Precise GNSS-2 Satellite Orbit Determination
Based on Inter-Satellite-Links

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Abstract

In the framework of the Global Navigation Satellite System 2 (GNSS-2), the achievable orbit determination accuracy of geosynchronous GNSS-2 satellites using Inter-Satellite-Links (ISL’s) is analyzed. The ISL geometry yields tracking conditions for the relative satellite distance, velocity and acceleration of up to 80,000 km, 6 km/s and 0.02 km/s$^2$. The geometrical dilution of precision of the GNSS-2 satellites is computed and kinematic position solution errors of 6 m in radial direction are derived, that violate the expected GNSS-2 requirement of 0.2 m. For dynamic orbit determination a GNSS-2 tracking concept is proposed that comprises a single ground station, that tracks a single master satellite, while the master tracks all slaves. A consider covariance analysis proves the feasibility of the concept, leading to radial position errors well within 0.1 m with total position errors less than 2 m. Thus, the proposed tracking concept serves as a highly accurate and conceptual simple system for GNSS-2.

Key words: Global Navigation Satellite System, Inter-Satellite-Links, Orbit Determination.

Introduction

The future GNSS-2 is a second generation satellite-based system providing an enhanced navigation service that fully meets the needs of the civilian community. In contrast to its predecessor GNSS-1, a satellite augmentation of the GPS and GLONASS systems, it is independent from GPS and is not controlled by a single nation.

While CNES and ALCATEL assume a LEO space segment for GNSS-2, AEROSPATIALE favors MEO concepts and this study is focused on a hybrid geosynchronous satellite concept, that is mainly considered at DASA and ESA. In particular, the investigations assume a sample European subset of the GNSS-2 space segment, comprising three geostationary satellites (GEO) at east longitude -20°, 15°, and 50° (G1, G2, G3), respectively, as well as four inclined geosynchronous satellites (IGSO) in four orbit planes (I1-I4) with 70° inclination and a common equator crossing at 15° east longitude (Figure 1).

Figure 1 Sample European GNSS-2 space segment.

Based on the considered GNSS-2 space segment the achievable satellite orbit determination accuracy is analyzed. In a first, purely kinematic, approach the investigations focus on the relative motion of GEO and IGSO satellites. An analytic model of the relative satellite motion is given and maximum relative position, velocity and acceleration figures are derived, that may become part of the ISL tracking system specifications. The computation of geometric dilution of precision values leads to an assessment of the accuracy of instantaneous kinematic position solutions from ISL tracking.

In a second approach, a consider covariance analysis is performed to cover both statistical and systematic errors of a dynamic orbit determination process and to provide realistic accuracy figures for the GNSS-2 satellite position and velocity. This approach is evaluated both for a complex tracking scenarios with ISL tracking links between all satellites as well as for a reduced master/slave concept. Comparing and evaluating the resulting accuracy differences leads to a proposed tracking concept for GNSS-2.
Relative Satellite Motion in the Orbital Frame

To analyze the relative motion of GEO-IGSO and IGSO-IGSO satellites, the relative position of satellite pairs in an orbital \((H, C, L)\)-frame is computed. This is accomplished using a triad, spanned by the unit vectors \(e_1\) (radial direction), \(e_2\) (cross-track) and \(e_3\) (along-track). A Keplerian approximation of the IGSO triad (GEO triad for \(i = 0\)) for negligible eccentricity \(e\) is given by

\[
\begin{align*}
e_1 &= \begin{pmatrix}
+ \cos \Omega M - \sin \Omega \sin M \cos i \\
+ \sin \Omega \cos M + \cos \Omega \sin M \cos i \\
+ \sin M \sin i
\end{pmatrix} \\
e_2 &= \begin{pmatrix}
+ \sin \Omega \sin i \\
- \cos \Omega \sin i \\
+ \cos i
\end{pmatrix} \\
e_3 &= \begin{pmatrix}
- \cos \Omega \sin M - \sin \Omega \cos M \cos i \\
- \sin \Omega \sin M + \cos \Omega \sin M \cos i \\
+ \cos M \sin i
\end{pmatrix}
\end{align*}
\]

Making use of the IGSO position unit vector \(e_1\), the relative \((H, C, L)\)-position of an IGSO satellite with respect to a GEO satellite is given by

\[
\begin{align*}
H_G^I &= -1 + \cos M_1' \cos M_2 + \cos i \sin M_1' \sin M_2 \\
C_G^I &= + \sin i \sin M_2 \\
L_G^I &= - \sin M_1' \cos M_2 + \cos i \cos M_1' \sin M_2
\end{align*}
\]

where the subscripts 1 and 2 refer to the GEO and IGSO satellite and \(M_1' = M_1 - \Omega_2\). Thus the radial and along-track position differences exhibit a 12 hour period, while the cross-component is characterized by a 24 hour periodicity.

A similar consideration is applied to compute the \((H, C, L)\)-position difference for IGSO-IGSO satellites assuming identical inclination \(i\), that leads to

\[
\begin{align*}
H_I^I &= \text{const.} + \sin^2 i (1 - \cos \Delta \Omega) \sin M_1 \sin M_2 \\
C_I^I &= \text{const.} + \sin i \cos i (1 - \cos \Delta \Omega) \sin M_2 \\
L_I^I &= \text{const.} + \sin^2 i (1 - \cos \Delta \Omega) \cos M_1 \sin M_2
\end{align*}
\]

where \(\Delta \Omega\) is the difference of the right ascension of the ascending nodes and \(M_1, M_2\) denote the mean anomaly of the IGSO satellites, respectively. Hence the IGSO-IGSO relative motion exhibits the same periodicity as the GEO-IGSO motion in the \((H, C, L)\)-components with 12 hours, 24 hours and 12 hours, respectively. The reason for the 24 hour period of the cross-track position component is the orbital normal vector \(e_2\), that is time-invariant, while the radial and along-track unit vectors \(e_1\) and \(e_3\) have a period of one orbital revolution.

Specifications for GNSS-2 Satellite Tracking System

The specifications for the satellite-satellite tracking system in the GNSS-2 constellation are closely related to the dynamics of relative satellite motion. This motion has been analyzed for all pairs of the sample GNSS-2 space segment. As result the relative motion of two IGSO satellites phased by 180° (i.e. I1-I3 and I2-I4) pose the highest demands for a tracking system. This is depicted in Figure 2, where the relative position (range), velocity (Doppler) and acceleration (Doppler rate) is shown for the satellites I2 and I4.

A candidate for a GNSS-2 ISL tracking system is certainly a system with general heritage from GPS. It is therefore instructive to compare the maximum values for GNSS-2 satellite-satellite distance, relative velocity and
acceleration with the maximum values of ground-based GPS receivers tracking GPS satellites or with the specifications for space-based GPS receivers. As example the GPS Motorola Viceroy Receiver is considered, that has been operated aboard the German scientific Equator-S spacecraft\(^1\) (working orbit \(h_p=500\) km, \(h_a=67000\) km) and the Russian MIR station\(^2\). While GPS signal acquisition for Equator-S has been demonstrated up to a distance of 61,000 km, maximum relative velocities of the MIR station and GPS satellites of 8 km/s could be supported.

### Table 1 Satellite tracking receiver characteristics.

<table>
<thead>
<tr>
<th></th>
<th>GPS rcv. on-ground</th>
<th>Motorola space-based</th>
<th>GNSS-2 ISL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>20,000 km</td>
<td>60,000 km</td>
<td>80,000 km</td>
</tr>
<tr>
<td>Doppler</td>
<td>4 km/s</td>
<td>8 km/s</td>
<td>6 km/s</td>
</tr>
<tr>
<td>Doppler Rate</td>
<td>0.0002 km/s(^2)</td>
<td>0.01 km/s(^2)</td>
<td>0.02 km/s(^2)</td>
</tr>
</tbody>
</table>

Although the Doppler shift for GNSS-2 ISL is moderate, the anticipated range values of 80,000 km provide important constraints for the required link margin and the Doppler rates exceed the maximum figures of the Equator-S experiment by a factor of 2. These conditions may require the onboard knowledge of the relative satellite motion for a dynamic tuning of the receiver tracking-loop and/or for an enhanced signal level.

### GDOP Analysis for GNSS-2 Inter-Satellite-Links

The purely kinematic GNSS-2 satellite position solution can be based on ranging measurements to other GNSS-2 satellites. The achievable position accuracy depends both on the accuracy of the range measurements \(\Delta \rho\) and on the observation geometry, given by unit vectors \(e_i\) of the GNSS-2 satellite under consideration to other satellites \((i=1,\ldots,k)\) in view.

Resulting from the observation equations for pseudorange measurements the state error \(\Delta \mathbf{x}=[\Delta \mathbf{r}, \Delta \mathbf{t}]^T\) as result of range measurements to \(k\) visible satellites may be described as

\[
G \Delta \mathbf{x} = \Delta \rho
\]

where the geometry matrix \(G\) is given as

\[
G = \begin{bmatrix}
\mathbf{e}_1^T & 1 \\
\mathbf{e}_2^T & 1 \\
\vdots & \vdots \\
\mathbf{e}_k^T & 1
\end{bmatrix}.
\]

The covariance matrix for the position is thus given as

\[
E(\Delta \mathbf{x} \cdot \Delta \mathbf{x}^T) = (G^T G)^{-1} \cdot E(\Delta \rho \cdot \Delta \rho^T) \cdot G^T (G^T G)^{-1}
\]

that collapses to \(\sigma_x^2(G^T G)^{-1}\), or

\[
\sigma_x^2 = \begin{bmatrix}
(XDOP)^2 & \text{cov terms} \\
(YDOP)^2 & \text{cov terms} \\
(ZDOP)^2 & \text{cov terms} \\
(TDOP)^2 & \text{cov terms}
\end{bmatrix}
\]

for uncorrelated range measurements. Here \(\sigma_x\) denotes the statistical error of the range measurements, that may be associated with the User Equivalent Range Error (UERE), while XDOP, YDOP, ZDOP, TDOP denote the individual dilution of precision (DOP) contributions to the geometrical DOP value GDOP. The final position error \(\sigma_x\) may thus be written as

\[
\sigma_x = \sigma_x \cdot \sqrt{XDOP^2 + YDOP^2 + ZDOP^2 + TDOP^2}
\]

In the following, the GDOP approach is applied to the sample GNSS-2 space segment, making use of ISL’s for tracking. It is noted that geosynchronous satellites tracked from ground yield GDOP values higher than 140 in a four-dimensional treatment, while realizing a satellite time with independent means leads to minimum GDOP values of about 8. If ISL’s are used for GNSS-2 tracking, the observation geometry benefits from the increased variation of the observation geometry as compared to Earth-based tracking. This is clearly demonstrated in Figure 3, that presents GDOP values for the geostationary satellite G1 as well as for the IGSO satellite I4.

Especially in the regimes of high northern and southern latitudes the tracking performance of IGSO’s is bad, due to lacking observation geometry from higher northern or southern locations. As the geostationary satellites are in the Earth equator plane and the IGSO satellites move within 12 hours from a given latitude to the corresponding latitude in the other hemisphere, the geostationary DOP evolution exhibits a 12 hour pattern. In contrast the 24 orbital period of the IGSO satellites is
visible also for the DOP values of IGSO satellites.

The lack of northern or southern observation geometry for IGSO’s is obvious at the northern or southern turning points of the IGSO orbit and thus appears every 12 hours with GDOP values of up to 14. This drawback may however be overcome by augmentation of the ISL links with terrestrial pseudolites (ground terminals), that radiate satellite-like navigation signals to the IGSO’s. The benefit of an additional pseudolite for the IGSO, located at 195° east longitude and 65° northern latitude (Alaska), is also presented in Figure 3. As result of the pseudolite the maximum GDOP value decreases from 14 to about 9, similar to the maximum GDOP value of the GEO satellite.

The main driver for the GNSS-2 position accuracy is, however, the radial position error, that may be deduced from RDOP. Hence the mean and maximum DOP contributions for GEO and IGSO satellites in the orbital frame are given in Table 2, where HDOP is the horizontal DOP and PDOP is the 3-dimensional position DOP value.

<table>
<thead>
<tr>
<th>DOP</th>
<th>GEO Mean</th>
<th>GEO Max</th>
<th>IGSO Mean</th>
<th>IGSO Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>RDOP</td>
<td>3.5</td>
<td>7.2</td>
<td>4.1</td>
<td>9.6</td>
</tr>
<tr>
<td>HDOP</td>
<td>2.3</td>
<td>4.3</td>
<td>3.1</td>
<td>5.9</td>
</tr>
<tr>
<td>PDOP</td>
<td>4.2</td>
<td>8.4</td>
<td>5.3</td>
<td>10.2</td>
</tr>
<tr>
<td>TDOP</td>
<td>1.6</td>
<td>3.1</td>
<td>3.8</td>
<td>9.2</td>
</tr>
<tr>
<td>GDOP</td>
<td>4.5</td>
<td>8.9</td>
<td>6.6</td>
<td>13.7</td>
</tr>
</tbody>
</table>

It is noted, that IGSO DOP values are systematically inferior to GEO figures, due to the bad observation geometry at high northern and southern latitudes. Furthermore, the radial or vertical DOP values are exceeding the horizontal values significantly. Considering ISL ranging measurements with 1 m statistical error hence leads to radial position errors of 7-10 m at maximum, that violate the expected accuracy requirements of 0.2 m for GNSS-2.

**GNSS-2 Tracking Concept and Analysis Approach**

In the previous sections, the purely kinematic approach of position solution has been studied based on simultaneous ISL range measurements from several satellites. Under conservative assumptions for the ranging accuracy the GNSS-2 accuracy requirements could not be met. As consequence, dynamic approaches using classical orbit determination are studied in the sequel that make use of the known laws of orbital dynamics. Such a dynamical approach introduces additional knowledge or constraints to the position reconstruction and thus stabilizes and improves the position adjustment in terms of accuracy.

In a later GNSS-2 software implementation phase, a purely dynamic approach may however be abandoned, in favor of a reduced-dynamic treatment. This transition could be forced by highly complex dynamical models, e.g. for solar radiation pressure or by requirements from rapid post-maneuver recovery. The basic approach to explore the benefits of dynamical orbit determination, that is followed in the sequel, is however not affected by these considerations.

The basic measurement type for the GNSS-2 space segment is ground-based range as well as ISL range. Here the ground-based ranging may either be derived from the PRARE (Precise Range and Range-Rate Equipment) or the SATRE (Satellite Time and Range Equipment) system. The PRARE system performs two-way links originating from the satellite, transponded by a ground terminal and received by the satellite, where the data could be processed in an automated onboard process, while the SATRE system transmits and receives signals at a ground station. Common to both approaches is the application of a Pseudo-Random Noise Code (PN) for high precision range measurements with a chip-rate of 10 MChips and 20 MChips for PRARE and SATRE, respectively. Alternatively, the ranging signal emitted by the GNSS-2 satellites that is received by the user may additionally be applied as primary tracking device. In this analysis typical ranging accuracy figures are taken from the operational experience with PRARE. The ISL ranging may be based on one-way or two-way optical or
radiometric tracking systems, that are assumed with conservative accuracy figures, as given in Table 3.

Ground-based tracking of the three GEO satellites G1, G2 and G3 and the four IGSO satellites I1, …, I4 may be based on a set of suitably selected ground stations,

Table 3 Error assumptions for GNSS-2 covariance analysis

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Force model errors</strong></td>
<td></td>
</tr>
<tr>
<td>Earth gravitational coefficient</td>
<td>4.3·10^{-10}</td>
</tr>
<tr>
<td>Earth gravitational field</td>
<td>10% GEMT2-GEMT3</td>
</tr>
<tr>
<td>Solar radiation pressure</td>
<td>20% a priori</td>
</tr>
<tr>
<td>Albedo</td>
<td>30%</td>
</tr>
<tr>
<td>Solid Earth tides</td>
<td>30%</td>
</tr>
<tr>
<td><strong>Ground-based tracking errors</strong></td>
<td></td>
</tr>
<tr>
<td>Range bias</td>
<td>40 cm a priori</td>
</tr>
<tr>
<td>Range noise</td>
<td>7 cm</td>
</tr>
<tr>
<td>Time tag error</td>
<td>3 µs</td>
</tr>
<tr>
<td>Troposphere</td>
<td>2%</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>0.3%</td>
</tr>
<tr>
<td>Station location longitude</td>
<td>8 cm</td>
</tr>
<tr>
<td>Station location latitude</td>
<td>8 cm</td>
</tr>
<tr>
<td>Station location vertical</td>
<td>32 cm</td>
</tr>
<tr>
<td><strong>Space-based tracking errors</strong></td>
<td></td>
</tr>
<tr>
<td>Range bias</td>
<td>100 cm a priori</td>
</tr>
<tr>
<td>Range noise</td>
<td>10 cm</td>
</tr>
<tr>
<td>Time tag error</td>
<td>300 µs a priori</td>
</tr>
</tbody>
</table>

with existing adequate station infrastructure. Potential locations are Bangalore (India), Hartebeesthoek (South Africa), Kerguelen (Indian Ocean), Kiruna (Sweden), Kourou (French Guiana), Malindi (Kenya), Santiago (Chile) and Weilheim (Germany). The ground-based tracking may be based on an interleaved schedule, where one station tracks several satellites within limited time slots and range data are accumulated with a sampling period of 600 s, when the satellite is above a 15° elevation threshold.

Space-based tracking is performed on a continuous schedule where in principle each of the satellites could track all others. Within the considered space segment no restrictions from signal obstruction of the Earth apply and ionospheric errors have not to be considered. The tracking system could be a heritage from the pseudo-range measurement principle applied by GPS. Although the range measurements are accumulated on-board, the orbit determination function could be executed on-ground as well as on-board and the location of this function has no consequences for the results, obtained within this analysis.

The analysis of dynamic orbit determination errors of the GNSS-2 space segment is based on a consider covariance analysis. To this end the multi-satellite error analysis software ORAN has been applied³, that supports the definition of a realistic tracking schedule and comprises systematic and statistic errors of the force and measurement models for all satellites and ground stations. The analysis has been conducted with emphasis to different tracking scenarios, but variations with respect to different sets of estimation parameters or modified error assumptions have also been considered⁴.

Results from a Distributed ISL Tracking Concept

The use of ISL’s for tracking purposes still requires the utilization of ground tracking stations. This is due to the fact, that the tesseral terms in the complex gravity field of the Earth can only fix the satellite position at geosynchronous altitude at a level of about 6 km. However, from a consider covariance analysis of a single satellite pair, consisting of a GEO and an IGSO satellite as well as a single ground station (Hartebeesthoek), satellite position errors at meter level are derived. Thus, single station tracking can be sufficient as baseline for the operational satellite-satellite tracking (SST) concept. It is noted, however, that robust mission operations may require more than a minimal ground station support, as part of redundancy and backup concepts.

A distributed concept for ISL tracking may be based on tracking links between all pairs of satellites. Thus a total of \( n(n-1)/2 \) ISL’s are available for orbit determination and no satellite has a specific centralized function. This tracking concept is of interest for a decentralized autonomous onboard orbit determination function. However, the inherent drawback of this approach is that tracking ISL’s require the adjustment of all satellite state vectors involved in the tracking. As those satellites states are determined from ISL tracking as well, the orbit determination process of the full space segment can not properly be split in processes for the individual satellites.

This may be demonstrated within a simplified scenario of 3 satellites (S1, S2, S3), where the IGSO satellite S1
is tracked from ground and there are three ISL’s, S2 and S3 being either GEO or IGSO satellites. Let

\[
\mathbf{A}_{ji}^{gi} = \begin{pmatrix}
\frac{\partial \mathbf{L}_i}{\partial \mathbf{L}_0} & \frac{\partial \mathbf{L}_i}{\partial \mathbf{L}_0} & \ldots & \ldots \\
\frac{\partial \mathbf{L}_i}{\partial \mathbf{L}_0} & \frac{\partial \mathbf{L}_i}{\partial \mathbf{L}_0} & \ldots & \ldots \\
\frac{\partial \mathbf{L}_i}{\partial \mathbf{L}_0} & \frac{\partial \mathbf{L}_i}{\partial \mathbf{L}_0} & \ldots & \ldots \\
\frac{\partial \mathbf{L}_i}{\partial \mathbf{L}_0} & \frac{\partial \mathbf{L}_i}{\partial \mathbf{L}_0} & \ldots & \ldots \\
\end{pmatrix}
\]

be the Jacobi matrix with the partial derivatives of the range measurements w.r.t. the state vector \( \mathbf{y}_i \). Here the superscript \( G \) denotes ground-based tracking and \( Sji \) the space-based tracking between satellites \( j \) and \( i \). Then the full Jacobi matrix \( \mathbf{A} \) includes the partials of the state vectors \( \mathbf{y}_1, \mathbf{y}_2 \) and \( \mathbf{y}_3 \) according to

\[
\mathbf{A} = \begin{pmatrix}
\mathbf{A}_{11}^G & \mathbf{A}_{12}^{S12} & \mathbf{A}_{13}^{S13} & 0 \\
0 & \mathbf{A}_{22}^{S12} & 0 & \mathbf{A}_{23}^{S23} \\
0 & 0 & \mathbf{A}_{33}^{S13} & \mathbf{A}_{33}^{S23}
\end{pmatrix}
\]

and exhibits the coupling of the satellite state vectors.

A formal solution to this detriment could be the mutual exchange of all ISL tracking measurements (or a priori covariance matrices), so that each satellite may solve for the states of the full space segment. However, even in this case, inconsistencies in the estimated satellite states, determined at each satellite independently, will remain.

Based on this distributed ISL tracking concept a multi-satellite consider covariance analysis has been performed. In total 86 parameters were estimated, comprising the satellite state vectors as well as the solar radiation pressure coefficients and the range and timing biases for the ISL tracking links.

The consider covariance results are shown in Table 4, where both statistical and total position errors, comprising statistical and systematic errors, are collated. Here the GEO and IGSO satellite with the maximum errors have been selected out of three GEO and four IGSO satellites. The error variations for different GEO satellites are about 7%, while the variations for IGSO satellites are up to 25%.

**Table 4 Maximum statistical (S) and total (T) satellite position errors for distributed ISL tracking concept.**

<table>
<thead>
<tr>
<th>Satellite</th>
<th>GEO</th>
<th>IGSO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>S</td>
<td>T</td>
</tr>
<tr>
<td>( \sigma_H ) [m]</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>( \sigma_C ) [m]</td>
<td>0.4</td>
<td>1.1</td>
</tr>
</tbody>
</table>

A remarkable level of less than 1.5 m is achieved for the position errors of all GNSS-2 satellites. The a priori sigma value of all estimation parameters could be significantly decreased in the orbit determination. Still, the total position error is governed by systematic errors, mainly due to modeling errors of the station location for Hartebeesthoek. This result calls for a precision model of the station location, including effects from solid Earth tides and plate tectonics. It is noted, that a subsequent one-day propagation phase does not lead to increases in the position accuracy as compared to orbit determination from the 2 day tracking arc, presented in Table 4.

**Results from the Master/Slave Tracking Concept**

The distributed tracking concept does not only require a variety of different ISL’s for tracking, but also implies serious drawbacks with respect to a rigorous treatment of state vector correlations, consistency and exchange of tracking data between all satellites of the space segment.

In the following, a master/slave tracking concept (cf. Figure 4) is proposed, that comprises a single ground-based link from one station (Hartebeesthoek) to one IGSO satellite, that serves as master for the GNSS-2 space segment. The master satellite performs SST with the other satellites, called slaves, that are permanently visible from the master, while slave-slave ISL’s are not required. Thus the total number of ISL’s is limited to \((n-1)\), as compared to a full SST concept with \(n(n-1)/2\) ISL’s. The selection of an IGSO satellite as master is required due to the varying observation geometry of an IGSO with respect to a ground station. If a GEO satellite were to be a master, two or three ground station should be used for tracking instead.

With the master/slave concept, the orbit determination function could be executed autonomously onboard the master, where all measurements are readily available. Hence the centralized approach does not lead to problems with state vector correlations or consistency and the Jacobi matrix in this concept is given by

\[
\mathbf{A} = \begin{pmatrix}
\mathbf{A}_{ij}^G & \mathbf{A}_{ij}^{S12} & \mathbf{A}_{ij}^{S13} \\
0 & \mathbf{A}_{ij}^{S12} & 0 \\
0 & 0 & \mathbf{A}_{ij}^{S13}
\end{pmatrix}
\]

The results from the master/slave concept are summarized in Table 5 for the GEO and IGSO satellite with the maximum error values.
The error variations for different GEO satellites are about 20%, while the variations for IGSO satellites are up to 35%. The reduction of ISL’s in this proposed operational concept leads to an increase of the slave satellite position errors of 80% at maximum, while the master satellite position errors increase by 30%. The significant increase of the slave position errors is largely caused by the reduction of number of measurements that considerably increases the statistical errors. In contrast the systematic errors increase only by 25% at maximum. If the results are scaled with respect to the same number of measurements, the differences of the position errors in the distributed and the master/slave concept are less than 50%. The moderate error growth is achieved by a remarkable reduction of the complexity of the space-based tracking system.

A further reduction of the GNSS-2 satellite position errors, especially in the height component, is achieved when the station location errors decrease. This could be realized with an improved station location modeling and is demonstrated by a reduction of the station location errors from (8 cm, 8 cm, 32 cm) to (3 cm, 3 cm, 3 cm) for the East, North and Zenith components. The result of the consider covariance analysis is given in Table 6, where maximum position errors of less than 2 m can be achieved for all GNSS-2 satellites, while all height errors are less than 9 cm.

Table 5 Maximum statistical (S) and total (T) satellite position errors for master/slave ISL tracking concept.

| Satellite | GEO | IGSO | | |
|-----------|-----|------| | |
| σH [m]    | 0.0 | 0.2  | 0.0 | 0.2 |
| σC [m]    | 1.1 | 1.6  | 1.3 | 1.4 |
| σL [m]    | 1.2 | 1.5  | 1.2 | 2.0 |
| σr [m]    | 1.7 | 2.2  | 1.8 | 2.4 |

Table 6 Maximum statistical (S) and total (T) satellite position errors for improved station location modeling.

| Satellite | GEO | IGSO | | |
|-----------|-----|------| | |
| σH [m]    | 0.0 | 0.1  | 0.0 | 0.1 |
| σC [m]    | 1.1 | 1.2  | 1.4 | 1.4 |
| σL [m]    | 1.2 | 1.2  | 1.3 | 1.4 |
| σr [m]    | 1.7 | 1.7  | 1.9 | 1.9 |

Conclusions

In the framework of the future GNSS-2, the achievable orbit determination accuracy of the GNSS-2 satellites using Inter-Satellite-Links has been analyzed. To this end an European subset of the GNSS-2 space segment has been defined, comprising three geostationary satellites as well as four inclined geosynchronous satellites.

To analyze the kinematic position solution accuracy the relative motion of GNSS-2 satellite pairs has been computed. As a result, tracking conditions have been found with maximum range, Doppler and Doppler rate values of 80,000 km, 6 km/s and 0.02 km/s², respectively, that contribute to the requirements for the design of the GNSS-2 ISL tracking system. Furthermore, the geometrical dilution of precision values of the GNSS-2 satellites have been computed using ISL’s, leading to values from 2 up to 9 for GEO and up to 14 for IGSO satellites. As consequence, instantaneous kinematic position solutions with a radial accuracy of 4 m–10 m may be derived, that obviously violate the demanding requirements of 0.2 m, expected
for GNSS-2.

The drawbacks of the kinematic satellite position solutions are avoided by a conventional dynamic or a reduced-dynamic orbit determination approach. To this end a consider-covariance analysis of the full space and ground segment has been conducted and the statistic as well as systematic force and measurement model errors have been treated in a rigorous manner, leading to realistic estimates of the GNSS-2 satellite position errors.

Using ISL’s, it has been shown that an adequate GNSS-2 tracking concept comprises a single ground station, tracking a single IGSO master satellite, while the master satellite tracks each of the slave satellites. As result, the radial position errors stay well within 0.2 m, while the total position errors are less than 3 m. An improved station location modeling even drives the accuracy to 0.08 m for the radial and 2 m for the total satellite position error. In contrast to the centralized master/slave concept, a complex distributed tracking system with ISL’s between all satellites of the space segment, improves the accuracy by only 50%. Thus, a cost-effective tracking concept with a single ground station, a master satellite and a number of slave satellite serves as a highly accurate and conceptual simple system, that should be of general relevance for the development of the GNSS-2 system.

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