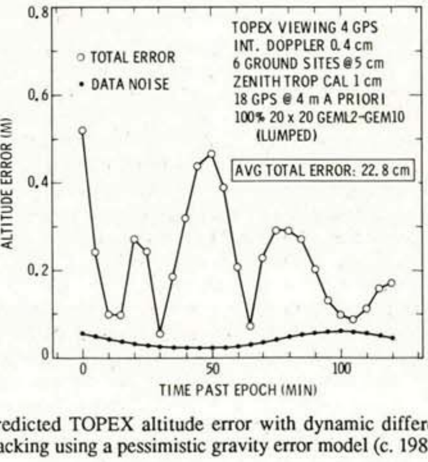

Fig 4. Predicted TOPEX altitude error with dynamic differential GPS tracking using a pessimistic gravity error model (c. 1983).

best gravity models c. 1983. We can see that a significant model improvement is needed to meet the TOPEX goal. A major effort now underway is expected to achieve this by TOPEX launch.

If this strategy were applied at much lower altitudes, such as the 250-350 km typical of shuttle flights, errors would soar. Gravity error would be surpassed by the error in modeling atmospheric drag; final orbit error could reach many tens of meters. At very low altitudes one is therefore led back to a geometric approach.

2.3 A Hybrid Strategy

This technique was proposed at JPL to address specific weaknesses of the two previous strategies. To eliminate modeling errors, the user solution is once more obtained geometrically, with a new solution at each time point. To prevent the proliferation of estimated parameters, the GPS satellite states are obtained dynamically at a single epoch. This can be thought of as a classical epoch state dynamic formulation with the user state treated as process noise with zero correlation time.

Although dynamics appear in this approach, they do so only for the high altitude GPS satellites for which dynamic modeling errors are negligible. Dynamic treatment of GPS orbits sharply reduces the number of estimated parameters, permitting use of a small reference network. Since the user solution is geometric, the customary dynamic error sources—gravity, drag, solar radiation, maneuvers, and venting, to name a few—are eliminated. In recognition of the geometric user solution, we refer to this technique as *non-dynamic* tracking. Note that for a non-dynamic position solution the pseudorange data type is required; however, if pseudorange alone is used, performance is again limited at the meter level by measurement error. The real power of this technique emerges when continuous carrier phase is introduced, allowing the smoothing of position solutions against observed position change. Several hours of smoothing can reduce data noise to a few centimeters.

An example from the extensive TOPEX error studies (Refs 9, 10) is shown in Fig 5. Over a 4-hr data arc the average altitude error is 7.3 cm with two peaks of about 12 cm. Several points are worth noting. Because dynamic errors are absent, this

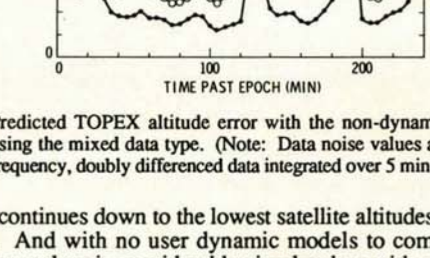

Fig 5. Predicted TOPEX altitude error with the non-dynamic strategy using the mixed data type. (Note: Data noise values are for dual frequency, doubly differenced data integrated over 5 min.)

accuracy continues down to the lowest satellite altitudes (roughly 150 km). And with no user dynamic models to compute, the solution procedure is considerably simpler than with a dynamic approach. The pseudorange data noise in Fig 5 corresponds to a single-channel precision of approximately 40 cm in one second. A new JPL receiver surpasses this by about a factor of two (Ref 8), while some current commercial receivers have about double this error. Doubling the data noise in Fig 5 increases the average error to 8.5 cm; halving it reduces the error to 7.0 cm. Note that multipath and other systematic errors must be contained so that after four hours of averaging their effect is below that of the data noise. The study assumed a six-receiver reference network and a flight receiver observing up to six satellites. Ground receivers were assumed to track all satellites above 10°. If the flight receiver is restricted to viewing four satellites, as may be the case for TOPEX, the observing geometry frequently breaks down and no solution is possible. Enlarging the ground network to 15 sites does not fully restore performance (Ref 10).

3. CARRIER RANGE

There is a data type called "carrier range" which is sometimes recoverable from differential GPS observations. Carrier range is obtained by determining the exact number of full cycles in the carrier phase observable differenced between two receivers. It is, in effect, a differenced pseudorange having the sub-centimeter precision of carrier phase measurements. The process of determining the integer cycle count is called cycle ambiguity resolution or "bias fixing," and a variety of techniques have been devised to carry it out (Refs 8, 11). Bias fixing is a demanding task that currently can be reliably achieved only between fixed ground sites no more than 100 km apart. Several groups are now trying to achieve bias fixing over continental distances.

Carrier range performance can often be approached with the combined carrier phase and pseudorange data types. Carrier phase has the precision of carrier range without the position information; pseudorange has the position information without the precision. Over long data arcs pseudorange error can be averaged down to bring the effective data noise near that of carrier range. Other errors tend to mask the remaining difference.

Consider, for example, the non-dynamic analysis with combined data types shown in Fig 5. The same case using carrier range, shown in Fig 6, yields an average improvement of 1.3 cm. (To optimize performance in Fig 6, three ground sites were adjusted.

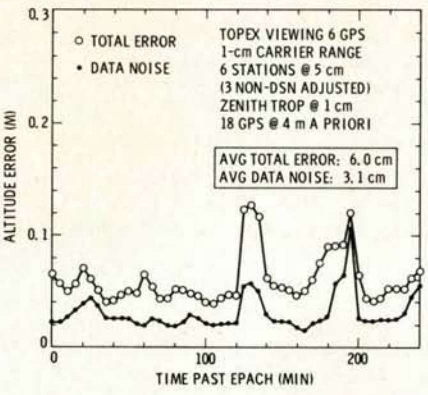

Fig 6. Predicted TOPEX altitude error with the non-dynamic strategy using differenced carrier range.

Without this refinement there is no net improvement.) When the data error is halved to correspond to the best GPS receivers, the carrier range advantage is reduced to 1.0 cm. Thus, if bias fixing should be unattainable over long distances, the combined data type may offer a practical alternative. Again we must stress that in precise applications of pseudorange, multipath and other systematic errors must be carefully controlled.

4. TWO ADVANCED STRATEGIES

The non-dynamic strategy has a significant limitation: Performance is dependent upon the momentary observing geometry, as evidenced by the error fluctuations in Fig 5. The momentary observing geometry depends in turn upon the ground network, the receiver viewing capacity, and the GPS satellite constellation. Loss of a key ground site, GPS satellite, or user channel can cause the solution to fail altogether. We can address this by reintroducing user dynamics, appropriately weighted according to model quality, while preserving the essential geometric technique. User dynamics will smooth the solution through trouble spots while adding information—and strength—throughout.

4.1 An Optimized Strategy

The approach is to restore the dynamic formulation and introduce geometry with added process noise in the user force models. Thus the GPS state solution is still fully dynamic while the user solution is now partly dynamic and partly geometric. The relative weighting of dynamics and geometry in the user solution can be continuously adjusted to maximize performance. We present the technique in a Kalman filter formulation.

Let $\hat{\mathbf{x}}_j$ be the state estimates (GPS and user) at time t_j; $\tilde{\mathbf{x}}_{j+1}$ the predicted state estimates at time t_{j+1}; $\mathbf{H_x}(j+1,j)$ the state transition from t_j to t_{j+1}; and $\mathbf{p}$ a set of process noise parameters for three-dimensional forces on the user. Then we have the following dynamic (state transition) model for the augmented state $\mathbf{X}=[\mathbf{x},\mathbf{p}]^T$ and its associated covariance $\mathbf{P}$ (Ref 12):

$$\tilde{\mathbf{X}}_{j+1} = \mathbf{H}_j\hat{\mathbf{X}}_j + \mathbf{B}\mathbf{w}_j \tag{3}$$

and

$$\tilde{\mathbf{P}}_{j+1} = \mathbf{H}_j\hat{\mathbf{P}}_j\mathbf{H}_j^T + \mathbf{B}\mathbf{Q}_j\mathbf{B}^T, \tag{4}$$

where

$$\mathbf{H}_j = \begin{bmatrix} \mathbf{H_x}(j+1,j) & \mathbf{H_{xp}}(j+1,j) \\ 0 & \mathbf{M}_j \end{bmatrix}; \tag{5}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{I_p} \end{bmatrix}; \tag{6}$$

$\mathbf{H_{xp}}(j+1,j)$ is the transition matrix relating $\tilde{\mathbf{x}}_{j+1}$ to the process noise parameters $\mathbf{p}_j$; $\mathbf{M}_j$ is a diagonal matrix with its i^{th} element

$$m_i = \exp[-(t_{j+1}-t_j)/\tau_i]; \tag{7}$$

$\mathbf{w}_j$ is a white noise process of covariance $\mathbf{Q}_j$ which is diagonal with its i^{th} element σ_i^2; and $\mathbf{I_p}$ is a unit matrix. Both σ_i and τ_i can be the same for all i. The relative weighting of dynamics and geometry can be varied by selecting different values for σ_i, p_o and τ_i. When $\sigma_i \to 0$ and $p_o \to 0$ this formulation reduces to the fully dynamic strategy; when $\tau_i \to 0$ and $\sigma_i \to \infty$ it becomes the non-dynamic strategy. The measurement model is the same as in the dynamic and non-dynamic strategies except that $\mathbf{X}$ and $\mathbf{P}$ are for the expanded state. For a Kalman filter we have

$$\hat{\mathbf{X}}_j = \tilde{\mathbf{X}}_j + \mathbf{G}_j(\mathbf{z}_j - \mathbf{A}_j\tilde{\mathbf{X}}_j) \tag{8}$$

and

$$\hat{\mathbf{P}}_j = \tilde{\mathbf{P}}_j - \mathbf{G}_j\mathbf{A}_j\tilde{\mathbf{P}}_j, \tag{9}$$

where $\mathbf{z}_j$ is the measurement vector at time t_j; $\mathbf{A}_j$ is the matrix of the corresponding measurement partials with respect to $\mathbf{X}_j$; and $\mathbf{G}_j$ is the Kalman gain given by

$$\mathbf{G}_j = \tilde{\mathbf{P}}_j\mathbf{A}_j^T(\mathbf{A}_j\tilde{\mathbf{P}}_j\mathbf{A}_j^T + \mathbf{R}_j)^{-1}, \tag{10}$$

with $\mathbf{R}_j$ being the error covariance of $\mathbf{z}_j$. These models are formulated in terms of current state for clarity. A pseudo-epoch state U-D factorized formulation (Ref 12) has been implemented in the GPS error analysis software known as OASIS (Orbit Analysis and Simulation Software) developed at JPL (Ref 13). We refer to this approach as the *reduced dynamic* strategy.

Determining the optimal weighting requires a careful error assessment. Figure 7 presents a simplified parametric analysis

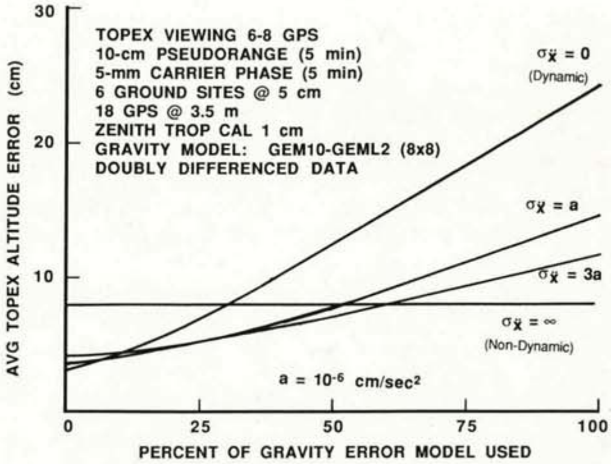

Fig 7. Relative performance of dynamic, non-dynamic, and reduced dynamic tracking strategies as a function of gravity model quality.

for TOPEX. Performance is shown for four weightings, including the pure dynamic and pure non-dynamic forms, as a function of gravity model quality. An 8x8 gravity model is used and TOPEX is assumed to view 6-8 satellites. In the gravity quality range expected for TOPEX (25-50% GEM10-GEML2) the two intermediate forms show a significant advantage over the two pure forms. This advantage is expected to increase if the TOPEX receiver is restricted to viewing four satellites. At the two extremes of the gravity quality range there is apparently little to be gained from an optimized reduced dynamic approach.

4.2 A Gravity Adjustment Strategy

Here we present a strategy specialized for a particular type of orbit and for a strictly post-processing application. The orbit must feature a regularly repeating ground track and be at an altitude between roughly 600 and 3000 km where gravity is the dominant dynamic error. Orbits for most earth observation satellites requiring high accuracy tracking, including Seasat, TOPEX, and ERS-1, fit these conditions. The technique features a novel gravity adjustment strategy which exploits the special character of the orbit to achieve accuracy and efficiency.

Ordinarily the earth's gravity field is represented by a spherical harmonic expansion, and ordinarily several hundred coefficients are needed to support precise dynamic tracking of a low orbiter. Gravity "tuning" usually involves the adjustment of selected harmonic coefficients as part of the orbit determination process. Resonant components with large effects on a particular satellite orbit can thereby be improved, with a resulting improvement in tracking accuracy. Here we dispense with the harmonic representation and substitute a set of local parameters or "bins," spaced evenly around each full ground track. For the fitting process, data from a large number of repeat ground tracks are collected in an ensemble. The user states at the beginning of each arc are estimated together with a single set of 3-D position corrections. These corrections correspond to the orbit perturbations in each bin, common to all repeat arcs, due to gravity mismodeling, as illustrated in Fig 8. Because the gravity pertur -

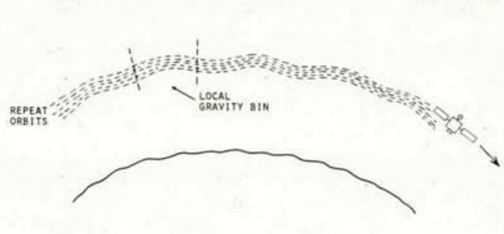

Fig 8. The gravity adjustment strategy estimates local gravity parameters using data from repeat ground tracks.

bations felt by the user are the same for repeat ground tracks, collecting repeat orbits in an ensemble permits accurate recovery of local gravity effects by averaging random and other non-repeating errors. The number of parameters needed for the entire globe, and hence the effective gravity resolution, is roughly the same as with a harmonic expansion. With this approach, however, only the relatively few parameters pertaining to a particular ground track are dealt with at one time.

The mathematical details are given in Refs 14 and 15. Here we present a brief summary. Let r(t) be the deviation of the orbiter position from a nominal trajectory and let t_j^i represent the j^{th} time point of the i^{th} trajectory in the ensemble, where 1=1,2,...,N and j=1,2,...,M. Then we can write the linearized expression

$$r(t_j^i) = \frac{\partial r}{\partial r_o^i} r_o^i + \frac{\partial r}{\partial v_o^i} v_o^i + d_j + \frac{\partial r}{\partial p^i} p^i, \qquad (11)$$

where r_o and v_o are the deviations in epoch position and velocity, p^i represents the effects of all non-gravitational dynamic parameters, and d_j is the local parameter, common to all arcs, representing the position correction due to gravity mismodeling at that point. Solving for the d_j in the fitting process constitutes the gravity adjustment performed by this technique. We can write the above equation in matrix form as

$$\mathbf{R}^i = (\mathbf{V}\mathbf{x}_o)^i + \mathbf{d}, \qquad (12)$$

where $\mathbf{R}^i$ is the vector of position corrections for each time point in the i^{th} arc, $\mathbf{x}_o$ is the correction to the epoch state vector for the i^{th} arc, $\mathbf{V}$ is the corresponding matrix of variational partials, $\mathbf{d}$ is the arc-independent vector of position corrections due to gravity mismodeling, and the non-gravitational terms $\mathbf{p}$ have been omitted. The a priori covariance of $\mathbf{d}$ can be derived from the gravity field used for the nominal trajectory by means of a transformation matrix of variational partial derivatives, as described in Ref 15. Combining measurement data from the multiple data arcs, we can write the standard regression equation

$$\mathbf{z} = \mathbf{A}\mathbf{x} + \mathbf{n}, \qquad (13)$$

where $\mathbf{z}$ is the measurement vector, $\mathbf{n}$ is the data noise vector, $\mathbf{x}$ is the vector of parameters (including GPS states) to be estimated, and $\mathbf{A}$ is the matrix of measurement partials. Note that both $\mathbf{A}$ and $\mathbf{x}$ can be partitioned into arc-dependent and arc-independent parts. An efficient method of solving the partitioned regression equation by application of the Householder transformation is given in Ref 14.

Though it isn't immediately evident, this is an extension of the non-dynamic strategy to repeating data arcs. As presented, the technique yields solutions for satellite epoch states in each arc plus arc-independent user position corrections in each gravity bin. In the degenerate case of a single data arc, however, the notion of arc-independent parameters collapses and we can obtain a simple set of geometric position corrections—the $r(t_i)$ defined above. This, in essence, is the non-dynamic technique. Analysis of this strategy applied to TOPEX is now underway.

A further application is to the improvement of gravity models. The arc-independent position corrections from all ground tracks in the repeat sequence can be collected together and, using appropriate transformations, a global gravity field produced which is tailored for the particular orbit. Because data from the multiple repeat ground tracks have been reduced to a small set of parameters, the total computation involved would be vastly less than with traditional techniques of global gravity solution.

5. SUMMARY AND CONCLUSIONS

Five approaches to differential GPS tracking of low earth satellites have been presented. The purely geometric strategy is far the most limited and is included here primarily for illustration and completeness. The general form, in which GPS orbits are adjusted, is capable in principle of near-decimeter performance but requires an inordinate number of ground sites. The simple form, without GPS adjustment, is practical but limited to meter level performance. Since it is operationally the simplest technique, for missions requiring meter level accuracy, geometric tracking may be the method of choice. Fully dynamic tracking, whether with GPS or another system, can offer decimeter accuracy only so long as dynamic modeling errors are adequately contained. For TOPEX the dominant error is in the earth gravity model and continued success in the current gravity improvement effort will be needed to reach a decimeter. For satellites at lower altitudes, such as NROSS, ERS-1, and EOS, decimeter gravity modeling will present a greater challenge. And at the lowest altitudes, where atmospheric drag is dominant, decimeter modeling is out of reach. The non-dynamic strategy, with its geometric user solution and dynamic GPS solution, is the first to offer practical subdecimeter accuracy at all altitudes and to dynamically active vehicles. And with no user models, it is operationally simpler than a dynamic approach. It suffers, however, from a geometric sensitivity, making it vulnerable to various forms of system degradation which can cause it to fail altogether.

Two extensions of the non-dynamic strategy bring more information to bear. The reduced dynamic strategy is a sophisticated hybrid combining dynamic and geometric techniques in a single user solution. Early analysis shows that it can be superior to either technique separately. The gravity adjustment strategy is designed to exploit efficiently the information in an ensemble of repeat ground tracks. In general, each arc of the ensemble will reflect a different pattern of GPS satellite formations. The resulting set of position corrections, common to all arcs, will therefore be less sensitive to momentary weaknesses in GPS geometry. (Geographically correlated weaknesses due to ground site distribution will of course persist.) This is a specialized technique which may be of benefit to missions like TOPEX with a suitable orbit and a delayed processing schedule. For general applications, however, the optimized reduced dynamic strategy appears to be the strongest option. Operationally no more complex than classical dynamic orbit determination, it offers subdecimeter accuracy to all low orbiters, minimal sensitivity to dynamic and geometric weaknesses, and the versatility to adapt to changing conditions.

6. ACKNOWLEDGEMENT

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