STUDY OF A SPLITTING METHOD FOR RENDEZVOUS MANEUVERS

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Abstract

Europe, through ESA, has decided the conception of an Automated Transfer Vehicle, ATV, to support the exploitation of the International Space Station ISS. Rendezvous between ATV and ISS was first studied on the base of a four maneuvers strategy satisfying Kuzmak rendezvous equations[1]. Injection orbits and motorization of ATV may imply such a long time of burn for the first maneuver application that it must be split. We developed an original maneuver splitting method which generates two subsystems of rendezvous equations and allows the use of more unknowns that are necessary to treat the splitting maneuver problem. We give numerical results in the case of the first maneuver splitting for ATV/ISS rendezvous showing the way our approach can generate 5 maneuvers new strategies dividing by two the first maneuver modulus with an over cost always insignificant.

***Key words**:* maneuver splitting, rendezvous strategy, Kuzmak equations.

Introduction

The first studies of optimal phasing strategies for rendezvous between ATV and ISS have shown that this problem belongs to a new class because ATV injection orbits are quite dangerous and need a very first perigee maneuver. In these circumstances, the first maneuver modulus of classical four maneuvers strategy can become very important, up to 150 meters per second. The specificity of the problem grows with the fact that vehicle motorization generates long times of burn, about half an orbital period for some first maneuvers application. As this can be an operational drawback, it becomes necessary to acquire an optimization method able to take into account inequality constraints under maneuver modulus.

To reach this goal, we choose to build an original method based on maneuver splitting. It constitutes a fitting of the approach used by KIAM specialists to treat rendezvous between Mir station and Soyouz or Progress vehicles[2], which allowed us to get the first results mentioned above.

Optimization of rendezvous strategies

The approach[2] used to realize our first studies on ATV/ISS rendezvous strategy is based on the 6 linearized rendezvous equations system drawn up by G. E. Kuzmak for a keplerian model with impulsive maneuvers[1]. It links maneuvers components to the deviation vector between vehicle and station in the station local orbital frame through the system (1).

$$\begin{cases} w\Delta r = \sum_{i=1}^{N} \left(2T_i \cos\varphi_i - N_i \sin\varphi_i\right) \\ \Delta V_r = \sum_{i=1}^{N} \left(-2T_i \sin\varphi_i + N_i \cos\varphi_i\right) \\ \Delta V_\tau = \sum_{i=1}^{N} \left(T_i\left(2\cos\varphi_i - 1\right) + N_i \sin\varphi_i\right) \\ w\Delta\tau = \sum_{i=1}^{N} \left(T_i\left(-3\varphi_i + 4\sin\varphi_i\right) + 2N_i\left(1-\cos\varphi_i\right)\right) \\ w\Delta z = \sum_{i=1}^{N} -W_i \sin\varphi_i \\ \Delta V_z = \sum_{i=1}^{N} W_i \cos\varphi_i \end{cases} \tag{1}$$

where:

- $\left(\Delta r, \Delta\tau, \Delta z, \Delta V_r, \Delta V_\tau, \Delta V_z\right)^t$ is the initial vector of radial, tangential, lateral position and velocity deviations,
- $\left(T_i, N_i, W_i\right)^t$ is the components vector of the i^{th} maneuver,

- φ_i is the long argument of latitude of the application point of the i[th] maneuver (counted from rendezvous point):

$$\varphi = u + (n-1)\cdot 2\pi - u_{RDV} - (n_{RDV} - 1)\cdot 2\pi \qquad (2)$$

where n (resp. n_{RDV}) is the current number (resp. at rendezvous) of turns described by the vehicle, u (resp. u_{RDV}), is the current (resp. at rendezvous) argument of latitude of the vehicle.

- N is the number of maneuvers of the strategy,
- w is the orbital velocity of the reference circular orbit considered in the linear theory of Kuzmak.

The optimization method we used lies on the resolution of system (1) for all admissible sets of application points $\{\varphi_i\}_{i=1,N}$. Note that such sets can be constructed taking into account some operational constraints.
For each fixed set $\{\varphi_i\}_{i=1,N}$, maneuvers components are obtained resolving the 6 equations of system (1).
Then, one can easily determinate the minimum propellant consuming strategy since the characteristic velocity F (3) represents this cost.

$$F = \sum_{i=1}^{N} \sqrt{T_i^2 + N_i^2 + W_i^2} \qquad (3)$$

As a consequence, the strategy must be constructed with no more than 6 maneuvers components: 4 radial or tangential, and 2 lateral. In particular, this fact doesn't allow to determinate strategies including more than 4 planar components needed for maneuvers splitting.

The main idea of our method is to split system (1) into 2 subsystems, each one supporting partially corrections of the initial deviations. This allows the introduction of 6 new unknowns including 4 new planar components.
A careful use of this idea enables the determination of a 5 maneuvers new strategies whose cost are very close to the minimum cost of a 4 maneuvers reference one.

Splitting the rendezvous equations system

Let's rewrite system (1) as:

$$\begin{cases} AX = B \\ A \in M_{6,6} \\ X \in \Re^6 \\ B \in \Re^6 \end{cases} \qquad (4)$$

where:

- A is the influence matrix,
- X is the maneuvers components vector,
- B is the initial deviations vector.

The splitting is then defined by:

$$\begin{cases} A_1 X_1 = \Pi \cdot B \\ A_2 X_2 = (Id - \Pi)\cdot B \\ \Pi \in M_{6,6}, \Pi = diag(\Pi_i), \Pi_i \in [0,1] \\ Id = diag(1,1,1,1,1,1) \\ X_1 \in \Re^6, X_2 \in \Re^6 \\ A_1 \in M_{6,6}, \{\varphi_i\}_1 \; given \\ A_2 \in M_{6,6}, \{\varphi_i\}_2 \; given \end{cases} \qquad (5)$$

It generates 2 subsystems and the 6 new unknowns are related to maneuvers applied at $\{\varphi_i\}_2$ set of long arguments of latitude.
One can notice that sets $\{\varphi_i\}_1$ and $\{\varphi_i\}_2$ can have common points. This property can be used to introduce less unknowns as well as to built strategies whose maneuvers have radial, tangential, and lateral components.

The diagonal matrix Π is called weight matrix and leads the system splitting to a particular type of solution. Choices of weight coefficients and of active components repartition over the 2 subsystems lead to an infinity of split subsystems and therefore to a great number of solutions.

Searching for a new rendezvous strategy

In the present study, we have developed the use of (5) to obtain the splitting of the first maneuver modulus of an optimal 4 maneuvers strategy taken as a reference.

Set $\{\varphi_i\}_2$ differs from set $\{\varphi_i\}_1$ only by a new value of the long argument of latitude for the application point of the second split part of the initial first maneuver T_1^{bis}.
Both coplanar and non-coplanar cases are investigated.

Coplanar rendezvous

For a coplanar rendezvous, we consider a strategy made of 4 purely tangential maneuvers.
The resulting subsystems are given by:

$$\begin{cases} A_1 X_1 = \Pi \cdot B \\ A_2 X_2 = (Id - \Pi) \cdot B \\ A_1 = A_1(\{\varphi_i\}_1) \in M \\ A_2 = A_2(\{\varphi_i\}_2) \in M_{4,4} \\ Id = diag(1,1,1,1) \\ X_1 \in \Re^4, X_2 \in \Re^4 \end{cases} \tag{6}$$

Maneuvers components of the new strategy are then deduced:

$$\begin{cases} T_1 = X_1(1) \\ T_1^{bis} = X_2(1) \\ T_2 = X_1(2) + X_2(2) \\ T_4 = X_1(3) + X_2(3) \\ T_5 = X_1(4) + X_2(4) \end{cases} \tag{7}$$

In this case, the total cost is:

$$F = \sum_{i=1}^{4} \sqrt{(X_1(i) + X_2(i))^2} + |X_1(1)| + |X_1(2)| \tag{8}$$

The long argument of latitude value for the application point of T_1^{bis} was chosen taking advantage of the fact that the influence matrix coefficients are all periodic except coefficients related to the phase equation (9) in which long argument of latitude acts directly.

$$w\Delta\tau = \sum_{i=1}^{N} \left(T_i(-3\varphi_i + 4\sin\varphi_i) + 2N_i(1 - \cos\varphi_i)\right) \tag{9}$$

This translates the prevalent status of the first maneuver and implies that φ_{12} must be as close as possible to $\varphi_{1,1}$ if one wants to keep the reference solution structure and cost. The best choice seems to be:

$$\varphi_{1,2} = \varphi_{1,1} - 2\pi \tag{10}$$

To determinate the weight matrix coefficients, we take into account the following remarks:

1- Phase equation structure implies that if one wants an efficient splitting of the first maneuver only, the tangential initial deviation $\Delta\tau$ correction must be supported by both subsystems.

2- Some numerical tests showed us that the separation of ΔV_r and ΔV_τ corrections over the 2 subsystems implies a less efficient splitting than the separation of Δr or $\Delta\tau$ corrections.

3- Furthermore, numerical tests showed that in all configurations of rendezvous, various weight matrix imply the same splitting effect but, as a global rule, deviations correction must be balanced over the 2 subsystems.

As a consequence, in the frame of our application, we use:

$$\Pi = diag(\alpha, \alpha, \alpha, \alpha) \tag{11}$$

Non coplanar rendezvous

For strategy including lateral corrections, we have to use the 2 equations governing rendezvous out of ISS orbital plan. The splitting method allows us to consider up to 4 lateral components in the new strategy.

Resulting subsystems can be:

$$\begin{cases} A_1 X_1 = \Pi \cdot B \\ A_2 X_2 = (Id - \Pi) \cdot B \\ A_1 = A_1(\{\varphi_i\}_1) \in M \\ A_2 = A_2(\{\varphi_i\}_2) \in M_{6,6} \\ Id = diag(1,1,1,1,1,1) \\ X_1 \in \Re^6, X_2 \in \Re^6 \end{cases} \tag{12}$$

maneuvers components can be given by:

$$\begin{cases} T_1 = X_1(1) \\ W_1 = X_1(5) \\ T_2 = X_2(1) \\ W_2 = X_2(5) \\ T_3 = X_1(2) + X_2(2) \\ W_3 = X_1(6) \\ T_4 = X_1(3) + X_2(3) \\ W_4 = X_2(6) \\ T_5 = X_1(4) + X_2(4) \end{cases} \tag{13}$$

and the new cost function is then:

$$F = \left|X_1(4)+X_2(4)\right| + \sqrt{(X_1(3)+X_2(3))^2 + X_2(6)^2} + \sqrt{(X_1(2)+X_2(2))^2 + X_1(6)^2} + \sqrt{X_2(1)^2 + X_2(5)^2} + \sqrt{X_1(1)^2 + X_1(5)^2} \quad (14)$$

In this case, the choice of the weight matrix can be done using the same rule as in the coplanar case because coefficients of the 2 equations introduced are 2Π periodic too.

The weight matrix to be used can be:

$$\Pi = diag(\alpha,\alpha,\alpha,\alpha,\alpha,\alpha) \quad (15)$$

If out of plan deviations to be corrected are not too important, the introduction of 4 lateral components is not necessary. Lateral corrections can then be support by less components, or only by one of the 2 subsystems. The weight matrix to be used can then be:

$$\Pi = diag(\alpha,\alpha,\alpha,\alpha,1,1) \quad (16)$$

Numerical results

We give in the present section examples of maneuver splitting in 3 different representative configuration of rendezvous between ATV and ISS. All rendezvous strategies have to be executed in 72 hours.

First case of coplanar rendezvous

We consider that ISS orbit is circular with an altitude of 350km and with an inclination of 51.6 degrees.
ATV injection orbit is given by:
-Perigee altitude: 50 km
-Apogee altitude: 300 km
-Inclination: 51.6 degrees.

The initial phasing distance is of 30.7 degrees.
Initials deviations in station local frame are:

$$w\Delta r = 34.491\,\text{km}$$
$$\Delta V_r = -67.501\,\text{m/sec}$$
$$\Delta V_\tau = 67.207\,\text{m/sec}$$
$$w\Delta\tau = -86335.837\,\text{km}$$

The 4 tangential maneuvers optimal strategy taken as a reference is given in Table 1.

Table 1: Reference 4 maneuvers optimal strategy

maneuver	n (turns)	u (deg)	φ (rad)	T(m/s)
1	1	130	295.55	93.024
2	1	320	292.55	4.432
3	47	315	3.61	0.954
4	48	150	0.21	3.288
Total cost F (m/s)			101.69	

The 2 subsystems are given by (6) and the total cost function F by the form (8). We used a weight matrix of the form (11) with a various values of α coefficient.
Resulting first modulus splitting and total cost of the new strategies are represented in figure 1.

Best splitting effect had been obtained for $\alpha = 0.5$ and the corresponding strategy is given in Table 2.

Table 2: 5 maneuvers new strategy

maneuver	n (turns)	u (deg)	φ (rad)	T(m/s)
1	1	130	295.55	46.579
1 bis	1	320	292.55	2.343
3	2	130	289.26	49.539
4	47	315	3.61	3.053
5	48	150	0.21	0.182
Total cost F (m/s)			101.69	

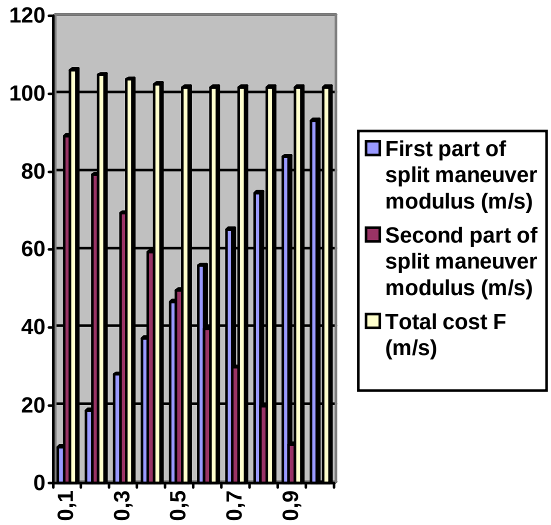

Figure 1: First tangential maneuver splitting and strategy total cost versus α weight coefficient.

Second case of coplanar rendezvous

We consider that ISS orbit is circular at the altitude of 460km and with an inclination of 51.6 degrees.

ATV injection orbit is given by:
Perigee altitude: -20 km
Apogee altitude: 450 km
Inclination: 51.6 degrees

The initial phasing distance is of 29.472 degrees.
Initials deviations in station local frame are:

$$w\Delta r = -50.792 \text{ km}$$
$$\Delta V_r = -208.262 \text{ m/sec}$$
$$\Delta V_\tau = 192.973 \text{ m/sec}$$
$$w\Delta\tau = -119708 \text{ km}$$

The 4 tangential maneuvers optimal strategy taken as a reference is given in Table 3.

Table 3: Reference 4 maneuvers optimal strategy

maneuver	n (turns)	u (deg)	φ (rad)	T(m/s)
1	1	135	288.58	149.647
2	1	330	285.18	-11.849
3	46	330	2.43	-19.212
4	47	100	0.16	23.594
Total cost F (m/s)			204.3	

The 2 subsystems are given by (6) and the total cost function F by the form (8). We used a weight matrix of the form (11) with various values of α coefficient. Resulting first modulus splitting and total cost of the new strategies are represented in figure 2.

Best splitting effect had been obtained for $\alpha = 0.5$ and the corresponding strategy is given in Table 4.

Table 4: 5 maneuvers new strategy

maneuver	n (turns)	u (deg)	φ (rad)	T(m/s)
1	1	135	288.58	74.858
2	1	330	285.18	-10.524
3	2	135	282.29	75.127
4	46	330	2.43	-20.454
5	48	150	0.21	23.173
Total cost F (m/s)			204.14	

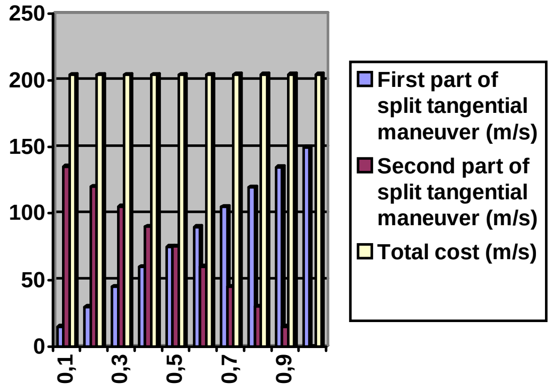

Figure 2: First tangential maneuver splitting and strategy total cost versus α weight coefficient.

Non coplanar rendezvous

We consider that ISS orbit is circular at the altitude of 460km and with an inclination of 51.6 degrees.

ATV injection orbit is given by:
-Perigee altitude: 50 km
-Apogee altitude: 300 km
-Inclination: 51.6 degrees.

The initial phasing distance is of 29.472 degrees.
Initials deviations in station local frame are:

$$w\Delta r = 107.91 \text{ km}$$
$$\Delta V_r = -96.138 \text{ m/sec}$$
$$\Delta V_\tau = 56.219 \text{ m/sec}$$
$$w\Delta\tau = -139027.5 \text{ km}$$
$$w\Delta z = -0.529 \text{ km}$$
$$\Delta V_z = -4.997 \text{ m/sec}$$

The 4 maneuvers optimal strategy taken as a reference is given in Table 5.

Table 5: Reference 4 maneuvers optimal strategy

maneuver	n	u (deg)	φ (rad)	T(m/s)	W(m/s)
1	1	116	288.91	135.485	-3.421
2	1	260	286.40	24.828	1.838
3	46	132	5.89	4.07	0
4	47	100	0.16	-0.253	0
Total cost F (m/s)				164.75	

The 2 subsystems are given by (12) and the total cost function F by the form (14). We used a weight matrix of the form (16) with various values of α coefficient.

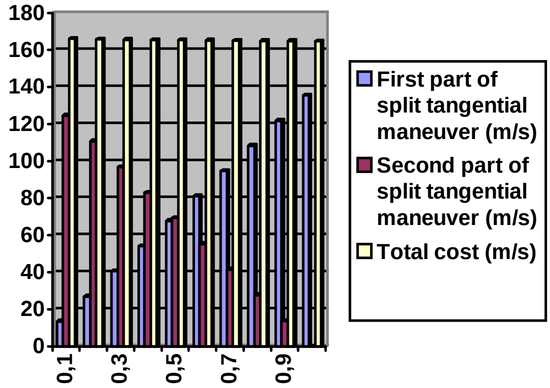

Figure 3: First tangential maneuver splitting and total strategy cost versus α weight coefficient.

Resulting first tangential modulus splitting and total cost of the new strategies are represented in figure 3. Best splitting effect had been obtained for $\alpha = 0.5$ and the corresponding strategy is given in Table 6.

Table 6: 5 maneuvers new strategy

maneuver	n	u (deg)	φ (rad)	T(m/s)	W(m/s)
1	1	116	288.91	67.74	-3.42
2	1	260	286.40	24.84	1.838
3	2	116	282.62	69.25	0.
4	46	132	5.89	2.86	0.
5	47	100	0.16	-0.58	0.
Total cost F (m/s)				165.45	

The use of a weight matrix of the form (15) splits lateral corrections over 4 maneuvers and give strategies of higher total cost. The 5 maneuvers strategy obtained with a weight coefficient α=0.5 is the result of the best splitting effect and is given in Table 7.

Table 7: 5 maneuvers new strategy

maneuver	n	u (deg)	φ (rad)	T(m/s)	W(m/s)
1	1	116	288.91	67.74	-1.71
2	1	260	286.40	24.84	0.918
3	2	116	282.62	69.25	-4.39
4	46	132	5.89	2.86	2.02
5	47	100	0.16	-0.58	0.
Total cost F (m/s)				166.11	

Conclusions and perspectives

Knowing the complex and tricky nature of the introduction of inequality constraints within classical optimization method, we developed a maneuver splitting method which has shown its efficiency and its simplicity of use and implementation through some ATV/ISS orbital rendezvous problem resolution. It allowed us to construct new strategies including reasonable times of burn. The two subsystems can be used to reach other goals than first maneuver splitting if one finds out adapted weighting rules. But in the frame of transfer problems, the use of our method can be easily made since they don't contain any phase equation.

References

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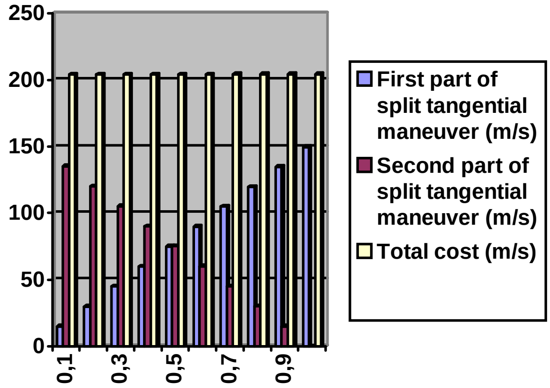

Figure 3: First tangential component splitting and strategy total cost versus α coefficient.

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