A STUDY OF THREE SATELLITE CONSTELLATION DESIGN ALGORITHMS

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Abstract

In the last few years, the number of satellite constellation concepts has increased dramatically, particularly in the commercial communications sector. This large growth in the number of satellite constellations has made good satellite constellation design imperative.

To address this need, GMV has studied and tested three satellite constellation design algorithms. These algorithms are: an inclined, symmetric constellation algorithm (often known as Walker constellations), a polar, non-symmetric constellation algorithm, and an adaptive random search algorithm used for satellite constellation design optimization. These three algorithms address various needs for the three main types of satellite constellations: navigation, earth observation and communications. Tests for all of the algorithms were performed using constellations which are either in use, or in the development phase.

These algorithms can be used either separately or together to form a powerful, two-step constellation optimization design tool.

Key words: Satellite Constellation Design, Constellation Algorithms, Constellation Design Optimization

Introduction

Until the last decade, a single spacecraft was usually sufficient to perform most space missions. Within the last decade, however, it has become clear that for some applications, a single spacecraft can not fulfil the mission objectives. One way to fulfill these mission objectives is to use a series of satellites in different orbits -- in orbit terminology, a "satellite constellation".

Over the last decade, satellite constellation concepts have been envisioned for a broad range of uses. Initially starting with navigation and positioning constellations (NAVSTAR/GPS, Glonass, GNSS), satellite constellations concepts have branched out into telecommunications for direct telephony, mobile message systems and broadcasting (Iridium, Globalstar, Teledesic, ICCO/Immarsat-p, Ellipso, ECO), and into Earth Observation (DMSP 5D-2, GOES, TIROS-N, FUEGO). The last few years have seen a virtual explosion of satellite constellation issues.

The mission analysis, design and planning of all these new satellite constellations has been carried out using in-house software tools and algorithms. There exist no general algorithms or software tool to do this. Towards the end of developing a commercial software tool – called ORION[2] -- for satellite constellation design and planning, GMV performed a study of constellation design algorithms for the ESA-GSTP contract. This study resulted in the development and testing of three of these algorithms.

GMV chose the three design algorithms to be incorporated in ORION based on the following criteria. First of all, the algorithms must cover the definition of satellite constellations whose primary service objectives are either:

- Global or regional earth observation
- Global or regional telecommunications
- Global navigation and location networks

These three encompass the most common mission objectives for satellite constellations.

Second of all, in order to take into account a wide range of possible mission requirements and objectives, the design algorithms must permit the definition of satellite constellations for the following:

- Continuous coverage of all the Earth’s surface
- Continuous coverage over an entire latitude band (i.e. within the δ to φ degrees of latitude,

where δ and φ may vary between –90 degrees and 90 degrees in latitude). If this latitude band is symmetric with respect to the equator (that is, φ is equal to -δ), the specific problem of equatorial coverage is analyzed.

- Continuous polar or high latitude band coverage
- Continuous or optimized coverage of a certain geographic area (i.e. a latitude-longitude box)

After investigation, GMV finally decided that the following three constellation design algorithms would be developed and tested:

1. A **symmetric, inclined constellation design algorithm** based on a Thomas Lang algorithm for Walker constellations,
2. A **non-symmetric, polar constellation design algorithm** based on the "*Streets-of-Coverage*" principle, and
3. An **adaptive random search constellation design algorithm** for optimizing constellation design with respect to such parameters as the DOPs (Dilutions of Precision), revisit time and satellite failures.

These three algorithms were developed and then tested using real constellation concepts. This was done in order to reconstruct the actual design processes of the original constellation designers, and to see if the final constellation design could be reproduced with the constraints which were known to GMV.

This paper will present an introduction to the theory behind each of the three satellite constellation design algorithms, an overview of the algorithms, the results obtained in testing the algorithms, and finally a comparison of each of the algorithms.

Symmetric, Inclined Constellation Algorithm

1. Theory

The traditional approach to the optimization of a satellite constellation has been formulated as the minimization of the number of satellites which satisfy a given geometrical coverage criterion. Typically this means guaranteeing the required level, or fold, of continuous global or zonal coverage of the Earth above a given minimum elevation threshold, or above a minimum altitude. The symmetric, inclined constellation algorithm is one of the traditional approaches. It provides an arrangement of symmetric, circular orbits which is often referred to as a *Walker Constellation*, based on the important contributions of J.G. Walker[9-10].

Walker developed a notation for labeling orbits that is commonly used as a starting point for constellation design. The **Walker Delta Pattern**[9-10] constellation is identified by 4 parameters:

- **i** , the constellation inclination
- **T,** the total number of satellites
- **P,** the total number of orbital planes
- **F,** the relative spacing between satellites in adjacent planes

The number of satellites per plane, S, is given as S=T/P.

Figure 1 displays a typical delta pattern arrangement of satellites. This delta pattern consists of four orbit planes (A, B, C and D) with a common inclination angle δ with respect to the reference plane.

In Walker constellations, all satellites are placed in circular orbits at the same altitude. All of the orbit planes have the same inclination i., and the ascending nodes of the P orbital planes are uniformly distributed around the equator at intervals of 360°/P. Within each orbital plane, the S satellites are distributed at intervals of 360°/S. The only remaining issue is the relative phase between satellites in adjacent orbital planes.

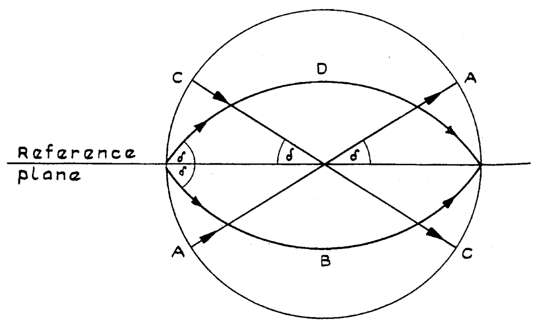

Figure 1: Walker Delta Pattern

For this purpose, Walker defined the phase difference (ΔΦ) in a constellation as the angle in the direction of motion from the ascending node of a satellite in one place to the nearest satellite in the next most westerly plane. The relative angular shift between satellites in adjacent orbital planes is equal to F*(360°/T). F may assume any value between 0 and (P – 1).

It should be noted that the symmetry of the orbital configuration leads to frequent recurrence of similar satellite patterns during each orbital period.

2. Algorithm

The Symmetric, Inclined Constellation Design Method optimizes a Walker constellation pattern using an algorithm developed by Thomas J. Lang in 1993[3-5]. This algorithm was chosen because it significantly reduces the CPU time needed to run.

The user needs very little information about the constellation in order to use this. He or she may know as little as what type of coverage is desired (either global or within a certain latitude band), and the Earth grid resolution. However, the more parameters a user knows, the more useful the results of the algorithm will be and the more quickly the algorithm will run.

The additional information that the user may specify consists of: minimum elevation angle, minimum altitude, minimum and maximum number of satellites, minimum and maximum orbital plane inclinations, desired fold(s) of coverage, and the number of orbital planes.

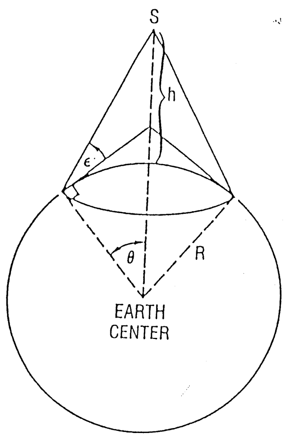

Figure 2: Single Satellite Viewing Geometry

Essentially, this algorithm optimizes the Earth central angle θ for constellations of T circular orbit satellites while still achieving continuous coverage in the desired latitude band. Figure 2 shows the satellite viewing geometry. The elevation angle ε of the satellite viewing cone, the central angle radius of earth coverage θ, and the satellite altitude h are related by the coverage equation as follows:

$$\cos(\theta + \varepsilon) = \cos\varepsilon / (1 + h / R_E) \qquad (1)$$

where R_E is the radius of the Earth. The following observations can be inferred from this equation:

- if h is fixed, ε decreases if θ increases;
- if ε is fixed, h increases if θ increases.

Thus, an increase in θ always brings about a negative effect, either in terms of altitude or minimum elevation angle.

The constellation with the lowest required value of θ will allow the lowest operating altitude for a fixed value of ε. Conversely, if satellite altitude is fixed, the lower operating limits on elevation angle ε will be maximized.

The value of the central angle radius of earth coverage θ, which is required for the constellation to achieve continuous global coverage, is used as a measure of the efficiency of the constellation configuration. This means that θ is the performance index that characterizes the global system quality and the optimization method consists of minimizing θ. The lower the value of θ for fixed T, the more efficient the constellation.

In order to obtain the lowest value of θ, the T satellites are propagated in time over an Earth grid. The smallest value of θ is then determined which ensures that all test points are visible to at least N satellites (where N is the desired multiplicity of coverage) for all times. The satellite constellation which results after many iterations is the optimal symmetric, inclined satellite constellations for continuous global or zonal coverage. One to four fold coverage can currently be handled.

It should be noted that for every prograde solution (T/P/F, i, θ), there is a mirror image retrograde solution (T/P/F*, i*, θ*) with:

$$F^* = P - F \qquad (2)$$

$$i^* = 180° - I \qquad (3)$$

$$\theta = \theta^* \qquad (4)$$

For most applications, only the prograde solution is of practical interest.

This algorithm, in spite of being the quickest, is still very CPU intensive. There is a nearly exponential relationship between the number of satellites in the constellation and the time it takes to determine the optimum θ and inclination. In other words, it takes less than a minute to optimize the 5/P/F constellations, about 10 minutes to optimize the 12/P/F constellations, about 1.5 hours to optimize the 70/P/F constellations, and about 5 hours to optimize the 100/P/F constellations. All of these times assume the same resolution of the Earth grid (4° in latitude) with global coverage, and result from using a Pentium PC.

3. Results

In order to test the efficacy of the symmetric, inclined constellation design algorithm, the GPS navigation constellation was used as a test case to see if it was possible to reproduce the various steps of the constellation design process. The main requirements applied to design the GPS constellation were as follows:

- Continuous 4-fold coverage of the entire surface of the Earth (needed for signal triangulation)
- An altitude of 20200 Km (a circular, MEO to avoid Doppler shift in the signal to the receiver and to have a 12 hour period)
- A 7,5-degree minimum elevation angle (needed for the receivers)

In order to verify the results, GMV first investigated the evolution of the GPS constellation design.

The first GPS constellation was a Walker constellation with 18 satellites in 3 planes, inclined to 55°. Although this pattern guaranteed worldwide continuous coverage by at least 4 satellites, it proved to be too sensitive to satellite failures. Thus, three spare satellites were added, one in each orbital plane, obtaining a configuration with 21 spacecraft in 3 orbital planes. Then, extensive computations with 1, 2 and 3 satellite failures led to the current constellation of 24 satellites in 6 planes characterized by an inclination of approximately 55°.

The results of the symmetric, inclined algorithm reproduce the various phases of the GPS constellation design.

As seen in Table 1, the first possible solutions with continuous, global, 4-fold coverage have 16 and 17 satellites in 16 and 17 planes respectively. These solutions are ruled out since they entail too large a number of orbital planes.

The first acceptable solution is the 18/3/1 inclined to 55.5°. Remembering this solution's susceptibility to satellite failure, however, the next acceptable solution is the 21/3/2 inclined to 54.3°. Finally, keeping in mind the extensive satellite failure analysis undertaken by GPS constellation designers, the current configuration, 24/6/4 inclined to 54.4°, emerges. It should be noted that there is another alternate configuration possible, 24/3/2 inclined to 54.5°. That this configuration was not chosen is probably due to the extensive satellite failure analysis.

Table 1: Results from Symmetric, Inclined Algorithm for Possible GPS Constellations

T	P	F	i (deg)	θ (deg)
16	16	10	56.6	68.516
17	17	11	52.7	66.093
18	2	*****	*****	*****
18	**3**	**1**	**55.5**	**63.544**
18	6	5	61.9	64.721
18	9	5	51.1	65.825
18	18	15	55.4	65.696
19	19	5	57.2	62.224
20	2	*****	*****	*****
20	4	3	56.5	64.664
20	5	4	58.0	62.207
20	10	7	58.6	61.756
20	20	8	55.7	61.396
21	**3**	**2**	**54.3**	**57.985**
21	7	6	61.9	61.382
21	21	5	59.8	60.999
22	2	0	45.0	67.431
22	11	5	56.0	58.987
22	22	16	57.6	57.366
23	23	14	54.2	56.094
24	2	0	45.0	63.409
24	3	2	54.5	54.455
24	4	2	59.4	59.440
24	**6**	**4**	**54.4**	**54.485**
24	8	4	57.3	56.674
24	12	5	55.4	54.359
24	24	20	55.8	54.698

**** indicates that no solution was possible

Polar, Non-Symmetric Constellation Algorithm

1. Theory

While developing the Symmetric, Inclined Constellation Design Algorithm, GMV realized that certain types of constellations are better optimized with a Polar, Non-Symmetric Design Method using a *Streets-of-Coverage* approach. In order to treat this weakness, GMV decided to develop a Polar, Non-Symmetric Design Algorithm based on the *Streets-of-Coverage* approach with multiple visibilities. This algorithm is especially tailored to provide good results for large constellations characterized by 1-fold coverage.

The *Streets-of-Coverage* concept is the following. Multiple circular orbit satellites at the same altitude are placed in a single plane so as to create a *Street-of-Coverage* which is continuously viewed (see Figure 3, which illustrates one street-of-coverage in the case of a non-polar orbit). The objective is then to determine analytically how many such streets (i.e., planes of satellites at the same inclination) are required to cover the zone of interest or the globe.

When the *Streets-of-Coverage* design technique is applied to a polar constellation, the resultant optimal configuration is a polar satellite network in which the motion of a spacecraft in one orbital plane is synchronized with that of the spacecraft in the adjacent planes (Phased Polar Constellation).

2. Algorithm

The Polar, Non-symmetric Constellation Design Algorithm implements an analytic method for identifying families of circular polar orbit constellations using minimal total numbers of satellites which can provide a desired fold of coverage n at or above a user-defined latitude (λ_n).
Because this algorithm is analytical instead of numerical (like the Inclined, Symmetric algorithm), it runs very quickly – typically in a matter of seconds.

The optimal configuration for a Polar, Non-symmetric satellite constellation is determined using the "*Streets-of-Coverage*" method. The algorithm proposed by Ullock and Shoen[8] for continuous 1-fold global coverage is implemented. In order to achieve multiple folds of coverage (2-, 3- and 4-fold), this algorithm has been integrated with Rider's algorithm[1,5-6]. The algorithm obtained provides very accurate results for 1-fold continuous global coverage and less accurate results for multiple levels of coverage. The results in terms of central angle radius of earth coverage θ tend to be very good.

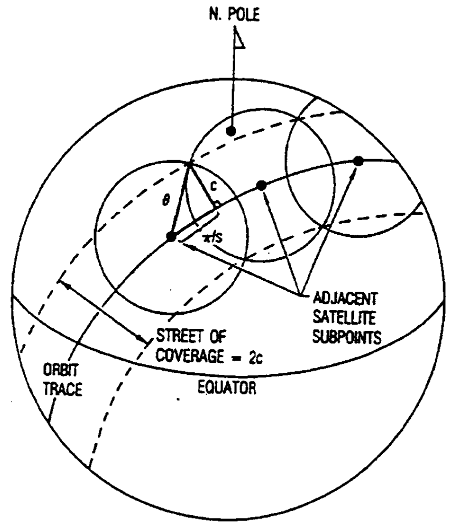

Figure 3: Continuous Street of Coverage from a Single Orbital Plane

The user needs to provide very little information about the constellation in order to use the Polar, Non-symmetric Constellation Design Algorithm. The only necessary input information is the type of coverage desired (either global or above a certain latitude). The additional information that the user may specify consists of minimum elevation angle and/or altitude, along with minimum and maximum values of the following parameters: number of satellites, desired fold(s) of coverage, and number of orbital planes (p).

The basic assumption of the optimization method stems from Rider's algorithm: the constellation can be arranged so that there are 2(p – 1) co-rotating interfaces and two counter-rotating interfaces. The resultant configuration is defined as "non-symmetric" because the orbit separation (delta RAAN) between co-rotating planes is different from the orbit separation between the two counter-rotating interfaces (see Figure 4).

The optimization methodology is carried out using a series of analytical relations which provide the values of the variables that define the constellation arrangement. Optimally phased polar constellations are derived by minimizing the central angle of coverage θ. As output, the program provides the optimal values of the angular spacing between co-rotating orbits (ϕ) and of the relative phase between satellites in adjacent planes (ω), along with the optimum θ. Note that, since a polar network is taken into account, the inclination of the orbital planes is assumed to be 90 degrees.

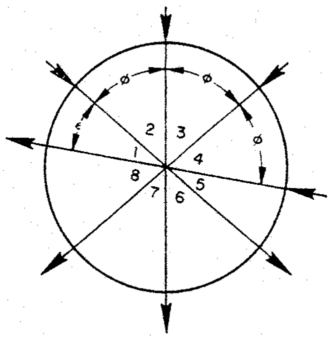

Figure 4: Non-symmetric Polar Network

The first step of the optimization process consists in determining the optimum central angle of coverage θ. This is obtained by solving equation (5) employing the bisection method of approximating roots, using an ending tolerance in the solution of 10^{-6} degrees.

$$(p-1)(\theta+c_j)+(c_1+c_j) = 2p\arcsin\left[\cos\lambda_n\cos\left(\frac{p-k}{2p}\pi\right)\right] \quad (5)$$

If a value of optimum θ is obtained (that is, if equation (5) has a solution), then the program computes the optimal value of ϕ and the optimal value of ω. These angular quantities are determined using some relations provided by Ullock and Shoen[8] and adapting them to the case of multiple folds of coverage.

For the co-rotational segments, reference is made to Figure 5. It is seen that the three satellites located at points A, B and C have fields of view θ which intersect at point T, located at latitude λ_T. The latitude of the three satellites are designated as λ_1, λ_2 and λ_3 respectively. Since it is desired to maximize the incremental longitude β between points T and C, the great circle arc TC must be perpendicular to the arc of longitude OC. The arc DT is a perpendicular to longitude OB and bisects AB. The derivation of the equations holds only in the region in which satellites 1, 2 and 3 are on the same "side" of the pole and thus is limited to $\theta \leq 90° - \lambda_T$.

In order to design the optimal polar network that provides a desired fold of coverage n at or above a specified latitude λ_n, the most critical condition occurs when $\lambda_T = \lambda_n$. In fact, the main requirement is to design the synchronization between co-rotating planes so as to ensure continuous coverage at or above latitude λ_n.

From spherical trigonometry relations applied to triangles DTB, DTO and CTO and considering DT = c_j, the following relations are obtained:

$$\phi = \arcsin\left\{\frac{\{1-[\cos^2\theta/\cos^2(j\pi/s)]\}^{1/2}}{\cos\lambda_n}\right\} + \arcsin\left(\frac{\sin\theta}{\cos\lambda_n}\right) \quad (6)$$

$$\omega = \lambda_3-\lambda_2 = \arcsin\left(\frac{\sin\lambda_n}{\cos\theta}\right) - \arcsin\left[\frac{\sin\lambda_n}{\cos\theta}\cos(j\pi/s)\right] + \pi/s \quad (7)$$

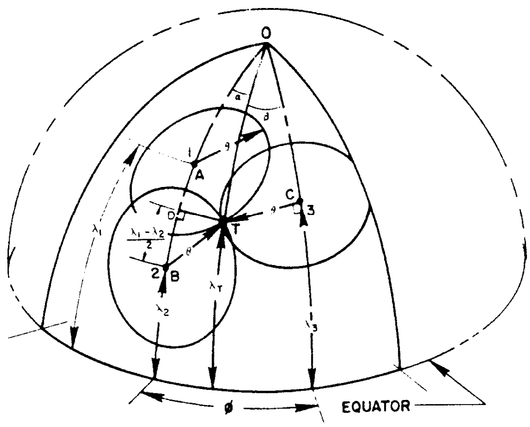

Figure 5: Synchronization of satellites in adjacent orbital planes

As underlined, these relations hold if $\theta \leq 90° - \lambda_n$. In addition, these relations provide more accurate results when $\sin\theta / \cos\lambda_n$ is small, that is, when a large number of satellites is taken into account (θ is small), particularly when λ_n is not 0 degrees. This was also pointed out in the paper by Adams and Rider[1] who analyzed arbitrarily and optimally phased polar orbit constellations and provided a much more complex algorithm to obtain minimum total numbers of satellites to achieve continuous single or multiple coverage above a specified latitude. Moreover, in their work Adams and Rider also notice that in general, to maximize the number of co-rotating interfaces in a polar network, their ascending nodes should be distributed over kπ radians with an approximate value for the angular spacing between co-rotating orbits (ϕ) of kπ/p radians.

This approximation is used to compute ϕ when equation (6) cannot be applied (because $\theta > 90° - \lambda_n$). Similarly, when equation (7) cannot be used, the relative phase between satellites in adjacent planes (ω) is considered to have an approximate value of π/s. In some cases these approximations are considerable and the corresponding values of ϕ and ω are not very accurate if compared with the results provided by Adams and Rider[1]. However, the results in terms of central angle θ tend to be very good and GMV has largely been able to reproduce the values of θ provided by Adams and Rider.

3. Results

In order to test the efficacy of the Polar, Non-Symmetric Constellation Design Algorithm, Motorola's telecommunications constellation Iridium was used as a test case. GMV wanted to see if it was possible to reproduce the various steps of the constellation design process. The main requirements applied to design the Iridium constellation were as follows:

- Continuous 1-fold coverage of the entire surface of the Earth
- A 10-degree minimum elevation angle, which is a very common requirement for satellite telecommunication systems.

In order to verify the results, GMV first investigated the evolution of the Iridium constellation design.

The first Iridium constellation consisted of 77 satellites, at 765 km, in a polar orbit, and the first plane had a RAAN of 27° which yielded a counter-rotating plane separation of 18°. There were seven planes and 11 satellites in each plane. This constellation evolved and became the current Iridium orbital configuration which is characterized by 66 satellites, arranged in six planes, and containing 11 satellites each. The orbital altitude is 780 Km and the inclination is 86.4° — probably to avoid the risk of collisions between spacecraft.

Using the Polar, Non-Symmetric algorithm and the constraints outlined above, GMV was able to reproduce the initial configuration of the Iridium constellation perfectly (see Table 2). Given the above constraints, the first constellation configuration possible with an altitude of 765 km is the configuration with 77 satellites in seven planes with a co-rotating planar separation of 27°.

Table 2: Results from the Polar, Non-Symmetric Algorithm for the Initial Iridium Constellation

T	P	S	θ(deg)	ϕ(deg)	ω(deg)	H (km)
77	7	11	18.457	27.114	16.364	766

In all of the following tables for Iridium, the notation is the same. T is the total number of satellites, P is the number of planes, S is the number of satellites per plane, θ is the Earth central angle, ϕ is the angle between two co-rotating planes, ω the relative phase between satellites in adjacent planes and H is the minimum altitude possible for the given configuration.

In attempting to reproduce the new configuration of Iridium, GMV assumed the change was brought about by a reduction in the minimum elevation angle. This worsens the minimum altitude somewhat, but results in a significant savings of spacecraft (66 instead of 77). In order to verify this, the optimization algorithm was run again with the following requirements:

- Continuous 1-fold coverage of the entire surface of the Earth
- An 8-degree minimum elevation angle, which is a common requirement for satellite telecommunications systems.

As seen in Table 3, the first satellite configuration which the algorithm produced, was the configuration of 66 satellites in six planes with 31.4 degrees between co-rotating planes. The altitude associated with configuration, however, was slightly lower than the current Iridium configuration.

Table 3: Results from the Polar, Non-Symmetric Algorithm for an Intermediate Iridium Constellation

T	P	S	θ(deg)	ϕ(deg)	ω(deg)	H (km)
66	6	11	19.907	31.402	16.364	769

In order to clear up this last bit of confusion regarding the Iridium configuration, GMV did a literature search on the exact elevation angle of the current Iridium constellation. A value of 8.2° was found. Running the Polar, Non-Symmetric algorithm again with an elevation angle of 8.2°, gave the following results (see Table 4).

Table 4: Results from the Polar, Non-Symmetric Algorithm for the Final Iridium Constellation

T	P	S	θ(deg)	ϕ(deg)	ω(deg)	H (km)
66	6	11	19.907	31.402	16.364	780

This gives the Iridium configuration exactly. In fact, if the algorithm is run with an altitude of 780 Km and a minimum elevation angle of 8.3° degrees, the first available solution is the one with 70 satellites and the solution with 66 satellites is no longer available.

Advanced Adaptive Random Search Algorithm

1. Theory

The two constellation design algorithms described above are both well-known and widely used methods of constellation design. However, both of these methods have certain limitations. The most important of these limitations are their inability to take anything but classical, geometric design factors into consideration, and their inability to consider a latitude-longitude box for optimal coverage. Both of the previous algorithms can be used for regional coverage, but the regional coverage must be over an entire latitude band. It cannot be of a latitude-longitude box. Nor can either of the previous algorithms take satellite failures or hybrid constellation possibilities into account in the constellation design. In order to address these limitations, GMV developed the Adaptive Random Search algorithm for constellation design.

The Adaptive Random Search algorithm is a variation of a genetic algorithm. The basic theory underlying genetic algorithms is the following. Given an individual with certain traits which the user wants optimized, the algorithm does a Monte Carlo simulation to propagate these traits in the offspring of the original individual. If one of the offspring is “better”, i.e. more optimized, than the parent individual, this offspring is then chosen to have offspring in order to see if the trait to be optimized can be improved upon again. If the parent individual is “better” than any of the offspring, then a Monte Carlo simulation generates more offspring. This cycle repeats either a certain number of times or until the trait has reached an optimum decided by the user. This process can be repeated any number of times for any number of traits to be optimized.

If several traits are to be optimized and one trait has a much stronger influence than any other trait on the optimization of the individual, then certain problems can arise. For example, if Trait 1 has a disproportionately large effect on the optimization as compared to Trait 2, then if both traits are optimized simultaneously, the results are heavily dominated by the optimization of Trait 1. This can result to the point that Trait 1 is optimized, but Trait 2 is not at all – it may be a local minimum or some other value.

In order to avoid this problem, a subset of genetic algorithms called *adaptive random search algorithms* was developed at GMV by Miguel Romay et al. [7] and consequently adapted for use in this study. In adaptive random search algorithms, Trait 1 of the above example is optimized first and completely separately from Trait 2 (or any other traits). The optimized value of Trait 1 is then used for the optimization of the other traits, allowing for slight variations of Trait 1 to see if this can be bettered when optimized with Trait 2. The adaptive random search method should be defined as a genetic algorithm where the population is reduced to one chromosome. Crossover is not possible and only mutation can occur.

After testing, satellite constellations were found to have one variable with a much stronger influence on the optimization process than any other: inclination angle. This was independent of the function to be optimized.

2. Algorithm

In any optimization problem the first step is to define clearly the function that has to be optimized. In order to be able to choose the best solution among two (or more) possible solutions. This function must take into account that different performances have to be provided at different locations. This algorithm allows for the optimization of the following functions:

- VDOP, for the vertical positioning accuracy
- HDOP, for the horizontal positioning accuracy
- PDOP, for the 3-dimensional positioning accuracy
- TDOP, for the time determination
- GDOP, for the total position and time accuracy
- Mean Revisit Time for a particular latitude-longitude box
- Maximum Revisit Time for a particular latitude-longitude box
- Satellite Failures

Any of the above functions can be used in the calculation of the cost function, CF (see eq. 8). The cost function only considers the variable to be optimized that is calculated in a set of points chosen by the user over a period of 24 hours. The cost function is calculated as the sum of the sums of the weighted, squared, maximum variables to be obtained at each of these points.

$$\begin{aligned} CF = {} & w_V \sum_{i=1}^{N_P} w_i VDOP_{MAX}^2 + w_H \sum_{i=1}^{N_P} w_i HDOP_{MAX}^2 \\ & + w_P \sum_{i=1}^{N_P} w_i PDOP_{MAX}^2 + w_T \sum_{i=1}^{N_P} w_i TDOP_{MAX}^2 + \\ & w_G \sum_{i=1}^{N_P} w_i GDOP_{MAX}^2 + w_{MXT} \sum_{i=1}^{N_P} w_i MAXT_{MAX}^2 + w_{AVT} \sum_{i=1}^{N_P} w_i AVGT^2 \end{aligned} \quad (8)$$

where N_p is the number of points on the earth surface, $VDOP_{MAX}$ is the maximum VDOP obtained on each of these points during the selected period (and so on for each of the possible cost functions listed above), and w_i

is the weight that the user gives to any of the points. An example of the set of points, and their associated weights, used for some preliminary optimizations is showed in Figure 6. The user can control the function to be optimized by putting more control points in the most important areas, or giving more weight to those points.

It should be noted here that the cost function evaluation requires a significant amount of computational time. A significant effort has been devoted to minimize the required time to evaluate this function, as every time that any of the parameters of the constellation (typically thousands of times) is changed, the cost function must be re-evaluated.

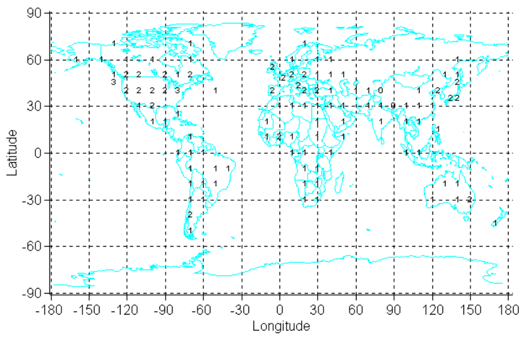

Figure 6: Map of Weighted Points Used as Input for the Adaptive Random Search Constellation Algorithm

In order to use this algorithm, the user must specify the six Keplerian orbital elements for all of the satellites in the constellation:

- Semi-major axis
- Eccentricity
- Inclination
- Right Ascension of the Ascending Node (RAAN)
- Argument of Perigee
- Mean Anomaly

If the user intends to use either the mean or maximum revisit time in the cost function, then he or she must also specify the elevation angle of each satellite.

As is obvious from the input necessary to use this algorithm, the Adaptive Random Search algorithm has a fundamental difference with the two previous algorithms described in this paper. Namely, the user must already have a very good idea of the constellation design desired and must have a very specific way in which he or she wishes to optimize the constellation. Because of this difference, the Adaptive Random Search algorithm is best used as a second step in constellation design after first using either of the previous algorithms.

The logic flow of this algorithm is shown in Figure 7.

3. Results

In order to test the efficacy of the Adaptive Random Search algorithm, the optimization of the Earth observation constellation, **Fuego**, was considered as a test case. Fuego is a satellite constellation envisioned primarily to detect and monitor forest fires in real time with coverage that is optimized over the Mediterranean basin (although a certain service is intended to be provided over land masses in the latitude band ± 60°).

The optimization of Fuego was done in two steps so as to show how the classical design algorithms and the advanced design algorithm can be integrated to perform a detailed study.

In the first step, the Symmetric, Inclined Constellation Design Algorithm was implemented to design a constellation with following requirements:

– continuous 1-fold coverage of the latitude band ± 60 degrees
– 12 satellites

The results of the Symmetric, Inclined Constellation Design Algorithm resulted in the configuration 12/3/2 inclined to 49 degrees being selected for further optimization. The current configuration of the Fuego constellation is 12/3/2 inclined to 47.5 degrees.

In the second step, the constellation has been optimized so as to minimize the maximum and the mean revisit time over the Mediterranean basin (prime service area) and most of Europe. The symmetric configuration provided by the previous step (12/3/2) is used as the input of the Adaptive Random Search Algorithm.

In the second step, the Adaptive Random Search Algorithm was used to optimize the constellation with the following constraints:

– All planes had to have the same inclination
– The number of planes was fixed at three
– The orbital altitude was fixed at 700 km
– A 20° minimum elevation angle for all satellites

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COMPUTE COST FUNCTION USING INITIAL CONSTELLATION

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OPTIMIZE CONSTELLATION WITH RESPECT TO INCLINATION (IF INCLINATION IS VARIABLE): RANDOM SEARCH WITH A UNIFORM DISTRIBUTION AND THE STANDARD DEVIATION PROVIDED BY THE USER

↓

OPTIMIZE LONGITUDE OF GEO S/C (IF ANY): RANDOM SEARCH WITH UNIFORM DISTRIBUTION AND USER-DEFINED STANDARD DEVIATION

↓

OPTIMIZE COST FUNCTION USING UNIFORM DISTRIBUTION ONCE

↓

→ OPTIMIZE COST FUNCTION USING A GAUSSIAN DISTRIBUTION UNTIL ENOUGH CONVERGENCE HAS BEEN DETECTED

↓

OPTIMIZE COST FUNCTION ONCE MORE USING UNIFORM DISTRIBUTION TO CHECK IF THE RESULT OF THE GAUSSIAN DISTRIBUTION OPTIMIZATION IS A LOCAL MINIMUM OR NOT

↓

IF UNIFORM DISTRIBUTION GETS BETTER RESULTS, RUN GAUSSIAN DISTRIBUTION AGAIN

←

↓

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Figure 7: Adaptive Random Search Algorithm Logic Flow

– Mean revisit time over the Mediterranean was the most important function to optimize

– Maximum revisit time over the Mediterranean was weighted 20% of the mean revisit time

The maximum and mean revisit time were computed over a simulation period of 28 hours and a region which covers the Mediterranean basin and most of Europe. This simulation period has been selected because the Fuego satellites complete an integer number of orbits in 28 hrs (17 revolutions).

The resulting optimized constellation was changed from the original constellation in the following ways:

- The inclination was changed from 49° to 45.73°.
- In the first orbital plane, the mean anomaly was shifted by 2.11°, in the second plane by 3.36°, and in the third plane by 15.72°.
- The RAAN of the first orbital plane was changed by 11.57°, the RAAN of the second orbital plane was changed by 25.79°, and the RAAN of the third orbital plane was changed by 40.87°.

Table 5 provides a comparison of the mean and maximum revisit times of the initial and final designs.

Table 5: Mean and Maximum Revisit Times Comparison for Fuego

	Mean Revisit Time	Max Revisit Time
Initial Design	22.867 min	320 min
Optimized Design	19.184 min	237 min

Comparison of the Algorithms

Comparing the results provided by the Inclined, Symmetric and the Polar, Non-Symmetric Constellation Algorithms the following conclusions were reached.

For single-fold continuous global coverage with more than 20 satellites, the optimally phased polar constellations appear to be more efficient. In all other cases (1-fold coverage with less than 20 s/c and multiple levels of continuous coverage), the symmetric, inclined constellations are more efficient. In most cases, for the same number of satellites, the symmetric, inclined constellations offer continuous global coverage at a lower altitude (correspondingly lower θ).

For both the Inclined, Symmetric and the Polar, Non-Symmetric Constellation algorithms, the user needs very little information about the constellation in order to use these algorithms. He or she may know as little as what type of coverage is desired (either global or within a certain latitude band). Neither of these algorithms will ever output a hybrid constellation configuration.

The Adaptive Random Search algorithm is fundamentally different from the Inclined, Symmetric and the Polar, Non-Symmetric algorithms. In order to use the adaptive random search algorithm, the user must already have a very specific constellation in mind. This constellation may be inclined, polar, symmetric, non-symmetric, LEO, MEO, HEEO, GEO or any hybrid combination thereof.

What is important is that the user knows and is able to define the six Keplerian orbital elements of each satellite in the constellation in the first instant of time. If the user does not know the orbital elements for each spacecraft, then this algorithm will not function.

Additionally, the user must know exactly where constellation coverage is desired and what weighting, if any, to give to each area, as well as how he or she would like to optimize the design of the constellation.

Fundamentally, the Adaptive Random Search algorithm is not a pure constellation design algorithm. It is a cross between a design algorithm and a performance algorithm. The results of the algorithm are an improved constellation design. In order to obtain these results, certain performances of the constellation are optimized. This is the fundamental difference between the Adaptive Random Search constellation design algorithm, and the Inclined, Symmetric and Polar, Non-Symmetric constellation design methods.

5. Conclusions

The Inclined Symmetric, Polar Non-Symmetric, and Adaptive Random Search algorithms form a powerful two-step tool for designing satellite constellations. First, to obtain a design using classical geometric methods, the user may run either the Inclined Symmetric or the Polar Non-Symmetric algorithm according to his or her requirements. Then, in order to further refine the constellation design with regard to certain performance parameters, the user may run the Adaptive Random Search Algorithm. The result of this two-step process is an optimized constellation.

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