

# LUNAR PROSPECTOR ORBIT DETERMINATION AND GRAVITY FIELD MODELING BASED ON WEILHEIM 3-WAY DOPPLER MEASUREMENTS

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## Abstract

In the summer of 1998, the Lunar Prospector (LP) spacecraft was tracked from DLR's Weilheim ground station. Over a period of six weeks passive 3-way Doppler data have been collected by the 30 m deep space antenna making use of the available uplink stations of the Deep Space Network (DSN). The paper describes the tracking campaign and the subsequent data analysis using both Geodyn II and DLR's in house orbit determination software DEEPEST. The original measurements exhibit a noise level of 0.5 mm/s at count times of 30 s. Taking advantage of the LP75G gravity model, the data could be modeled to an accuracy of typically 6 mm/s rms in weekly orbit determinations. Attempts were also made to model the 0.2 rev/s rotation of the LP spacecraft, which is evident as a periodic signal with a  $\sim 7$  mm/s amplitude in tracking data sampled at a 1 s interval, but proved to be unsatisfactory due to an apparent instability of the rotation rate. Finally all tracking data were processed using the GEODYN II & SOLVE program available at TU Delft to generate partial derivatives for gravity field coefficients and demonstrate the basic capability of solving for a global 75x75 gravity model.

**Key words:** Lunar Prospector, Doppler Tracking, Orbit Determination, Gravity Field

## Lunar Prospector Tracking

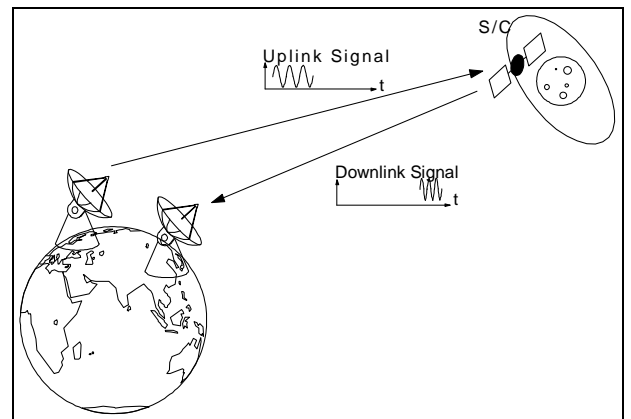
A six weeks tracking campaign of the Lunar Prospector (LP) spacecraft was conducted at DLR's Weilheim ground station (Fig. 1) in summer 1998 with special permission of the LP project. Tracking started on July 6<sup>th</sup> (DOY 187) and was terminated on August 15<sup>th</sup> (DOY 227). Throughout this period, passive 3-way Doppler measurements (Fig. 2) were collected by DLR's 30 m antenna whenever LP was visible from Weilheim



and uplinks were provided by NASA's Deep Space Network (DSN).

## Figure 1: DLR's 30m Deep Space antenna at Weilheim

The uplink signal generated by the 26 m and 34 m stations at Madrid (DSS 61, 66), Canberra (DSS 42, 46) or Goldstone (DSS 16, 24, 27) was received by the Lunar Prospector spacecraft, retransmitted coherently, and finally received by the 30 m antenna in Weilheim. Here, the downlink signal with a nominal carrier frequency of 2273 MHz was processed by an ESA standard Multi Purpose Tracking System (MPTS)<sup>1</sup> after passing the low noise amplifier and a recently installed low-Earth S-band receiver.



## Figure 2: 3-way Doppler tracking principle

The measurement principle, which is known as 3-way Doppler tracking, differs from common 2-way measurements by using separate uplink and downlink stations. In contrast to 2-way measurements, drifts or biases in the uplink frequency generation are not, however, cancelled by corresponding errors in the frequency standard of the Doppler measurement unit. A hydrogen maser atomic clock operated at the Weilheim ground station<sup>2</sup> was therefore connected to the MPTS system to provide a highly accurate and stable reference for measuring the carrier frequency of the received downlink signal.

Non-destructive Doppler measurements were registered at a count interval of 1 s, but later-on combined into effective 30 s count intervals for orbit determination purposes. The measurements exhibit a  $1\sigma$  noise level of about 3 mm/s at 1 s count intervals or, equivalently, 0.5 mm/s at count times of 30 s. Besides a notably reduced data noise, the integrated measurements are essentially free of periodic Doppler variations caused by the 5 s/rev spacecraft rotation.

The Doppler counts were subsequently converted to average range rate measurements using the nominal LP uplink frequency. Furthermore, the data preprocessing comprised the assignment of the proper uplink station identification to the data records, based on the forecast of the DSN operations schedule. Additionally a residual monitoring was performed, followed by a manual data editing to remove records affected by uplink station transitions or occultation phases.

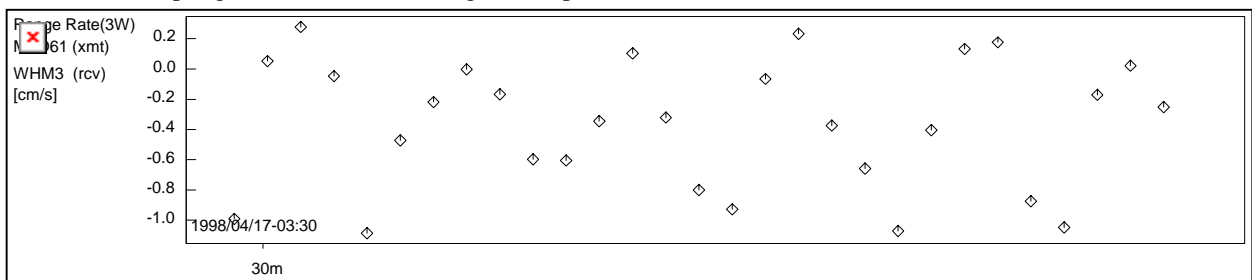
### Spacecraft Rotation

Lunar Prospector is equipped with helical low-gain and medium gain antennas, which are nominally aligned with the s/c spin axis. Any radial offset between the spin axis and the symmetry axis of the receiving and/or transmitting antenna results in an oscillation of the phase center with a period equal to the spin period of 5 s. At short Doppler count intervals (1 s), the s/c rotation with a amplitude  $\delta v$  of about 7 mm/s is clearly discernible from the tracking data (Fig. 3). This finding is in agreement with Beckmann and Concha<sup>3</sup>, and indicates an antenna offset of  $\delta v/\omega$  or 6 mm, where  $\omega$  is the s/c angular rotation rate. Using a data arc of 30 min duration with 1 s count interval, a s/c rotation period of  $P=5.02\pm 0.02$  s was derived by a Fourier analysis of the tracking data residuals. Besides the main signal a superimposed oscillation with twice the frequency and about  $1/3^{\text{rd}}$  the amplitude was found. It indicates a notable asymmetry of the periodic phase center variation but remains otherwise unexplained.

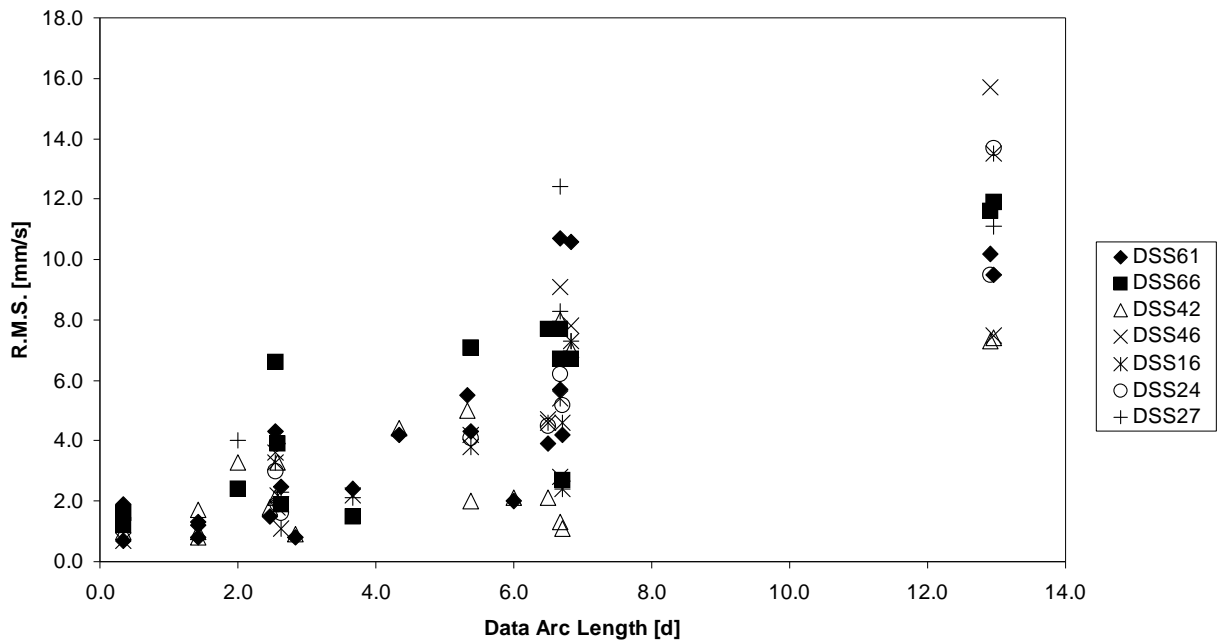
Since the spin period differs slightly from an ideal value of 5 s, it cannot fully be compensated by choosing count or sampling intervals that are integer multiples of

5 s. A pronounced beat pattern with a typical period of 30 min may e.g. be observed by selecting every 5<sup>th</sup> measurement from a set of 1 sec count interval data. The corresponding spin period of  $P = (1/5s - 1/1800s)^{-1} = 5.014$  s is compatible with the value derived above from Fourier analysis. In case of a 30 s count interval the same beat period applies but the amplitude of the averaged Doppler variation amounts to a mere 0.02 mm/s, which may be neglected in comparison with the overall data noise.

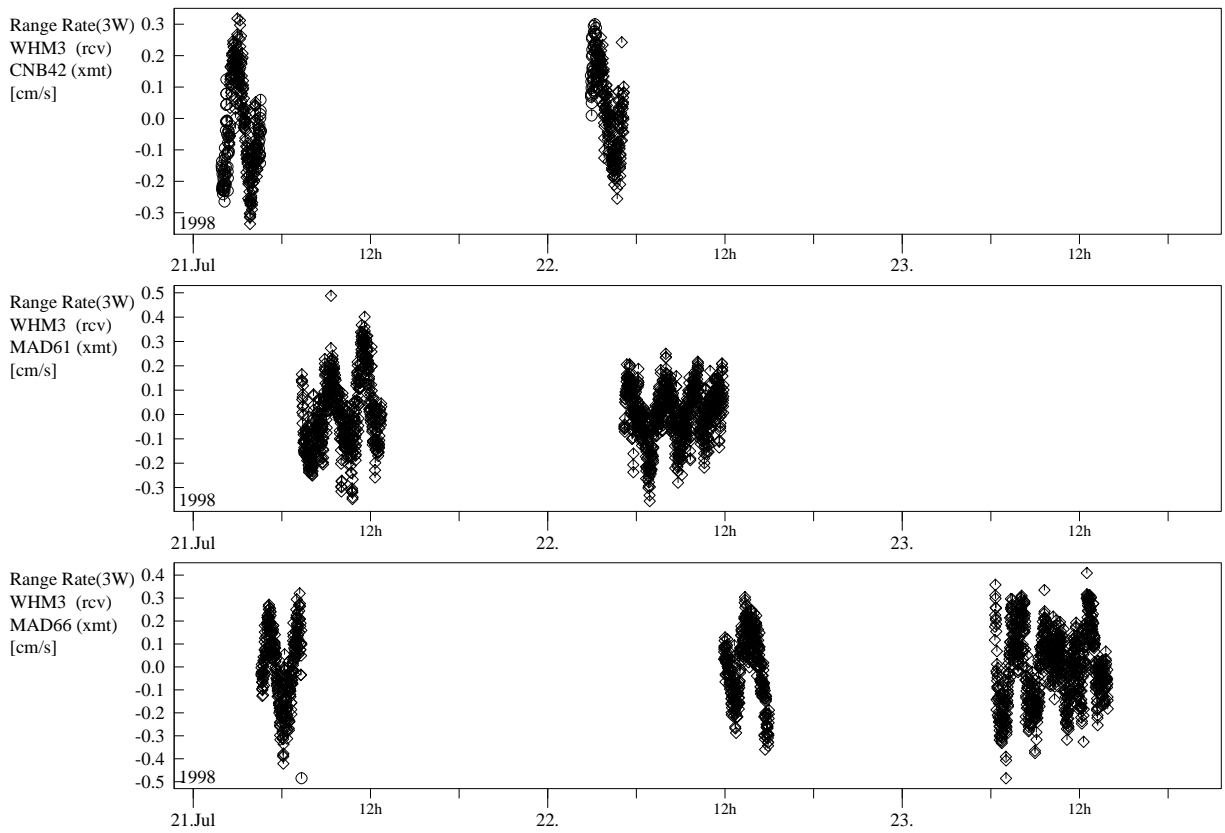
It is furthermore noted that the rotation induced Doppler variation could not be modeled in a satisfying manner over long data arcs, even when applying a bi-harmonic sinusoidal antenna rotation model. Apparently the rotation period is not strictly constant but subject to minor variations. These may be attributed to variations of the moment of inertia caused by thermal expansion of the booms of Lunar Prospector. Similar effects have been observed for other spinning s/c (McElrath, priv. comm.) but could not be quantified for LP in this study due to the non-availability of telemetry information.



**Figure 3:** A 30 s data arc showing the 5 s rotation period of Lunar Prospector. The periodic Doppler shift of  $\pm 7$  mm/s indicates a 6 mm offset between the antenna phase center and the actual spin axis.



**Figure 4:** Orbit determination range rate residuals (rms) from Madrid (DSS61, DSS66), Canberra (DSS 42, DSS46) and Goldstone (DSS16, DSS24, DSS27) for varying data arcs.



**Figure 5:** Sample Lunar Prospector range rate residuals. Receive station is Weilheim (WHM3), transmit stations are Canberra (DSS 42) and Madrid (DSS61, DSS66).

## Orbit Determination

The LP tracking data collected at Weilheim served as a test bed for GSOC's newly developed DEEPEST orbit determination software for deep space missions and planetary orbiters.

In case of a lunar orbiter, the force model accounts for a high degree and order gravity field of the Moon, the gravitational perturbations of the Sun, Earth and, optionally, planets as well as solar radiation pressure. Positions of the respective bodies are taken from precomputed JPL ephemerides. The (non-relativistic) equation of motion and the associated variational equations are integrated by a variable order, variable step-size multistep method, which is self-starting and supports interpolation as well as dense ephemeris output. For consistency with JPL's LP75G gravity model<sup>4</sup> applied in the data analysis, the numerically integrated DE403 libration angles were used to convert from EME2000 to the lunar body-fixed, principal axis system. All other transformations are based on conventional IAU expressions (precession, nutation, sidereal time) and IERS Earth rotation parameters.

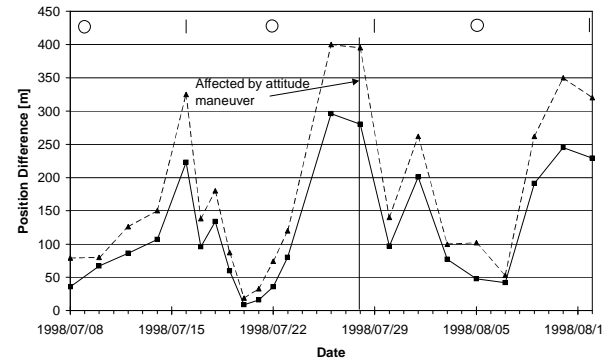
Using the above models, data were processed in batches varying in duration from several hours up to two weeks. In Fig. 4 the rms residual values from 25 orbit determination runs for different data arcs and stations are depicted. The Madrid antennas DSS61 and DSS66 share the highest common visibility with Weilheim among the DSN stations and thus imply long data arcs. As consequence, the rms of the residuals of Madrid tend to higher values than the residuals from Goldstone or Canberra. Otherwise, no systematic anomaly related to a specific station or antenna is apparent.

Under favorable conditions (face-on geometry) range rate residuals of less than 1 mm/s were obtained for 30 s count intervals and short and medium arcs, thus confirming both the accuracy of the tracking system and the good quality of the LP75G gravity model.

For long data arcs (Fig.5) with multiple uplink stations involved, individual Doppler biases were estimated, due to an incomplete knowledge of the actual DSN uplink frequency and the simplifying assumption of a nominal reference frequency in the tracking data preprocessing and range rate computation. The correlation of the bias parameters and the state vector components typically is at a level of 20% while the standard deviation of the bias estimates is typically 0.1 mm/s.

From Fig. 4 it is evident that in general both the mean and the maximum rms residuals increase with increasing data arcs. This phenomenon is clearly related with force modeling deficiencies and can most probably be attributed to restrictions of the LP75G gravity model.

Further improvements may be expected from higher degree and order models, that will become available after moving Lunar Prospector into a lower altitude orbit (Konopliv priv. comm.).



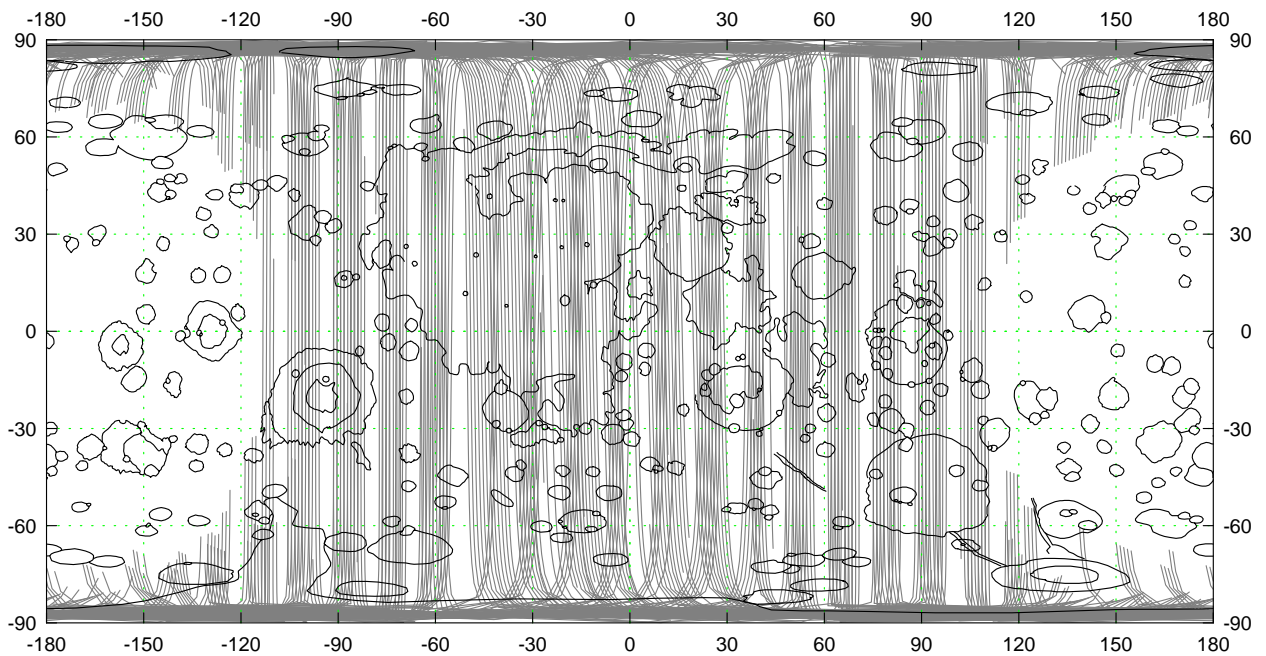
**Figure 6:** LP ephemeris overlap results using LP75G. Shown are maximum (dashed, triangle) and rms values (solid, rectangle) of the ephemeris differences. Face-on geometry is indicated as circles, edge-on geometry as vertical lines.

To assess the quality of the orbit determination solutions, ephemerides from different solutions are numerically compared. To this end, data arcs of about 2.5 days have been applied for orbit determination and 3 day ephemerides have been generated. Two subsequent trajectories overlap by 1 day and the maximum position and rms values have been computed. The results are depicted in Fig. 6 for an interval of one month.

It is noted that an attitude trim maneuver has been conducted, starting on 1998/07/27 16:10:17 UTC, to erect the spacecraft spin axis normal to the ecliptic plane. The maneuver has been modeled and estimated within the DEEPEST s/w and yields an effective velocity increment of about 2 cm/s. The mapping of remaining maneuver modeling deficiencies on the ephemeris overlap at 1998/07/28 is estimated at the order of 100 m.

The maximum rms value of the position difference is 300 m, as compared to an operational GSFC requirement for the definitive LP ephemeris of 1000 m rms in each component. The maximum position error is exceeding the rms value by less than 50%.

The temporal variation of the overlap results in Fig. 6 is obviously related to the observation geometry with higher errors during edge-on geometry (orbit normal perpendicular to line of sight) and smaller errors near face-on phases. This is caused mainly by increased gravity model errors on the lunar far-side that are relevant for edge-on geometry, while the lunar gravity field for face-on geometry is reasonably well known.



Thus the orbital variation of the overlap ephemeris differences is much smaller for face-on geometry ( $\sim 10$  m) than for edge-on geometry ( $\sim 200$  m). The number of tracking measurements for edge-on phases is reduced by about 30% as compared to face-on phases, but does not significantly affect the results, since systematic errors are dominating. Furthermore, the singularity of the face-

**Figure 7:** Coverage of the lunar surface with Weilheim 3-way Doppler tracking data from 1998/07/05–1998/08/16.

on observation geometry is removed by extended tracking arcs of 2.5 day duration that provide sufficient variation in parallax. On 1998/07/29, a slightly higher error maximum is observed than during the edge-on phases 14 days before and after. This may partially be attributed to a residual mismodeling of the attitude trim maneuver, that occurred during this face-on period.

Prior to the Lunar Prospector mission, one of the state-of-the-art gravity models was the GLGM-2 model of the GSFC. It is a spherical harmonic model complete to degree and order 70 that has been developed from S-band Doppler tracking data from the Clementine mission, as well as historical tracking data from the Lunar Orbiters 1-5 and the Apollo 15 and 16 subsatellites<sup>5</sup>.

The tracking of Lunar Prospector has contributed substantially to an improved knowledge of the lunar gravity field. This may be demonstrated by comparisons of ephemeris overlaps using either GLGM-2 or LP75G. Overlaps from orbit determination runs using GLGM-2 for face-on geometry lead to differences of 9 km and 4 km, in contrast with 0.04 km and 0.07 km for LP75G. The orbit determination runs were even found to diverge for edge-on geometry.

### Gravity Field Determination

In Fig. 7 the coverage of the lunar surface solely from Weilheim 3-way Doppler data is given. Although a coarse coverage of the lunar near-side has been

achieved, a sparse coverage of the pole regions is obvious as well as uncovered meridional stripes with about 150 km width. As a global 75 degree and order field corresponds to a surface resolution of 75 km, the collected tracking data do not suffice for the restitution of a high degree and order gravity field.

Nevertheless, to demonstrate the principal capability

and skill for the development of such a complex model, first attempts were finally made to adjust a 75 degree and order lunar gravity field model solely using the Weilheim 3-way Doppler measurements. At the time of the analysis, LP75D was the most advanced gravity model, that was therefore used as reference and a priori model. The relevant computations were carried out at TU Delft using a local version of GSFC's GEODYN II<sup>6</sup> and SOLVE<sup>7</sup> programs installed on a Cray J90 computer. To ensure consistency with the LP75D reference frame, a semi-analytical libration series compatible with DE403 (J. Williams, priv. comm.) was implemented into GEODYN. Partial derivatives and normal equations for a set of 75x75 gravity field coefficients were generated for three subsequent one week data batches (covering the weeks 28-30, where Weilheim data were available at that time) and later on combined within the SOLVE program. Kaula's rule for the a priori uncertainties of all coefficients of degree  $n$ , given by<sup>8</sup>  $1.5 \cdot 10^{-4}/n^2$ , was, furthermore, used to constrain the gravity field solution, since the given measurements alone do not allow the complete set of coefficients to be determined.

As result, the sample 75x75 gravity model JLGM-03 (Joint Lunar Gravity Model) was obtained and orbit determination runs for the three week batches were rerun with the new model. While the Doppler residual rms values using the LP75D model were 3 mm/s, 9 mm/s and 5 mm/s for the weeks 28-30, respectively, the corresponding figures reduced dramatically to 0.5 mm/s

for the full three-week arc while using JLGM-03.

To assess the derived lunar gravity model further, orbit determinations have been performed with independent data from calendar week 31. Surprisingly, it turned out, that the derived Doppler residual values are at an unacceptable level of 10 cm/s, inferior to the results from LP75D and clearly showing the limitations and the preliminary nature of JLGM-03. An unambiguous explanation for this finding is currently lacking and work is under way to identify and solve the problem. As described above, the coverage of the lunar surface with Doppler data is definitely not sufficient for a resolution of such a complex gravity field. Therefore attempts are currently under way to incorporate additional a priori information for the gravity field into SOLVE, that has been derived from the LP75G development process. Alternatively, a reduction of the order and degree of the estimated parameters within the 75x75 gravity model is possible. As third option, tests based on simulated tracking data are foreseen.

It is noted, that beyond all efforts for gravity field recovery from near-side tracking, a substantial improvement of the lunar gravity field is expected from far-side tracking of a lunar orbiter, as foreseen within the Japanese Selene project.

### Summary

A six week tracking campaign of the Lunar Prospector spacecraft has been conducted in summer 1998 with DLR's 30 m deep space antenna at Weilheim.

The collected 3-way Doppler tracking data were applied to qualify GSOC's newly developed orbit determination software DEEPEST for planetary orbiters and deep space probes. As result, range rate residuals were achieved with less than 1 mm/s rms for short tracking data arcs and a count interval of 30 s. A consistency analysis of the orbit determination results yielded position errors less than 300 m rms, when using the LP75G gravity model.

To demonstrate the development of a high degree and order lunar gravity model, the sample 75x75 gravity field JLGM-03 was derived, despite the lacking coverage of the lunar surface with Weilheim Doppler data. While the resulting gravity model is by no way competitive with models derived from continuous tracking, it marks an important step in European lunar gravity field analysis and demonstrates the readiness for more detailed investigations within the upcoming Selene and LunarSat missions.

### Acknowledgement

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