OPTIMAL USE OF ELECTRIC PROPULSION FOR FUTURE LEO CONSTELLATIONS POSITIONING

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Abstract

This paper describes an advanced method for future electric constellations positioning design. The main problem is to minimize the duration of such low-thrust constellations deployment. For each satellite, the orbit raising corresponds to many orbital revolutions and its optimization leads to a so-called "rapidly rotating" control problem, numerically bad conditioned. An averaging tool has been developped to compute optimal thrust law achieving the final rendezvous under technological and operational constraints such as power limitation and attitude specification. Considering the global constellation positioning, the specific problems of target plane acquisition and satellites phasing can be solved combining the previous low-thrust optimization tool with drift periods without thrust. We propose furthermore innovative strategies to accelerate the deployment, in return for consumption and operational overcost. Finally, the results of different trades-off in terms of injection orbit, pointing modes and positioning strategies are presented in the cases of big LEO constellations.

***Key words**:* Constellations, Electric Propulsion, Positioning strategies, Optimization.

Introduction

Electric propulsion will be used in the very near future for LEO telecommunication constellations such as Skybridge and Teledesic. The success of this new propulsion compared to classical chemical one comes from its low fuel consumption due to its high specific impulse. It thus allows to increase the payload or the number of satellites launched simultaneously. However, the thrust being low, the constellation deployment duration can be rather long. A general trade-off[1] must hence be realized for each project to decide about electric propulsion interest.

Apart from such commercial considerations, this new propulsion leads to several technical problems rarely addressed and yet of prime interest, like orbit restitution and trajectory optimization. Contrary to impulsional trajectories, continuous low-thrust ones have to be controlled at each time to get the final rendezvous, minimizing a certain criterion (first of all the duration but also the fuel consumption for a fixed-time transfer) under several constraints. We have thus to solve an ***optimal control problem***, so-called ***rapidly rotating*** because of the "rapid" angular revolutions compared to the "slow" orbit change (see Fig. 1). Such a problem is hard to solve by classical shooting methods, especially for full-orbital rendezvous (in both eccentricity, inclination and phase).

Averaging techniques[2], whose principle consists in eliminating the rapid oscillations, appear to be well-adapted to solve this kind of problems. An "averaged problem"[2] is introduced, better conditioned than the initial one and whose optimal control law can be used to command the satellite with high accuracy. These techniques have been first applied to geostationnary transfers[3-4]. In the case of constellations positioning, the problem is to perform multiple-plane acquisition and multiple-satellites phasing in the same plane for single launch. We will see that for certain strategies, such a rendezvous leads to multiple-satellite control problem.

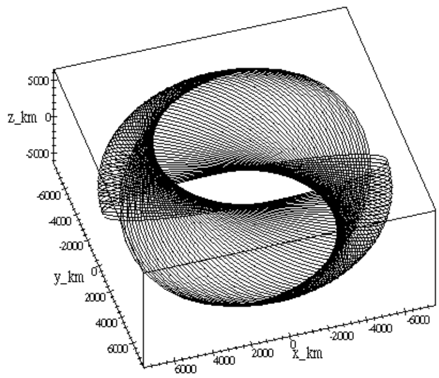

Figure 1: Low-thrust trajectory profile

Low-Thrust Orbit Raising Optimization

For homogeneous constellations, the different satellites have to reach the same operational orbit in terms of semi-major axis, eccentricity and inclination, whatever the injection orbit and the dispersions depending on the launcher. The final right ascension and phase are let free for the moment. The injection orbit can be either at a low-Earth altitude ("indirect" injection) or just below the final orbit ("direct" injection), and in the first case it can be either circular or elliptical.

Let us denote u the thrust vector, u_{max} the maximal thrust modulus, γ the acceleration due to environmental perturbations (assumed reduced to Earth zonal effect), x the five-state vector (a, $e_x = e \cos \omega$, $e_y = e \sin \omega$, i, Ω), $\alpha = v + \omega$ the angular phase, m the satellite mass, g_e the gravitational acceleration at sea-level and Isp the specific impulse of the thruster. Considering the minimum-time criterion, each satellite orbit raising can be modeled by the following optimal control problem:

$$\begin{cases} \min\limits_{\|u\| \le u_{max}} t_1 \\ \dfrac{dx}{dt} = f(x,\alpha)\left[\dfrac{u}{m} + \gamma(x,\alpha)\right] \\ \dfrac{dm}{dt} = -\dfrac{\|u\|}{g_e Isp} \\ \dfrac{d\alpha}{dt} = g_0(x,\alpha) + g_1(x,\alpha)\left[\dfrac{u}{m} + \gamma(x,\alpha)\right] \\ x(t_0) = x_0 \quad \varphi(x(t_1)) = 0 \\ m(t_0) = m_0 \quad m(t_1)\,free \\ \alpha(t_0) = \alpha_0 \quad \alpha(t_1)\,free \\ t_1\,free \end{cases} \qquad \textbf{(1)}$$

where f, g_0 and g_1 are coming from the Gauss's equations, and the constraint $\varphi(x(t_1)) = 0$ characterizes the final operational orbit (fixed values except for Ω, that is: $\varphi_i(x(t_1)) = x_i(t_1) - x_i^1 \;\; \forall i \ne 5$).

In order to apply Chaplais's averaging method[2], the equations must be written in terms of angle instead of time. Considering the same transformation as in [4], the problem (1) falls under the following rapidly rotating form at the first order in ε (a small parameter related to the low thrust modulus u_{max}):

$$(P)\begin{cases} \min\limits_{\tilde{u} \in U_{ad}} h(\tilde{x}(\tilde{\alpha}_1)) \\ \dfrac{d\tilde{x}}{d\tilde{\alpha}} = F\left(\tilde{x}, \tilde{u}, \frac{\tilde{\alpha}}{\varepsilon}\right) \\ \tilde{x}(\tilde{\alpha}_0) = \tilde{x}_0 \quad \Phi(\tilde{x}(\tilde{\alpha}_1)) = 0 \\ \tilde{\alpha}_1\,free \end{cases} \qquad \textbf{(2)}$$

The averaging method[2-4] consists in approximating this problem by the following averaged problem:

$$(\bar{P})\begin{cases} \min\limits_{\tilde{v} \in V_{ad}} h(\tilde{X}(\tilde{\alpha}_1)) \\ \dfrac{d\tilde{X}}{d\tilde{\alpha}} = \overline{F\left(\tilde{X}, \tilde{v}(.),.\right)} \\ \tilde{X}(\tilde{\alpha}_0) = \tilde{x}_0 \quad \Phi(\tilde{X}(\tilde{\alpha}_1)) = 0 \\ \tilde{\alpha}_1\,free \end{cases} \qquad \textbf{(3)}$$

with the classical averaging notation[2] for any function F ω-periodic in the "rapid" movement:

$$\overline{F(\chi,.)} = \frac{1}{\omega}\int_0^{\omega} F(\chi,\theta)\,d\theta \qquad \textbf{(4)}$$

Constraints Accounting

The satellite design may induce technological and operational constraints such as power limitation and attitude specification. These constraints have to be expressed in a rather simple way in order to be taken into account in the previous low-thrust optimal control problem.

Power constraint

Whatever the electric thruster (ionic as UK 10 or RIT 10 ones, or plasmic as SPT 100 one), it requires a significant power budget which can be incompatible with the power available on the platform. Therefore, thrust can not be delivered continuously but within a certain ***duty-cycle*** ratio, depending on Sun elevation (directly through to the shadowing cycles or indirectly through the battery status). This constraint can be expressed more precisely as follows: thrust duration on several revolutions Δt^{ON} must represent less than the duty-cycle ratio DC of these revolutions duration Δt, that is:

$$\Delta t^{ON} \le DC \times \Delta t \qquad \textbf{(5)}$$

Shadow effect has been included in the averaging optimization tool[4-5] but not yet the battery modeling. It is thus impossible for the moment to get the thrust law with optimal coasting arcs fulfilling the constraint (5). Two sub-optimal solutions can be considered:

1. impose arcs without thrust symetrically placed on odd/even orbits so that the ON/OFF cycles have the slightest effect on eccentricity (see Fig. 2),
2. keep continuous thrust with reduced maximal modulus ($u'_{max} = DC \times u_{max}$), and simulate the real trajectory by introducing in the previous trajectory coasting arcs fulfilling the duty-cycle constraint (with u_{max} instead of u'_{max}).

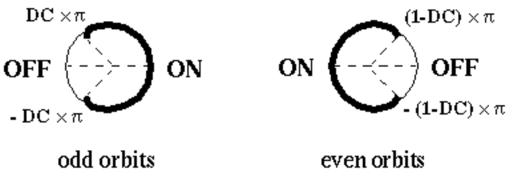

Figure 2: ON/OFF duty-cycle constraint

The first solution consists in replacing the thrust constraint of problem (1) by the following:

$$\begin{aligned} &if\ \ \alpha \in I_\alpha\ \ then\ \ \|u\| \le u_{\max}\ \ else\ \ u = 0 \\ &with\ \ I_\alpha = [-DC\times\pi, DC\times\pi]\ \ for\ odd\ orbits \\ &and\ \ I_\alpha = [-(1-DC)\times\pi, (1-DC)\times\pi] \\ &\qquad\qquad\qquad for\ even\ orbits \end{aligned} \quad \textbf{(6)}$$

Attitude constraint

AOCS specifications may impose specific pointing modes during the transfer (such as inertial, or with low variations in pitch and yaw around the speed direction). In this case, the thrust direction is no longer free as previously but constrained. Indeed, the thrust orientation results most of time from the satellite attitude control, especially for single thruster with fixed nozzle. The best way to handle this constraint would be to replace the thrust by the specified law in the initial problem and to optimize the new control parameters. We propose two alternative methods in the case of a low-variation attitude constraint:

1. ***add cone constraint***[4] (see Fig. 3) on thrust direction in the optimal control problem in order to limit the variations in pitch and yaw,
2. ***modify the orbit raising target*** in order to get low-variation thrust profile without cone constraint during the transfer, and perform the residual corrections to get the effective target with dedicated pointing modes at the end of the transfer.

The first method consists in introducing the following constraint in problem (1), where $C(\alpha)$ is the cone of angle α in the speed direction (as depicted in Fig. 3 in the tangential/normal orbital frame):

$$u \in C(\alpha) \quad \textbf{(7)}$$

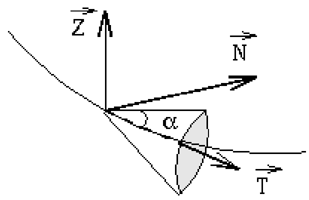

Figure 3: Cone constraint

Constellation Positioning Strategies

The global constellation consists of several planes including several satellites each. The deployment scenario depends on the constellation: ***plane-by-plane*** launches (so-called “direct” launches) for little LEO constellations and ***multiple-plane*** launches (so-called “indirect” launches) for big LEO ones. In the second case, several clusters of satellites are injected at a low-Earth altitude, in order to take advantage of the relative plane drift between the injection altitude and the target one to fill in different planes. To separate these clusters in plane, two strategies can be considered (see Fig. 4):

1. the ***drift strategy***, transferring each cluster successively after a drift phase adjusted to perform the target plane acquisition (absolute positioning),
2. the ***parallel strategy***[6], transferring the clusters simultaneously in opposite directions to accelerate the relative plane drift (relative positioning).

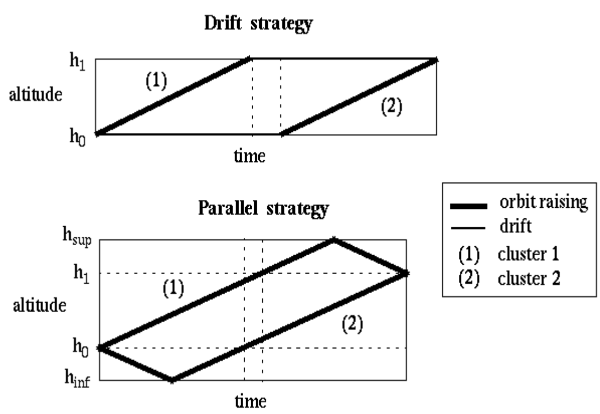

Figure 4: Positioning strategies

In return for its efficiency in terms of deployment duration, the second strategy leads to fuel and operational overcost (anti-tangential maneuvers) and requires an extended flight domain[6] (altitudes below and above the injection and operational ones). It requires furthermore ***multiple-satellite rendezvous optimization***, whose resolution is rather difficult (except for circular transfers in nominal case leading to simple tangential and anti-tangential thrust laws[6]).

Whatever the injection scenario or the positioning strategy, right ascension is assumed to be corrected with minimum consumption overcost:

- using natural plane drift for indirect injection (during the drift phase for the drift strategy and the orbit raising phase for the parallel strategy),
- adjusting the initial right ascension (accounting for possible launcher dispersions) for direct injection.

In fact, we will see below that out-of-plane maneuvers will also be required during the orbit raising phase for right ascension correction due to the phasing.

Low-thrust Phasing Strategies

Let us consider the cluster satellites phasing during the orbit raising phase of the drift strategy. A ***rendezvous in both phase and plane*** has to be performed in order to phase different satellites in the same plane, because of the coupling between phase and right ascension. Mainly two phasing strategies can be considered (see Fig. 5):

1. ***strategy 1***, introducing an additional drift phase for phasing and correcting the induced right ascension deviation during the orbit raising phase, within an adjustment loop on the drift phase duration,
2. ***strategy 2***, realizing both phase and right ascension rendezvous during the orbit raising phase.

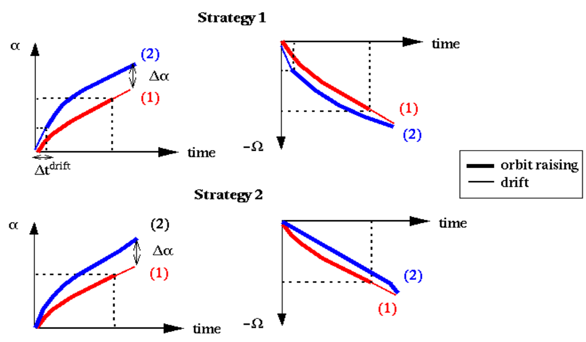

Figure 5: Phasing strategies

The curves (1) correspond to the so-called "reference" orbit raising trajectory, without phase nor right ascension rendezvous (solution of problem (1)). The associated final orbit is used to defined the target one:

- the final reference time t_1^{ref} is taken as the target time t_1^{target},
- the initial right ascension Ω_0 is chosen so that the final reference one Ω_1^{ref} corresponds to the target one Ω_1^{target},
- the target phase α_1^{target} is adjusted so that the difference with the final reference one α_1^{ref}, denoted $\Delta\alpha$, ranges from 0 to 360 °.

The curves (2) correspond to the modified trajectory, reaching the rendezvous in both phase and plane (after a drift phase for strategy 1 and directly for strategy 2).

Strategy 1

Denoting Δt^{drift} the drift phase duration and considering the previous notations, the first strategy leads to the following optimal control problem:

$$\left\{\begin{array}{l} \min\limits_{\|u\|\le u_{max}} t_1 \\ \dfrac{dx}{dt} = f(x,\alpha)\left[\dfrac{u}{m}+\gamma(x,\alpha)\right] \\ \dfrac{dm}{dt} = -\dfrac{\|u\|}{g_e Isp} \\ \dfrac{d\alpha}{dt} = g_0(x,\alpha)+g_1(x,\alpha)\left[\dfrac{u}{m}+\gamma(x,\alpha)\right] \\ x(t_0')=x_0' \quad \varphi'(x(t_1))=0 \\ m(t_0')=m_0 \quad m(t_1)\ free \\ \alpha(t_0')=\alpha_0' \quad \alpha(t_1)\ free \\ t_1\ free \end{array}\right. \tag{8}$$

where:

$$\left|\begin{array}{l} t_0'=t_0+\Delta t^{drift} \\ x_0'=x_0+\Delta t^{drift} f(x(t_1),\alpha(t_1))\ \gamma(x(t_1),\alpha(t_1)) \\ \alpha_0'=\alpha_0+\Delta t^{drift}\dot{\alpha}_1 \\ \dot{\alpha}_1 = g_0(x(t_1),\alpha(t_1))+ \\ \qquad g_1(x(t_1),\alpha(t_1))\ \gamma(x(t_1),\alpha(t_1)) \end{array}\right. \tag{9}$$

and:

$$\left|\begin{array}{l}\varphi'_i(x(t_1)) = \varphi_i(x(t_1)) \quad \forall i \neq 5 \\ \varphi'_5(x(t_1)) = \Omega(t_1) - \Omega_1^{target} - \\ \qquad\qquad (t_1 - t_1^{target})\dot{\Omega}_1 \\ \dot{\Omega}_1 = f_5(x(t_1),\alpha(t_1))\ \gamma(x(t_1),\alpha(t_1))\end{array}\right. \quad \textbf{(10)}$$

Finally, the drift phase duration Δt^{drift} is determined by the following phase rendezvous equation:

$$\alpha(t_1^*) - \alpha_1^{target} - (t_1^* - t_1^{target})\dot{\alpha}_1 = 0 \quad \textbf{(11)}$$

where t_1^*, the optimal final time of problem (8), is function of Δt^{drift}.

Strategy 2

In this case the following optimal control problem gives directly the phasing solution:

$$\left\{\begin{array}{l}\min\limits_{\|u\|\leq u_{max}} t_1 \\ \dfrac{dx}{dt} = f(x,\alpha)\left[\dfrac{u}{m} + \gamma(x,\alpha)\right] \\ \dfrac{dm}{dt} = -\dfrac{\|u\|}{g_e Isp} \\ \dfrac{d\alpha}{dt} = g_0(x,\alpha) + g_1(x,\alpha)\left[\dfrac{u}{m} + \gamma(x,\alpha)\right] \\ x(t_0) = x_0 \quad \varphi'(x(t_1)) = 0 \\ m(t_0) = m_0 \quad m(t_1)\ free \\ \alpha(t_0) = \alpha_0 \\ \rho(\alpha(t_1),t_1) = 0\end{array}\right. \quad \textbf{(13)}$$

where the constraint $\rho(\alpha(t_1),t_1) = 0$ characterizes the phase rendezvous (see (11)):

$$\rho(\alpha(t_1),t_1) = \alpha(t_1) - \alpha_1^{target} - (t_1 - t_1^{target})\dot{\alpha}_1 \quad \textbf{(14)}$$

This strategy is more optimal than the first one in terms of rendezvous duration and consumption, since it induces less right ascension correction during the transfer. But it appears to be rather difficult to apply in the case of near-circular orbits. Indeed, the resolution of problem (13) leads in this case to numerical convergence problems, which can be explained physically by the fact that circular minimum-time trajectories provide less phase maneuvering margin than elliptical ones.

Numerical results

We present below the results of different trades-off in the cases of big LEO constellations such as Skybridge and Teledesic ones, as an application of the previous optimization methods. We have considered here a unique study case which can be applied to both constellations, corresponding to the following assumptions:

- shift of 45 ° between two adjacent planes,
- SPT 100 thruster (F = 83 mN, Isp = 1450 s),
- m_0 = 1100 kg,
- circular operational orbit with frozen perigee (h_1 = 1400 km, $e_1 = 7.86 \times 10^{-4}$, ω_1 = 90 °, i_1 = 55 °),
- DC = 80 %.

Concerning the launcher dispersions, only inclination dispersion of 0.12 ° has been taken into account in order to see the effect of plane correction during the transfer (the effect of eccentricity correction being observed even in the nominal case because of frozen orbit acquisition).

The duty-cycle constraint has been treated in a rather simple way, that is considering a reduced thrust modulus (F' = 66.4 mN) to get a first dimensioning idea of duration/consumption budgets and attitude law. Within an operational context, the real trajectory including coasting arcs should be simulated as described above (second solution for power constraint accounting). It should be noticed that the duty-cycle ratio, which has been fixed here to its mean value, could have been taken variable during the transfer (as a function of Sun elevation).

The first trade-off deals with the injection orbit. Two injection cases have been considered both at a low-Earth altitude: a circular one and an elliptical one (as described in Fig. 6).

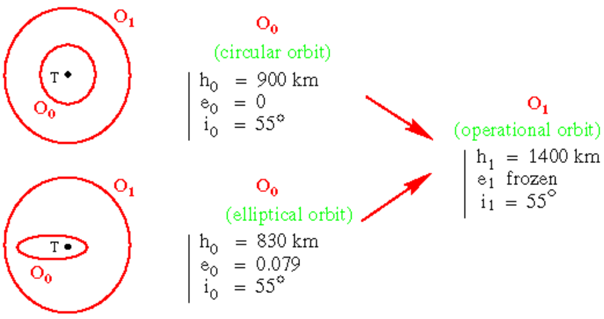

Figure 6: Injection cases description

In each case, we present the results of the rendezvous problem (8) (corresponding to the orbit raising phase of the drift positioning strategy with the first phasing strategy), within the following asumptions:

- no attitude constraint,
- $\Delta\alpha = 360\,^{\circ}$ (worst phasing case).

We adopt moreover the following notations:

- $\Delta t = t_1 - t_0$ (orbit raising duration),
- $\Delta m = m_1 - m_0$ (orbit raising consumption),
- $\Delta V = -g_e \; Isp \; \ln(m_1/m_0)$,
- (ψ,ξ): attitude law in pitch (in-plane direction) and yaw (out-of-plane direction) (see Fig. 7 for the definition of pitch and yaw angles in the tangential/normal orbital frame),
- (ψ^*,ξ^*): maximal deviations in pitch and yaw.

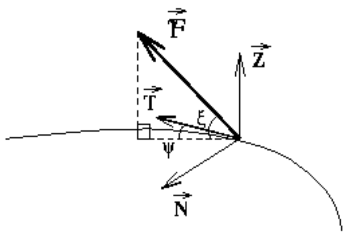

Figure 7: Pitch and yaw definition

Table 1: Injection trade-off

Injection case	Δt (days)	Δt^{drift} (h)	Δm (kg)	ΔV (m/s)	(ψ^*,ξ^*) $(^{\circ},^{\circ})$
Ariane 5 (circular)	49	13	19.5	253	(8,65)
Soyouz (elliptical)	108	147	41	540	(180,80)

The figures 8 and 10 give the temporal evolution of the minimum-time trajectory in terms of orbital parameters (absolute evolution except for right ascension and phase which are plotted relatively to the target). The figures 9 and 11 give the angular evolution of the minimum-time thrust law in pitch and yaw (with a zoom on the first and the last revolutions).

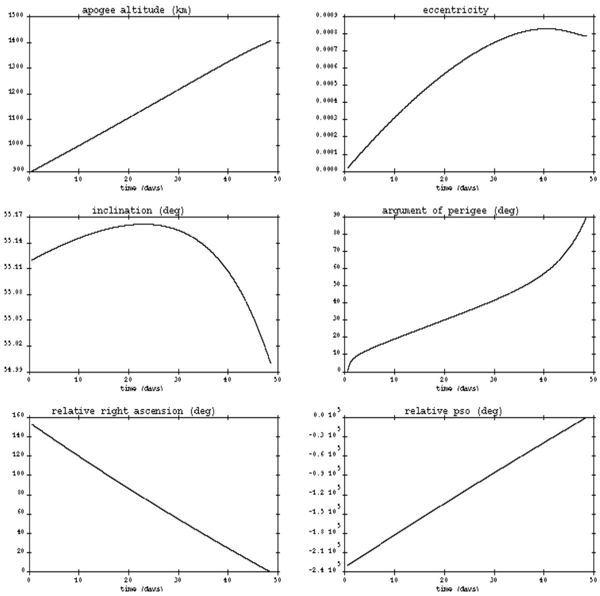

Figure 8: Circular optimal trajectory

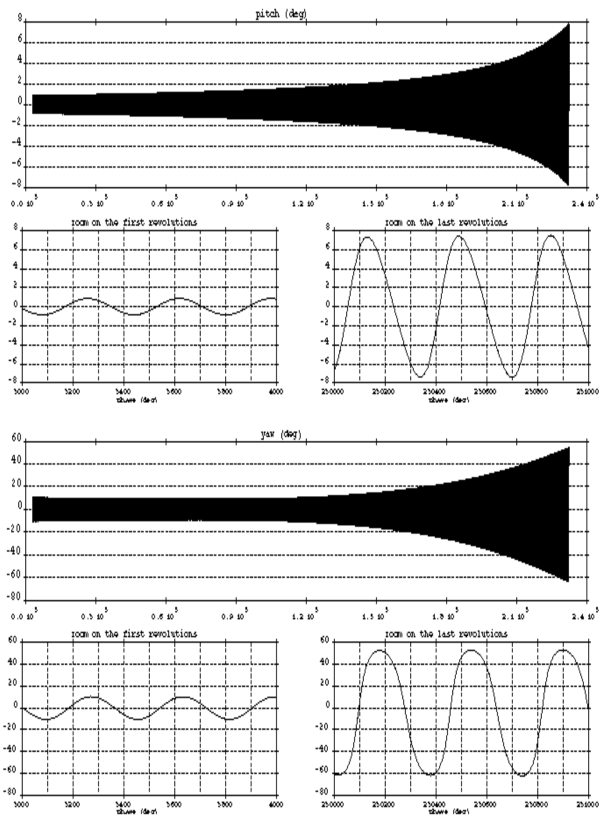

Figure 9: Circular optimal thrust law

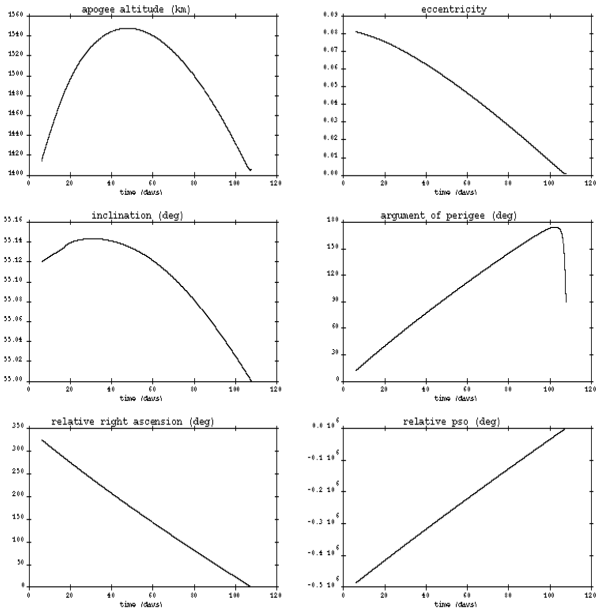

Figure 10: Elliptical optimal trajectory

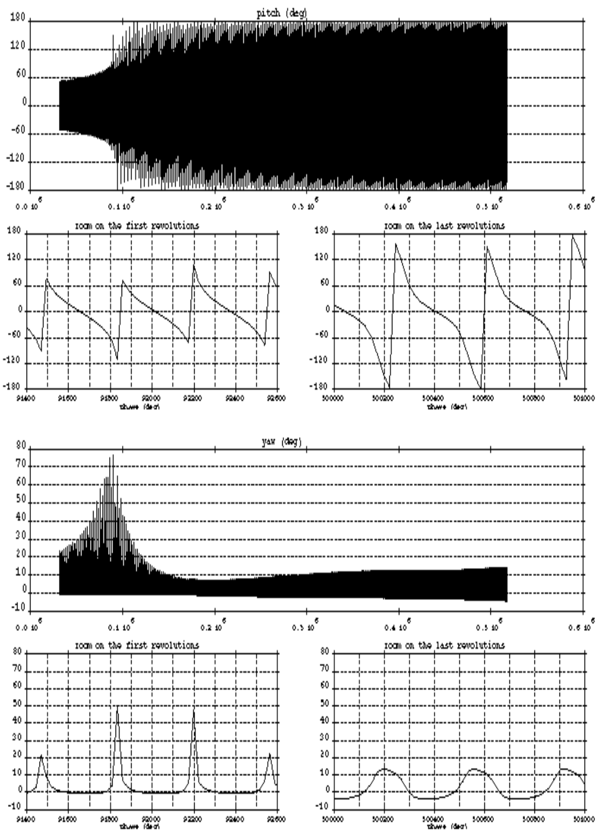

Figure 11: Elliptical optimal thrust law

We see that the ***elliptical*** injection leads to a so-called ***supersynchronous trajectory***, characterized by an apogee altitude higher than the final altitude during the transfer (see Fig. 10). The associated thrust law includes two phases (see Fig. 11):

1. *an acceleration phase*, propelling the satellite in the speed direction (apogee altitude increase),
2. *a deceleration phase*, propelling the satellite in the opposite speed direction (apogee altitude decrease),

the maximal out-of-plane maneuvers being performed at maximal apogee altitude. Such a strategy allows to accelerate eccentricity and plane corrections (as for geostationary transfers[3-4]). The transfer remains however costly in both duration and consumption (about twice as much as the circular injection one, see Table 1).

As a result, elliptical injection appears to be incompatible with big LEO constellations requirements in terms of duration/consumption budgets and pointing mode complexity.

The second trade-off concerns the attitude constraint accounting. Only the circular injection case will be handled here, since the elliptical one would lead to even higher duration considering additional constraint. Let us assume a maximal deviation of 30° allowed in pitch and yaw during the transfer. The table 2 gives the results obtained applying either the cone constraint or the method which consists in modifying the target and performing the final correction with dedicated pointing mode, as described previously.

Table 2: Attitude constraint trade-off

Method	Δt (days)	Δt^{drift} (h)	Δm (kg)	ΔV (m/s)
Cone constraint	48	16	19	247
Final correction	53	18	22	285

The second case includes a final correction of 0.3° in right ascension with perpendicular-to-plane maneuvers (requiring about 6 days and 3 kg). This table shows that cone constraint is more efficient than final correction. However, it leads to complex attitude law which may be incompatible with certain AOCS specifications (requiring for instance pure sinusoïdal laws).

The table 3 compares finally the drift and the parallel positioning strategies for two planes filling, in the case of circular injection without attitude constraint.

Table 3: Positioning strategy trade-off

Strategy	total duration (days)	drift phase duration (days)	total consumption (kg)	total ΔV (m/s)
Drift	110	61	19	253
Parallel	90	0	36	475

The parallel strategy allows about 20 % duration gain in return for about 80 % consumption overcost. It is thus attractive (especially for multiple-plane filling), considering that duration is more critical than consumption for electric propulsion. However, the satellite design must take into account additional constraints, such as satellite rotation of 180 ° during the transfer and extended flight domain.

Conclusion

In this paper, the complex problem of electric constellations positioning with full-orbital rendezvous has been investigated from both theoretical and operational viewpoints. Different strategies accounting for electric propulsion characteristics have been presented and applied to big LEO constellations such as Skybridge and Teledesic. The low-thrust optimization method which had been developped initially for geostationary transfers appears to be rather flexible since it has been generalized to treat the specific constellations problems, such as combined rendezvous in phase and plane, and power/attitude constraints. However, the optimal control form should be adapted in this case to model more precisely the global positioning problem with constraints: multiple-satellite state instead of single-satellite one to treat the parallel strategy, mixed power/consumption criterion instead of duration one to combine the duty-cycle and the mission drift phases, and thrust law adaptation to pointing mode requirements.

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