DEFINITION OF A REFERENCE ORBIT FOR THE SKYBRIDGE CONSTELLATION SATELLITES

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Abstract

This paper presents the definition and the calculation method of the reference orbit for the SkyBridge constellation of satellites. The feasibility of the station keeping of the SkyBridge satellites has already be shown in a previous paper[1].

It begins with the calculation of a frozen and phased orbit with a first order theory where only secular variations of the orbital parameters are considered. Then the reference orbit is computed in mean parameters with an analytical method where only zonal harmonics coefficients up to 16 are considered. The short period perturbations are added in order to provide a reference osculating orbit. Finally the accuracy of this orbit is estimated with a numerical integration.

The choice of a unique reference orbit for the whole constellation is the best solution to ease the station keeping, to minimize the computation load in the user terminals and to minimize the ground segment operations.

The study has been performed jointly by CNES and ALCATEL Space Industries.

Key words: *LEO constellation, station keeping, reference orbit, frozen orbit, phased orbit, mean parameters, analytical method.*

1-Introduction

SkyBridge intends to provide a telecommunication service by means of an homogeneous constellation of 80 satellites Low Earth Orbit. This constellation architecture impose the same semi-major axis, the same inclination and the same eccentricity for all the satellites. Only mean orbital parameters allow to define the same reference orbit for the whole constellation while keeping the homogeneity properties with high accuracy. Moreover SkyBridge has to provide a simple orbit model which allows each user to localize easily the satellite from its terminal. The use of an analytical model for the reference orbit is well adapted for users because it minimizes the time and the size of the orbit computation.

A reference orbit should provide at every time the position of one satellite to the ground station network. For the usual station keeping of only one satellite, most of the predictable orbital perturbations are taken into account in the dynamical reference orbit model. This orbit is built specifically for a particular mission; so a change of one parameter of this orbit needs of course a complete new calculation of the reference orbit. Some forces which are difficult to forecast, like the drag, the radiation pressure and also the thermal effects, are usually not included into the dynamical model of the reference orbit. Some maneuvers are realized to maintain the satellite close to its theoretical position defined by the reference orbit.

The station keeping of a whole constellation needs a new approach. The most convenient solution in that case is to calculate the same reference orbit for all the satellites of the constellation. We have to define a dynamical model where the semi-major axis, the inclination and the eccentricity of the reference orbit is only dependent of the phase of the satellite position and not of the time. To respect such a definition of the reference orbit, only zonal coefficients of the Earth gravity field have to be included in the dynamical model. This orbit is firstly calculated in terms of mean parameters with an analytical method in an iterative process where the cycle duration, the semi-major axis and the eccentricity of the orbit are recomputed each time. We obtain a frozen and phased orbit very stable over every cycle. Short orbital perturbations due to the zonal coefficients are added in last step of the process to provide an osculating orbit. The periods of those short perturbations are sub-multiples of the revolution period of the satellite, the final reference orbit preserves the stability properties of the mean reference orbit.

The definition of an unique reference orbit for the constellation simplifies the tracking of the satellites by the ground control segment and by the user terminals. The computation of the reference orbit for one satellite to another is direct. All the satellites move on a same reference grid and the only difference from one satellite to an other is a constant shift of phase imposed by the initial conditions, the ascending node and the mean anomaly.

2-Definition of the reference orbit

When defining the same reference orbit for all the satellites of the constellation, the difficulty is to choose a realistic dynamical model which is independent of the initial conditions of each satellite. A dynamical model which includes only the main zonal deformations of the Earth gravity field can provide the desired reference orbit.

The zonal harmonic coefficients represent the zonal deformations of the Earth gravity field in a geocentric frame where the z-axis is oriented toward the north pole. According to Kaula's theory[2] those coefficients produce three types of orbital perturbations on a satellite moving around the Earth:

- some secular perturbations, due to the even coefficients, on the ascending node, on the argument of perigee and on the mean anomaly,
- some long-period perturbations, mainly due to the odd coefficients, on all the orbital elements except on the semi-major axis at the revolution period of the perigee and at the sub-multiples of this period,
- some short-period perturbations on all the orbital elements at the revolution period of the satellite around the Earth, the mean anomaly, and at the sub-multiples of this period.

The phasing of the orbit should consider all these perturbations. But the cycle duration of an orbit is a whole number of satellite revolutions, so the calculation of the phased orbit can be accomplished in mean parameters, independently of the short period perturbations. Moreover it is convenient to use that type of parameters, because with a judicious choice of the mean parameters it is possible to cancel the effects of the long perturbations due to the odd zonal coefficients, that is to say to freeze the orbit. Finally the phasing of the orbit can be first executed by only considering the secular variations of the mean parameters.

2.1-Frozen orbit

A frozen orbit is an orbit where the mean eccentricity, the mean inclination and the mean argument of the perigee keep their initial value. To calculate such an orbit, only the zonal coefficients have to be considered.

By using the Kaula's theory, it is has been proved[3] that the motion of the mean eccentricity $\mathbf{e_m}$, of the mean inclination $\mathbf{i_m}$ are cancelled on the Earth for a mean argument of perigee $\boldsymbol{\omega_m}$ equal to **90 degrees**. The 90 degrees value of the mean argument of perigee is kept constant for a particular combination of the triplet (a_m, e_m, i_m), where $\mathbf{a_m}$ is the mean semi-major axis. The orbit is so called frozen.

In the case of SkyBridge, where the mean semi-major axis was initially fixed to **7845 km** and the mean inclination to **53 degrees**, the calculation of the frozen orbit is accomplished for a mean eccentricity of $\mathbf{0.8454\ 10^{-3}}$.

This calculation has been executed with the JGM3[4] Earth gravity field model. The choice of the JGM3 model for the definition of the reference orbit is justified by the fact that JGM3 is the operational gravity model used for the TOPEX/POSEIDON satellite which orbits close to the SkyBridge altitude.

2.2-Phased orbit

An Earth phased orbit is an orbit which periodically passes over the same ground points. As it was explained before, the calculation of the phased orbit is firstly made in terms of mean parameters where only secular variations are considered. Moreover the orbit is frozen, so the secular variation of the argument of perigee has been cancelled. In fact the secular drift of the satellite position on its orbit is equal to the secular drift of the mean anomaly.

The initial phasing of the orbit is calculated with a first order dynamical model where only J_2 is taken into account. The secular drifts of the ascending node $\Delta\Omega_S$, of the mean anomaly ΔM_S and of the argument of perigee $\Delta\omega_S$ are given by the following expressions:

- $\Delta\Omega_S = - 3/2\ (a_e/a)^2\ J_2\ n/(1-e^2)^2 \cos i\ (t-t_0)$
- $\Delta M_S = [\ n + 3/4\ (a_e/a)^2\ J_2\ n/(1-e^2)^{3/2}\ (2-3\sin^2 i)](t-t_0)$
- $\Delta\omega_S = 0$ *(for a frozen orbit)* ==> $\Delta(\omega+M)_S = \Delta M_S$

where a_e is mean radius of the Earth, n is the mean motion of the satellite and J_2 is the amplitude of the second zonal harmonic in the JGM3 model.

The secular drifts are calculated by using these expressions with the following initial conditions:
- a_0 =7845 km, e_0 = 0.8454 10^{-3}, i_0 = 53 deg,
- ω_0 = 90 deg, Ω_0 = 0 deg, M_0 = - 90 deg,
- any initial value of the mean sidereal time: T_{SID_0}.

The drift of the mean sidereal time which has been considered is $d(T_{SID})/dt$=0.72921150902 10^{-4} rad/s.

Different cycle durations are found by looking for all the dates where the orbit plan has accomplished a complete revolution with regard to the Earth. This condition is expressed by: $\Omega(t) + T_{SID}(t) - T_{SID_0} = 2k\pi$, where k is the number of sidereal days. The number of revolution of the orbital plan in a fixed geocentric frame is disregarded because in less than 36 days, the satellite plan has not yet accomplished a complete revolution

The SkyBridge specifications have chosen a cycle duration close to 36 days. For the closest corresponding cycle duration estimated with this method, the value of mean anomaly is calculated. The value of the semi-major axis is adjusted by a dichotomy method in order to cancel the shift position of the satellite with respect to its initial position (M = -90 deg) at the beginning of the cycle. We finally obtain:

k = 36 sidereal days
a_0 = 7845.083615 km
T_{cycle} = 35.6150402 days

The satellite has accomplished 445 revolutions around the Earth during this cycle duration.

The value of the semi-major axis has to be adjusted again in order to take in account the second order secular drifts of the ascending node and of the mean anomaly due to the other zonal harmonics and to coupling effects of the zonal harmonics. The eccentricity value has to be corrected simultaneously with the semi-major axis to stabilize the frozen orbit. That is the subject of the next paragraph.

3-Adjustment of the mean reference orbit

The previous paragraph describes the calculation of the frozen and phased orbit with a first order theory where only J2 is taken in account. We have now to introduce additional accuracy on the orbital perturbations in order to provide a realistic reference orbit. At an altitude of 1500 km, the knowledge of Earth gravity field truncated to the order 16 is quite enough for the mission analysis. The orbital perturbations due to the zonal harmonics of this potential have to be considered for the calculation of the phased orbit. We now use an analytical method where the orbit trajectory is calculated in terms of mean parameters. The reference orbit is so calculated with a high accuracy. The phasing error of the orbit and the residuals of the long period perturbations are easily estimated. We have to calculate successively some corrections on the mean semi-major axis and on the mean eccentricity until we obtain a stable solution.

3.1-Mean parameters

A satellite orbit is classically calculated with a numerical integrator where all the orbital perturbations due to a dynamical model are taken in account without any distinction. With the analytical theories, it becomes possible to identify separately the amplitudes and the periods of the orbital perturbations due to one Earth gravitational coefficient. In an analytical computation of satellite trajectory, the satellite mean parameters are calculated taking into account the different orbital perturbations due a considered model.

The SkyBridge reference orbit is determined with mean parameters, the dynamical model used for is the JGM3 Earth gravity field limited to 16 zonal harmonic coefficients. The orbit is tabulated with a 100 seconds step on a duration of 10 cycles. The period of the main long period perturbations is near 180 days, so a calculation of the orbit on 10 cycles minimize the influence of the residuals of the long period perturbations.

3.2-Correction of the semi-major axis

The ephemeris is obtained in mean parameters with the new initial conditions where a_0 =7845.083615 km. The formula $\Omega(t) + T_{SID}(t) - T_{SID_0} = 2k\pi$, with k=36, allow us to estimate the accuracy $\Delta\Omega$ (*figure 1*) of the phased orbit. The new cycle duration is calculated by considering the new value of the drift orbit plan which has been shifted to a more realistic value. The corresponding new value of the mean anomaly is considered as the new phasing error ΔM (*figure1*).

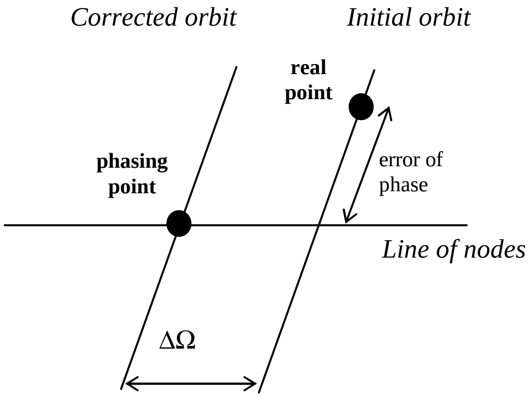

figure 1: Correction of phase

To cancel out this error, the semi-major axis a_0 is adjusted by a linear approximation based on the mean Keplerian motion of the satellite. The mean anomaly is a linear function of the time, M = nt, with n the mean motion of the satellite, so the correction on the semi-major axis is:

$$\Delta a = ([M_0 / (M_0+\Delta M)]^{2/3} - 1)\, a_0,$$

where $M_0 = 2\pi N_{rev}$ corresponds to the corrected value of the ascending node to phase the orbit, and N_{rev} is the whole number of satellite revolution around the Earth over the whole cycle duration.

The orbit is then computed with this new semi-major axis value. But the correction Δa is a linear approximation, so it has to be done many times. And between each iteration, the value of the eccentricity has also to be corrected in order to maintain the frozen properties of the orbit.

3.3-Correction of the eccentricity

The calculation of the value of the mean eccentricity to freeze the orbit has been done with a Kaula algorithm where only first order secular variations are taken in account. The coupling effects between zonal

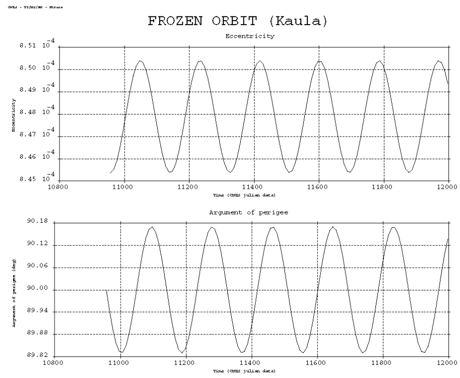

figure 2: Frozen orbit

coefficients have not be considered. So some residual long-period perturbations appears in the tabulated orbit. The amplitude of those residuals are:

$\Delta e = 5.\ 10^{-6}$ and $\Delta\omega = 0.4$ deg.

It is possible to improve the accuracy of the frozen orbit. For that we have to calculate the mean value of the eccentricity plotted in *figure 2*, the result obtained is: $e = 0.847915\ 10^{-3}$ with $\Delta e = 5.\ 10^{-9}$ and $\Delta\omega = 10^{-4}$ deg.

The successive semi-major axis corrections to phase the orbit are weak. The resulting perturbations on the eccentricity and on the argument of perigee can be corrected by centering the eccentricity value at each iteration.

3.4-Results with mean parameters

Successive adjustments of the semi-major axis and of the correction of the eccentricity are executed until we obtain a stable solution which leads to an accurate phased and frozen mean orbit. The following values are finally:

- a = 7847.3978918 km
- $e = 0.8405\ 10^{-3}$
- T_{cycle} = 35.615318481 days

The phasing error corresponding to this new set of mean parameters is $\Delta M = 10^{-8}$ deg on 10 cycles.

We have finally obtained a very accurate mean phased and frozen reference orbit. We have now to provide an osculating reference orbit.

4-Final reference orbit

This paragraph describes the computation of the osculating reference orbit where the short-period perturbations are added. Then the accuracy of the orbit is estimated by using a numerical integrator.

4.1-Addition of the short perturbations

The occurrences of the short period perturbations due to the zonal coefficients are the revolution period of the satellite and its sub-multiples. So the cycle duration of the phased orbit is independent of those perturbations. Nevertheless, the short period perturbations due to the odd zonal coefficients are not phased on the equator like the ones due to the even zonal coefficients. The real position of the satellite is shifted of $6.\ 10^{-5}$ degree on the orbit and on the ascending node. Only the short period perturbations of J2, J3, J4 have been added on the mean parameters. The initial parameters of the osculating orbit are:

$a_{osc} = 7852.7736368$ km, $e_{osc} = 1.0419\ 10^{-3}$,
$i_{osc} = 53.01476$ deg, $\Omega_{osc} = 359.99994$ deg,
$\omega_{osc} = 53.72314$ deg and $M_{osc} = -53.72308$ deg.

The phasing error corresponding to this computation over 10 cycles is 10^{-3} degree instead of the 10^{-8} degree obtained previously. This difference can be attributed to a small shift of the mean inclination which adds a secular drift on the ascending node. But this result is still very good and quite enough with respect to the station keeping constraints. We are now going to calculate a new orbit with a numerical integrator in order to check the accuracy obtained.

4.2-Numerical integration of the orbit

The initial osculating parameters of the reference orbit calculated with the analytical model have been introduced in a numerical integrator. The orbit is so calculated with a 10 seconds step on a 10 cycle duration We plot the value of the ascending and the value of the satellite position on the orbit. The accuracy obtained is 10^{-3} degree on both parameters which corresponds to the accuracy obtained with the analytical method. So this reference orbit can be used for the operational station keeping.

5-Conclusion

In this paper, we have described the definition of the reference orbit conducted by CNES and ALCATEL Space Industries. The different step of this study are summarized on the *figure 3*.

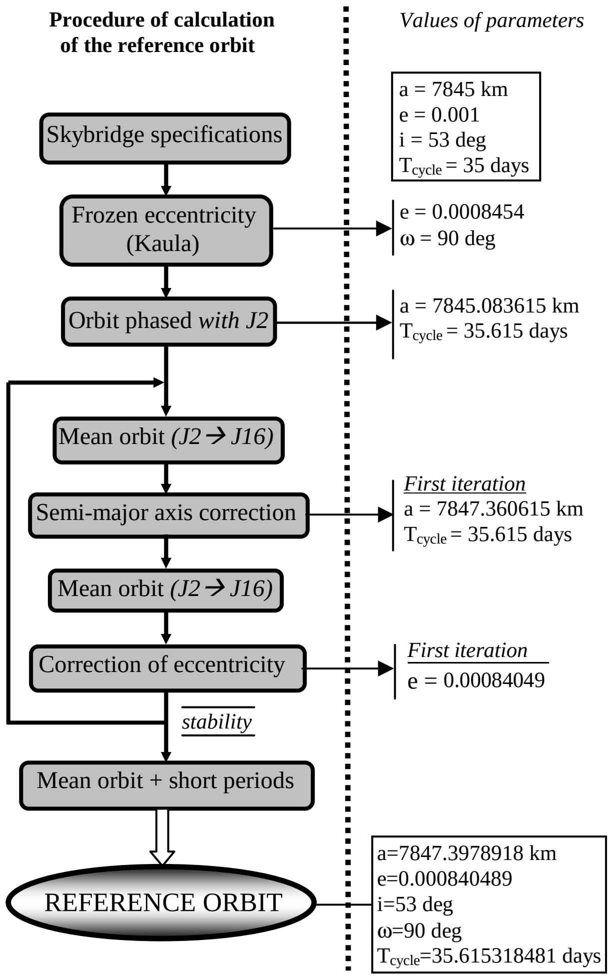

figure 3: Algorithm of calculation of the reference orbit

The experience acquired has given confidence in the feasibility of the station keeping with a reference orbit defined in mean parameters, which is a key of the SkyBridge project.

The analytical orbit model which will be loaded in the terminals is still on study and final refinements could be added.

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