# STATION ACQUISITION OF A HOMOGENEOUS CONSTELLATION BY LINEAR AND NONLINEAR OPTIMIZATION

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### Abstract

Constellations of satellites raises an increasing interest shown by the numerous prospects concerning mainly the communication but also related to positioning and earth observation... Of course, one of the main objectives in designing a constellation is, given an expected service, to find a strategy reducing the cost as much as possible.

One of important phase in building a constellation is the station acquisition phase which is fuel consuming so that it is an interesting goal to define optimal or nearly optimal strategies for that. The purpose of this paper is to develop such strategy for LEO (Low Earth Orbit) constellations, by using some beneficial effects of the Earth oblateness (and thus the irregularity of the gravitation) on the orbital parameters.

The optimization is performed in two steps. A first one consists in satellite affectation, i.e. after the launching of the satellites, bring each of them on specific orbits constituting together the desired constellation. A first optimization is performed by solving a combinatorial optimization problem, working on a simplified model and linear oriented procedure. Then, since the satellite orbit affectation is fixed, an optimal orbit transfer is performed on each satellite to provide, for each of them, the optimal sequence of maneuvers, taking into account the operational constraints and working on a more realistic model.

Finally, the above approach is illustrated through numerical experiments.

**Key words** : satellite constellation, station acquisition, orbital transfer, linear and nonlinear programming methods.

### Introduction

The station acquisition consists in positioning different satellites on several orbits, in order to form a given constellation and, therefore, to enable each satellite to carry out its mission. The studied constellation geometry is defined thanks to Walker parameters[1].

The build up of a constellation can be split in two important steps : first, the affectation of the different satellites and then, the transfer of each satellite from its initial position to the final one.

The proposed build up strategy consists on carrying out the transfer of each satellite in two stages, using the effect of the Earth oblateness. Each satellite will be transferred to two intermediary drift orbits before reaching the target one.

Such strategy supposes that we are treating the case of low-orbits and that the available drift time is sufficient to obtain an effective parameter's correction. The relative duration of each stage, and thereby the thrust dates, has to be optimized.

Both the affectation and the maneuver's dates are determined in order to minimize the global consumption of the constellation. In order to express the criterion, the dynamical model of the satellite has been simplified and the affectation problem which is combinatorial in nature has been solved through some linear programming procedure[4]. Out-of-plane maneuvers can be scheduled.

Once the affectation of the satellites has been solved, the next step consists in performing an optimal orbit transfer for each satellite, from its after launch initial orbit to the final one determined in the affectation phase. So, one has to solve, for each satellite, an optimal control problem which will be undertaken in an successive approximation approach, by performing successive refinements on the model, the constraints and the cost.

In this multistage multilevel procedure, the “optimal’’ solution provided by one level is used to initialize the new iteration which will provide an enhancement of the solution.

So, the global problem of station acquisition is treated step by step, by successive approximation and increasing complexity.

The paper will mainly focus on the second step (i.e. optimal orbit transfer) of the procedure.

## The Dynamical Model

This paper deals with homogeneous constellation : satellites composing it have the same altitude and inclination but with different anomalies and various orbital planes. Orbits are assumed to be quasi-circular. For a given time, satellite’s position is characterized with four parameters : the orbit semimajor axis (a), the right ascension of the ascending node ($\Omega$), the anomaly ($\alpha$) and the inclination (i). These parameters are affected by different perturbations. Indeed, several forces act on the satellite. Let us express the effects of a perturbation $\gamma$ on these parameters. Let us note $\gamma_n$, $\gamma_t$, $\gamma_w$ the components of this perturbation along the tangent, out-of-plane and normal directions (i.e. the direction of the velocity, direction of the angular momentum vector of the orbit and the normal direction that completes the trihedron). Afterwards, we will not consider the normal acceleration ($\gamma_n$=0). The gauss equations[2] for a near-circular orbits are given by :

$$\begin{aligned}
\frac{da}{dt} &= \frac{2a\gamma_t}{V} \\
\frac{di}{dt} &= \cos\alpha \cdot \frac{\gamma_w}{V} \\
\frac{d\Omega}{dt} &= \frac{\sin\alpha}{\sin i} \cdot \frac{\gamma_w}{V} \\
\frac{d\alpha}{dt} &= n - \frac{\sin\alpha}{\tan i} \cdot \frac{\gamma_w}{V}
\end{aligned} \tag{1}$$

n represents the orbit mean motion ($n=(\mu/a^3)^{1/2}$ with $\mu$ the Earth gravity constant). For quasi-circular orbits, the velocity V is given by :

$$V = a \,.\, n \tag{2}$$

We will consider two perturbations : Earth oblateness and thrust delivered by the propellant.

### J2 Effects

As this problem deals with LEO satellites, the inclusion of the effect of the first-order harmonic is important. We will assume that only the anomaly and the RAAN will be affected. Their variation in an interval of time $\Delta t$ is given by :

$$\frac{\Delta\Omega}{\Delta t} = -\frac{3\sqrt{\mu}\, J_2\, a_e^2}{2a^{3.5}} \cos i \tag{3}$$

$$\frac{\Delta\alpha}{\Delta t} = n + \frac{3\sqrt{\mu}\, J_2\, a_e^2}{2a^{3.5}} (4\cos^2 i - 1) \tag{4}$$

where $a_e$ represents the equatorial radius of the Earth.

Thereafter, we introduce **A** and **B** defined by :

$$A = -\frac{3\sqrt{\mu}\, J_2\, a_e^2}{2} \cos i \tag{5}$$

$$B = \frac{3\sqrt{\mu}\, J_2\, a_e^2}{2} (4\cos^2 i - 1) \tag{6}$$

### Thrust Effects

The thrust will be considered instantaneous. From the set of equations (1), we can express the evolution of the orbital parameters resulting from velocity increments ($V_t$,$V_w$) :

$$\Delta a = \frac{2}{n} \cdot V_t \tag{7}$$

$$\Delta i = \cos\alpha \cdot \frac{\sqrt{a}}{\sqrt{\mu}} \cdot V_w \tag{8}$$

$$\Delta\Omega = \frac{\sin\alpha}{\sin i} \cdot \frac{\sqrt{a}}{\sqrt{\mu}} \cdot V_w \tag{9}$$

The effects of the out-of-plane thrust on the anomaly will be neglected, so that this parameter is only affected by the variation of the semimajor axis (via the variation of the orbit mean motion).

From the above equations, we can express the evolution of the different parameters.

### The Orbit Acquisition Strategy

The positioning of each satellite on its final position will be performed in three steps of maneuvers. The first one brings the satellite on an orbit so that the differential drift between the target and the satellite is mainly used to reach the nominal and final orbital plane (it is first assumed that the inclination is nominal). This phase is intended to correct first the RAAN and out-of-plane maneuvers can be added when the differential drift is not sufficient.

The second maneuver is then produced to define an orbit so that, during the remaining time for station acquisition, the final anomaly is achieved, always using a differential drift. The third and final maneuver is given to zero this differential drift, i.e. bringing the satellite on this final position. If used, the out-of-plane maneuver will be constrained to be applied at the same time than the tangential one. Thus, in this strategy one can see two main phases : the first is intended for RAAN acquisition and the second for anomaly one.

Let us now express the evolution of the orbital parameters with the proposed strategy. Equations (3), (5) and (9) give the variation of the RAAN and equations (4) and (6) the evolution of the anomaly. The final values of these two parameters are:

$$\Omega_f = \Omega_0 + A\frac{\Delta t_1}{a_1^{3.5}} + A\frac{\Delta t_2}{a_2^{3.5}} + \sum_{k=1}^{k=3}\left(\frac{\sin\alpha_k}{\sin i}\cdot\frac{\sqrt{a_{k-1}}}{\sqrt{\mu}}\cdot V_{w_k}\right) \qquad \textbf{(10)}$$

$$\alpha_f = \alpha_0 + \sqrt{\mu}\,\frac{\Delta t_1}{a_1^{1.5}} + \sqrt{\mu}\,\frac{\Delta t_2}{a_2^{1.5}} + B\frac{\Delta t_1}{a_1^{3.5}} + B\frac{\Delta t_2}{a_2^{3.5}}$$

Where :

$\Omega_0$ and $\alpha_0$ represent the initial position, $a_0$ is the initial semimajor axis, $a_1$ (resp. $a_2$) the semimajor axis of the first (resp. the second) phase. $\Delta t_1$ (resp. $\Delta t_2$) is the duration of the first (resp. the second) phase. $Vw_k$ is the $k^{th}$ out-of-plane thrust and $\alpha_k$ is the $k^{th}$ anomaly where the thrust is applied. The efficiency of this thrust on the correction of the RAAN depends on this anomaly. Afterwards, $\alpha_k$ will be taken equal to $\pi/2$ when the out-of-plane thrust aims to correct only the RAAN.

Each satellite has to reach a given position that can be characterized by its anomaly ($\alpha_c$), its RAAN ($\Omega_c$) and its semimajor axis ($a_c$). Therefore, the orbit acquisition consists in the minimization of the error between the real final position and the target one. This represents the constraints of our problem. The figure (1) visualizes the proposed strategy.

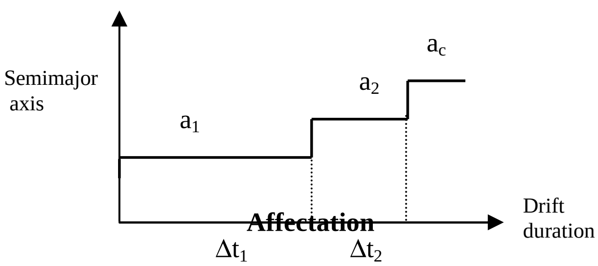


**Figure 1 : station acquisition strategy**

**Affectation**

First of all, when positioning a constellation, one deals with the affectation of the different satellites.

The best assignment of the different satellites is the one that minimizes the total build up cost. Once the target constellation is well defined and for a given relative duration of the two stages, this problem can be formulated as a linear optimization problem :

$$\underset{X_{j,k}}{\mathrm{Min}}\ \left(\sum_{j=1}^{N}\sum_{k=1}^{N} X_{j,k}\cdot C_{j,k}\right) \qquad (11)$$

where $X_{j,k}$ is equal to **1** when the $j^{th}$ satellite has to reach the $k^{th}$ position, and **0** if not. $C_{j,k}$ represents the cost of the transfer of the $j^{th}$ satellite to the $k^{th}$ position. It is given by :

$$C_{j,k} = \sum_{s=1}^{s=3}\sqrt{V_{ts}^2 + V_{ws}^2} \qquad (12)$$

s represents the index of the maneuver. We assume that the tangential and out-of-plane thrusts are applied at the same time and that only the anomaly and the RAAN are corrected.

The solution of this problem is computed by the Hungarian method developed by H.W. Khun[3]. For that, the elementary cost $C_{j,k}$ has to be computed. A linear formulation has been proposed in a previous work[4]. However, as the dynamics of our system are nonlinear, such assumption can lead to wrong estimation of the optimal cost. That is why a nonlinear formulation of the model has been developed (equations (10)).

Now, let us present the resolution of an orbit acquisition.

**Calculation of the Transfer Cost (Initial Guess)**

As seen previously, the differential drift between the target and the satellite is used to correct the two parameters. The target position and the set of equations (10) give the constraints to respect. In order to compute the cost, we have to find the two intermediary semimajor axis and the three out-of-plane thrusts that satisfy the constraints. In a first step, we will first not consider the out-of-plane thrusts ($V_w$=0) . The constraints become a set of two nonlinear equations depending on two variables. A Newtonian method[5,6] has been used to find a solution of this problem. However, such method assumes that we have a good guess of the solution. This initial guess is found as follows :

1) the target position is generally given at the end of the station acquisition. But, it can be useful to have it at any moment of the station acquisition.
   Thanks to the equation (3), we can compute the target position at any moment.
   For instance, the initial target position is given by:

$$\Omega_c^0 = \Omega_c - \left( A_c \frac{\Delta t}{a_c^{3.5}} \right) \quad (13)$$

$$\alpha_c^i = \alpha_c - \left( \sqrt{\mu} \frac{\Delta t}{a_c^{1.5}} + B_c \frac{\Delta t}{a_c^{3.5}} \right) \quad (14)$$

where $\mathbf{A_c}$ and $\mathbf{B_c}$ are given by the equations (5) and (6) (where the inclination is taken equal to the target one ($i_c$)). $\alpha_c$ and $\Omega_c$ characterize the final target position.

2) we assume that the first stage is sufficient to correct the RAAN. This mean that we reach the nominal position at the end of the first stage (i.e. $a_2 = a_c$). The initial error on the RAAN is corrected thanks to the differential natural drift between the target and the satellite.

$$\Delta\Omega = \delta\Omega_{target} - \delta\Omega_{real} = \Omega^0_{real} - \Omega^0_{target} \quad (15)$$

Where, namely, $\delta\Omega_{target}$ and $\delta\Omega_{real}$ represents the natural drift of the target and real RAAN, during the total build up duration. The right part of the equations (15) represents the initial error between the target and the satellite.
From the equations (15) and (3), we can determine the first semimajor axis and so, we can compute the tangential increments of velocity needed to realize this transfer.

3) However, the nominal anomaly is generally not reached at the end of the first stage. That is why the second stage is necessary ($a_2 \neq a_c$). We can calculate the real final anomaly reached at the end of the first stage. As for the case of the RAAN, the difference between the target anomaly and the real one, at the end of the first stage, has to be corrected by the differential natural drift in anomaly.

$$\Delta\alpha = \delta\alpha_{target} - \delta\alpha_{real} = \alpha^1_{real} - \alpha^1_{target} \quad (16)$$

Where, namely, $\delta\alpha_{target}$ and $\delta\alpha_{real}$ represents the natural drift of the target and real anomaly, during the second stage. The right part of the equations (16) represents the error between the target and the satellite at the end of the first stage. From the equations (16) and (4), we determine the value of the second semimajor axis.

4) As the semimajor axis of the second stage changes, the differential drift in the first stage can be insufficient to correct the RAAN. The three out-of-plane thrusts can be applied in order to correct the final error in the RAAN.

This initial guess gives a good estimation of the cost of the transfer of each satellite from its initial position to a given one. As this initial solution is faster than the Newtonian one, it will be used in the global optimization that determine the optimal assignment.

### Drift Times and Target Constellation

Our strategy divides the build up in two stages. We have proposed a method to optimize the affectation for a given relative drift duration. But, the relative duration of the two stages remains unknown (no constraints impose it). Thus, we have to determine the best distribution of the drift time.

Since the variation of $\Omega$ is the slowest and the most expensive, the duration of the first stage will be the longest. The relative duration of the two stages has to be optimized in order to minimize the cost. However, since the total duration of the station acquisition is bounded, it is enough to optimize only the first one.

For each value of the drift times, we can optimize the affectation of the different satellites as seen previously. Sometimes, only the global geometry of the constellation is given. In that case, we can also optimize one target position. The others are determined so that the final geometry is respected.

The drift time and, possibly, one target position are optimized using the Nelder & Mead simplex[8]. Compared to the optimization of the affectation, this optimization is in an upper level. Indeed, for each intermediary drift time and target constellation, the optimal affectation is determined and its cost will be the one used as a criterion of the Nelder method.

### Optimization of Satellite's Transfer

Previously, we presented the global optimization that gives us the relative duration of the two stages, the target positions, and the affectation of the different satellites. Now, we have to transfer each satellite from its initial position to the nominal final one. The initial guess used before (when optimizing the affectation) is not optimal. Indeed, the out-of-plane thrusts have been computed to correct the RAAN without optimizing them. In this section we will propose a method of refinement of the obtained solution.
The orbital transfer can be formulated as a minimization of a nonlinear criterion (the cost) with nonlinear equality constraints (the set of equations (10)). Two formulations of the constraints are possible : in the first one, one

needs to know the number of revolutions done by the angles. Indeed, the drift presented in (10) does not give the parameter's variation ranging between 0 and 2 π. We can initialize the number of revolutions of the variation with the one given by the initial solution. In this case, the constraints are :

$$\begin{aligned}\Delta\Omega &= \delta\Omega_{target} - \delta\Omega_{real} = \Omega^0_{real} - \Omega^0_{target} + 2\pi\cdot k_1 \\ \Delta\alpha &= \delta\alpha_{target} - \delta\alpha_{real} = \alpha^0_{real} - \alpha^0_{target} + 2\pi\cdot k_2\end{aligned} \qquad (17)$$

where the natural drift is computed during the two stages (as in (10)). k1 and k2 are the number of revolutions given by the initial solution.

The second formulation of the constraints is more simplified. The set of equations (10) gives the constraints : the final real parameters have to be equals to the target ones. The difference between them will be expressed between 0 and 2 π.

The variables of optimization are the two semimajor axis ($a_1$,$a_2$) and the three out-of-plane thrusts.

Since an initial solution is available, we use the Generalized reduced Gradient[5,7]: The out-of-plane thrusts vary so that the cost is minimized. Then, we compute the two semimajor axis that realize the new constraints (when applying these out-of-plane thrusts). As the variables of the optimization have to be greater or equal to zero, We have to pay attention to the sign of the out-of-plane thrusts. Here, the relative duration of the two steps is constant and equal to the value given by the global optimization.

Once the optimization of the orbit acquisition of each satellite is completed, we can verify the validity of the found affectation.

## Refinement of the Solution

We have already presented a basic solution of the problem of the station acquisition. For each satellite composing the constellation, the proposed method gives the optimal orbital acquisition. Only the RAAN, the anomaly and the semimajor axis have been corrected. In addition, the same drift times have been considered for all the satellites. However, the found solution could be sub optimal. Let us now try to improve it.

### Relaxation on the Thrust Dates

Previously, the thrust dates were optimized so that the total cost of the constellation station acquisition is minimized. The previous optimization of the relative duration of the two stages is equivalent to optimize the date of the intermediary thrust. It was the same for all the satellites. Now, let us optimize the thrust dates for each satellite. The first one will be always applied at the beginning. We can optimize either the date of the intermediary thrust only, or those of the two last ones.

First, we will optimize the second thrust date. The problem we have to solve is a constrained optimization: The constellation operator defines the interval I to which belongs the second thrust date (and thus the relative duration of the two stages). We used the Nelder simplex[8]. As this method is a unconstrained optimization method, the constraints on the time will appear in the criterion:

$$\mathrm{Min}(J)$$

$$J = \begin{cases} \mathrm{optimal\_cost}(a_1, a_2, V_{w1}, V_{w2}, V_{w3}) & \text{if } \Delta t_1 \in I \\ +\infty & \text{if not} \end{cases} \qquad (18)$$

where optimal_cost indicates the cost for orbital transfer. This cost is the result of the optimization of the transfer for a fixed time (using the GRG method).

Thus, each satellite will have its own intermediary thrust date that minimizes its consumption.

In what precedes, the last thrust was scheduled at the end of the build up. As for the intermediary thrust date, this thrust date can be *relaxed* so that the solution of the optimal cost is improved. We use the same method as before, where we add a constraint on the final thrust date. So the criterion of optimization becomes :

$$\mathrm{Min}(J_2) \;\; / \;\; J_2 = \begin{cases} \mathrm{optimal\_cost}(a_1, a_2, V_{w1}, V_{w2}, V_{w3}) \\ \qquad \text{if } (\Delta t_1 \in I_1)\&(\Delta t_2 \in I_2) \\ +\infty \qquad \text{if not} \end{cases} \qquad (19)$$

The constellation operator will define the intervals $I_1$ and $I_2$. The last thrust has to be scheduled at the latest at the end of the total built up duration.

### Hohmann's Transfer

In what precedes, we have presented a solution of the optimal orbit acquisition of a satellite where we assume that we deal with circular orbits. The used optimization can lead to important semimajor axis differential gaps, so that the eccentricity is affected too much. Indeed, one needs, at least, two tangential thrusts for transferring a satellite between two circular orbits. Thus, the station acquisition is achieved in six steps of thrust (instead of three). We have to introduce the evolution of the RAAN and the anomaly during the Hohmann's transfer. This changes the constraints (17) and the criterion (12) since we have three additional thrusts.

As previously, this problem has been solved using the GRG method. The thrust dates could be also optimized.

In fact, The previous methods remains valid, only the cost and the constraints change, being more involved.

**Correction of the Inclination**

We have not corrected the inclination yet. The satellite is transferred to a final orbit that can have a different inclination than the target one. Such a constellation could not be able to carry out correctly its mission. Thus, we have to introduce this new constraint in our problem. However, the effect of the Earth oblateness depends also on the inclination. Thus, depending on inclination's value, the drift of the orbital parameters can be more or less important and so, the cost can be sub optimal. In fact, we have not only to correct the inclination but also to find the solution that costs the least. To solve directly such a problem will be too complicated. Thus, we used the following strategy : the inclination is corrected either at the beginning of the station acquisition, or at the end or at the intermediary thrust date. For each pattern, the cost will be optimized using the methods presented above. Then, the three costs will be compared and the least one will be chosen as a solution of the problem. Let us now express how the global problem is affected.

Previously, we have optimized the orbital transfer of a satellite in two stages, by transferring it to intermediary orbits with the same inclination. Here, the constraints remains the same with different inclination : each intermediary orbit has its own inclination. Depending on when it is corrected, the inclination is equal to the initial one or to the target one. The formulation of the criterion remains the same (12). However, from (8) and (9), we notice that the efficiency of the out-of-plane thrust depends on where it is applied. Before, we assume that $\alpha$ was equal to $\pi/2$. Thus, for correcting both of the RAAN and the inclination, the set of equation (8) and (9) gives the out-of-plane thrust and the place where it will be applied :

$$\begin{aligned} V_w &= \pm V\sqrt{\Delta i^2 + (\Delta\Omega \cdot \sin i)^2} \\ \alpha &= \arctan\left(\frac{\Delta\Omega \cdot \sin i}{\Delta i}\right) \end{aligned} \tag{20}$$

**More Refinement**

We presented an optimization for the problem of station acquisition using a simplified model of the satellites dynamics. However, we can obtain a more accurate solution by successive iterations, using an accurate tool of simulation. Thus, the proposed model is a rather good one and could be improved in order to achieve the station acquisition with the imposed precision.

Since we deal with low Earth orbits, we have introduced the effect of the atmospheric drag for different values of solar activity. The effect of the atmospheric drag on the semimajor axis has been discretized so that we achieve the station acquisition in two stages with several intermediary semimajor axis. Thus, the constraints will be similar to (10) but with several under stages. Successive iterations can be added to improve the precision of the station acquisition.

In addition, normal thrusts can be introduced to correct the eccentricity. In fact, the Hohmann's transfer assumes transfer between circular orbits. We used it for quasi-circular orbits, which can lead to errors. Sometimes, the imposed precision in eccentricity can not be realized. Thus, normal thrusts has been optimized using the Nelder simplex.

**Results and Numerical Comparison**

In this section, we will applied the proposed methods to the following example: the station acquisition of a 6-satellites constellation on 3 orbital planes and with no phase gaps between planes (Walker 6/3/0). All the satellites have an inclination of 53° and an altitude of 1000 km. The build up duration will be taken equal to 360 days. Initially, the satellites are on a circular orbit, with a semimajor axis of 6900 km and at an inclination of 53°. The initial anomaly and RAAN are zero. However, launchers generally inject the satellites with some errors. This will be taken into account by introducing stochastic errors in the initial positions (table 1).

**Table 1: initial positions of the satellites**

| N. sat. | Semimajor axis | $\Omega_0$ | $\alpha_0$ | $i_0$ |
|---|---|---|---|---|
| 1 | 6900.22 km | 359.9° | 359.96° | 52.99° |
| 2 | 6899.74 km | 0.07° | 0.04° | 52.98° |
| 3 | 6900.11 km | 0.15° | 359.93° | 52.96° |
| 4 | 6899.17 km | 0.17° | 0.13° | 52.97° |
| 5 | 6899.92km | 359.98° | 0.07° | 52.96° |
| 6 | 6900.16 km | 0.04° | 359.98° | 52.96° |

First of all, the global optimization gives us the affectation of the satellites and the drift duration. We optimized one target position ($\Omega_{c0}$ ,$\alpha_{c0}$ ), the others will be deduced from it. We found :

$\Delta t_1$ = 343.799 days
$\Omega_{c0}$ = 96.0°
$\alpha_{c0}$ = 117.67°

Global cost =1566.33 m/s

Thus, the other target RAAN will be either 216.08° or 336.08° (orbital planes spaced with 120°). The other target anomalies can only be worth to 297.67°. The best affectation of each satellite is given in the table (2).

**Table 2: target positions of the satellites**

| N. sat. | $\Omega_c$ | $\alpha_c$ |
|---|---|---|
| 1 | 216.08° | 117.67° |
| 2 | 216.08° | 297.67° |
| 3 | 96.08° | 117.67° |
| 4 | 96.08° | 297.67° |
| 5 | 336.08° | 297.67° |
| 6 | 336.08° | 117.67° |

The cost used to determine this assignment is computed from the initial guess showed on the following figure.

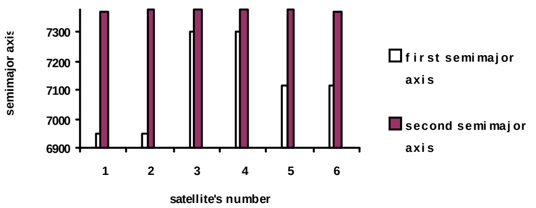


**Figure 2: semimajor axis found by global optimization**

From the figure 2, we notice that the satellites are clustered into three groups in the first stage. This is due to the fact that this stage is mainly used to correct the RAAN. Indeed, the target constellation is composed of three orbital planes, at the rate of two satellites per orbit. In the second stage, the semimajor axis are close to the target one. Thus, out-of-plane thrusts we have added to achieve the RAAN are rather small. The final RAAN and anomaly have been achieved with a precision of $10^{-3}$ degrees. This allow us to have an idea about the optimal affectation and about the relative duration of the two stages. Now, we will realize the orbit acquisition of each satellite. We will treat the case of only one satellite (for instance the first one).

The orbit transfer is first determined by the global optimization and then, optimized. It is given in the table below.

**Table 3: solution for the orbit transfer**

| | **Initial guess** | **Optimized solution** |
|---|---|---|
| $a_1$ | 6951.40 km | 6953.94km |
| $a_2$ | 7373.50 km | 7311.57km |
| $V_{w1}$ | 4.23 m/s | -3.96 m/s |
| $V_{w2}$ | 8.42 m/s | 0.00 m/s |
| $V_{w3}$ | 1.36 m/s | -3 .43 m/s |
| V | 261.24 m/s | 258.27 m/s |

We notice that the differential semimajor axis gaps are more important than those found before. For the first semimajor axis, its value has increased, so the differential drift of the target and the satellite has decreased. On the contrary, the second semimajor axis has decreased and thus the differential drift has increased. This is due to the fact that we have introduced a more accurate model. Indeed, we correct both of the anomaly and RAAN during the two stages (see equations (17)). Therefore, we need a less important variation of the RAAN in the first stage and a more important one in the second one. As a result, the out-of-plane thrusts decrease and so does the cost. These above results are given for the relative duration of the two stages found by the global optimization. This time can be relaxed. The interval of optimization will be taken centered around the previous second thrust date, with a variation of ±5 days. The optimization of these times gives :

$t_1$ = 338.799 days
V = 256.91 m/s.

We notice that the found date is the beginning of the imposed interval. This could be explained by the fact that the function is monotonous on this interval.

The last thrust date can be also relaxed. We impose that it is scheduled at more 5 days before the end of the build up. The dates of the two last maneuvers are given by:

$t_1$ = 338.798 days
$t_2$ = 355.00 days
V= 256.08 m/s

Thus, the satellite reaches its nominal orbit 5 days before the end of the station acquisition. This means that the found solution is a local optimum. We can probably improve the cost if the imposed intervals were larger.

We will now use the Hohmann transfer for the orbit acquisition. The found cost would be less important than that found by the previous method. This is due to the fact that the efficiency of a tangential thrust depends on the semimajor axis where it is applied (see equation (7)). Indeed, we found a cost about 251.27 m/s.

In addition, such a transfer permits to keep the eccentricity close to zero and thus, to have a more operational solution.

If the inclination is also corrected, we should have a more important cost. Indeed, the out-of-plane thrusts have to correct both of the RAAN and the inclination. In this example, the inclination is corrected by a thrust at the beginning of the second stage (previously, this thrust was found equal to zero). The error in inclination (about

-0.004°), is corrected by a thrust of an amplitude of 0.55 m/s. Thus the orbit acquisition cost increases and is equal to 258.34 m/s.

The atmospheric perturbation can also be added. Without taking into account this perturbation in the considered model and for a weak solar activity level, the nominal final position is achieved with an error of about 1° for the anomaly, 0.1° for the RAAN and 2 m for the semimajor axis. In this solution, out-of-plane thrusts have not been used. However, when introducing the effect of the atmospheric drag in our model, a more accurate solution is found and thus the precision is improved (about $10^{-7}$ degrees).

These results show well the efficiency of the successive refinements in solving the problem.

## Conclusion

We have proposed an optimal (or nearly optimal) strategy for the station acquisition of a homogeneous constellation of LEO satellites. A build up of the constellation is achieved in two phases. First, the affectation of the satellites is optimized. Then, the optimal orbit transfer of each satellite is performed.

The proposed strategy split the station acquisition into two stages, enabling to profit the effects of the Earth oblateness on the orbital parameters. Once the optimal affectation is fixed, we have performed an optimal orbit transfer of each satellite. It has been performed by successive refinements on the model, the cost and the constraints.

We finally gave some numerical results of our multistage multilevel strategy. An enhancement of the solution found is possible by using a simulation tool that works on a more accurate model.

Afterwards, it will be interesting to take into account the real duration of burns, and thus, to try to perform a solution of a low-thrust transfer problem, which will works on the ”optimal“ solution provided by the proposed methods.

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