ORBIT MANEUVERS OF SATELLITES IN NEAR-EARTH QUASI-CIRCULAR ORBIT

Huang Fuming
China Xi'an Satellite Control Center
P.O.Box 505, Xi'an 710043, Shaanxi, China

Abstract

A near-earth quasi-circular orbit is often used for earth observation satellites. In this paper, the method that the ground tracks are restricted in a given limit is adopted in order to study the track positioning and the ground track maintenance of the satellites in the near-earth quasi-circular orbits. The dynamics models of orbit maneuvers are established. On the basis of the simplified models, various parameters for orbit maneuvers are calculated.

Key Words**:** Orbit Maneuver, Dynamics, Satellite.

Introduction

In earth observation missions, the satellite is required to pass over a same area on the ground periodically, and the altitudes of the satellite above the ground are required to be almost the same so as to compare the sequential images of the same area taken from a same angle of view. The near-earth quasi-circular orbit with the recurrent and frozen features can meet the above requirements.

In order to ensure the imaging quality and the coverage of the earth, the orbit of the satellite should be adjusted to the nominal operation orbit in time after the satellite is put into orbit by a launch vehicle.

At the end of the orbit acquisition, the appropriate time is selected to adjust the orbit, to complete the track positioning and to establish the normal operation orbit, so that the satellite can meet the restriction of the track grids in order to ensure the imaging quality and the coverage of the earth.

During the lifetime, the satellite may drift from the nominal orbit because of various perturbations. Therefore, the orbital parameters (a, e, ω) in plane should be adjusted in time. So, the errors between the actual tracks and the nominal tracks are not beyond a given limit when the satellite passes over the ground tracks repeatedly.

Perturbation Movement Equations of the Satellite in Quasi-Circular Orbit

The orbit perturbations of the satellite include: the earth non-spherical gravity perturbation, the atmospheric drag perturbation, the luni-solar gravity perturbation, the solar radiation pressure perturbation and so on.

For a quasi-circular orbit, in order to make the orbit perturbation movement equation to avoid having the singular points, the following standard orbital elements are adopted:

$$(a, e_x, e_y, i, \Omega, \lambda)$$

where

$$e_x = e\cos\omega$$

$$e_y = e\sin\omega$$

$$\lambda = \omega + M$$

Its corresponding orbit perturbation movement equations are:

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\left\{\frac{P}{r}T + (e\sin f)S\right\}$$

$$\frac{de_x}{dt} = \frac{1}{na}\left\{(2\cos u)T + (\sin u)S\right\} + \frac{e\sin\omega\sin u ctgi}{na}W$$

$$\frac{de_y}{dt} = \frac{1}{na}\left\{(2\sin u)T - (\cos u)S\right\} - \frac{e\cos\omega\sin u ctgi}{na}W$$

$$\frac{di}{dt} = \frac{\cos u}{na}W$$

$$\frac{d\Omega}{dt} = \frac{\sin u}{na\sin i}W$$

$$\frac{d\lambda}{dt} = n - \frac{1}{na}\left\{\frac{1}{2}e\sin f(1+\frac{r}{P})T + (\frac{e\cos f}{2} + 2\frac{r}{P})S\right\} - \frac{\sin u ctgi}{na}W$$

Here, $n=\sqrt{\frac{\mu}{a^3}}$; *S* is the acceleration component of the gravity perturbation, the atmospheric perturbation and the orbit maneuver thrust in the radial direction in the orbit coordinate system, *T* is in the transversal direction , and *W* is in the normal direction.

Dynamics Models for Orbit Maneuvers

During the track positioning and the track maintenance, the on-board thruster for orbit maneuvers is used to generate the transversal thrust *T* so as to change the orbital parameters (a, e, ω) in plane simultaneously. Now, it is assumed that F_t is the constant thrust without errors in attitude, Δt_{xb} is the firing time length of the thruster, and ΔV_x is the velocity increment in the direction *T*. If the arc of the orbit maneuver is short, the thrust can be considered to be of pulse. Therefore, the change in the parameters of the quasi-circular orbit can be expressed in the following simplified formula:

$$\Delta V_x = F_t \Delta t_{xb} / m$$

$$\Delta a = 2a\sqrt{\frac{a}{\mu}}\Delta V_x$$

$$\Delta e_{cx} = 2\sqrt{\frac{a}{\mu}}\cos(\omega + E)\Delta V_x$$

$$\Delta e_{cy} = 2\sqrt{\frac{a}{\mu}}\sin(\omega + E)\Delta V_x$$

The above expressions are the dynamics models used to control orbital parameters. Here, $\Delta a, \Delta e_{cx}$ and Δe_{cy} are the target bias.

Now, there is a question: How are the least control times and the least fuel used to make the orbital parameters *a*, *e* and *ω* (or *a*, e_x, e_y) meet the requirements simultaneously? Generally, two tangential pulses are used for completing the coordinated control of the three parameters. Under special conditions, only one pulse is used. In order to complete the coordinated control of the three parameters, the control point is seldom at the apsis.

a_0 is used to indicate the nominal mean semi-major axis, and θ_c is used to indicate the directional cosine of the modified vector $\Delta\vec{e}_c$ in plane (e_x, e_y). When $\Delta V>0$, $\theta_c = \omega + E$; when $\Delta V<0$, $\theta_c = \omega + E + \pi$. If ΔV_1 and ΔV_2 are the velocity increments generated by the first and second tangential pulses respectively, and θ_{c1} and θ_{c2} are the directional cosines of the modified vectors $\Delta\vec{e}_{c1}$ and $\vec{e}_{c2}$ in the plane (e_x, e_y) respectively, then

$$\left(\frac{\Delta \mathbf{a}}{\mathbf{a_0}}\right)^2 = 4\frac{\mathbf{a_0}}{\mu}(\Delta \mathbf{V}_1^2 + \Delta \mathbf{V}_2^2 + 2\Delta \mathbf{V}_1 \Delta \mathbf{V}_2)$$

$$\Delta(\ \mathbf{e_c})^2 = \left(\frac{\Delta \mathbf{a}}{\mathbf{a_0}}\right)^2 + 8\frac{\mathbf{a_0}}{\mu}\Delta \mathbf{V}_1 \Delta \mathbf{V}_2 \left[\cos(\theta_{c2} - \theta_{c1}) - 1\right]$$

Here, there are three cases:

- $\left(\frac{\Delta a}{a_0}\right)^2 > (\Delta e_c)^2$

The two pulses of a same symbol are used to modify *a* and $\vec{e}_c$.

The position θ_{c1} of the first pulse can be selected at discretion. The parameters ΔV_2 and θ_{c2} of the second pulse are related with the parameters ΔV_1 and θ_{c1} of the first pulse.

- $\left(\frac{\Delta a}{a_0}\right)^2 < (\Delta e_c)^2$

The two pulses of the opposite symbols are used to modify *a* and $\vec{e}_c$.

- $\left(\frac{\Delta a}{a_0}\right)^2 = (\Delta e_c)^2$

Only one pulse can complete the coordinated control of *a*, *e* and *ω*.

Mathematical Models for Track Prediction

The geographical longitude at the descending node is used to express the position of the nominal track. If the nominal track of the descending pass over a specific area is defined to be the first track which is marked as L_{B1}. The northern nominal track adjacent to the first one is the second track, and the eastern nominal track adjacent to the first one is the *N*th track (i.e. the last track), then the distance between two adjacent nominal tracks is 360/N degrees. The longitude of the *K*th nominal track at the descending node is:

$$L_{Bk} = L_{B1} - (k-1)\Delta L_B$$

Where, $\Delta L_B = \dfrac{360^\circ}{N}$. L_{Bk} is transformed within (-180°, +180°] (positive in the east longitudes).

It is assumed that the longitude of an actual track at the descending node is *L* which value is within (-180°, +180°], and the nominal track nearest to it is the *j*th track, therefore we obtain

$$j = \text{int}(\frac{L_{B1} - L}{\Delta L_B} + 0.5)$$

Now, using *j* to replace *k* and inserted into the calculation formula of L_{Bk} , we obtain the corresponding longitude L_{Bj} of the nominal track. Then, the bias of the actual track relative to the nominal track is gained by means of the following formula:

$$\Delta L = L - L_{Bj}$$

Mathematical Models for Track Control

If T_1 is the time of the satellite passing over its certain descending node, the longitude bias between the actual track and the nominal track is $\Delta L_1 = L_1 - L_B$ in degree, the semi-major axis bias between the actual and nominal values is $\Delta a = a - a_0$ in meter, and the decay rate of the semi-major axis is $\dot{a}$ in meter/day , then the drift longitude at the descending node after *t* days is:

$$\Delta L_t = -3\pi \frac{a_e}{a}(\Delta at + \frac{1}{2}\dot{a}t^2) \quad \text{in meter}$$

$$= -540\,(\Delta at + \frac{1}{2}\dot{a}t^2)/a \quad \text{in degree}$$

From the above formula, it is obvious that, by adjusting the semi-major axis, the track positioning can be performed or the track can be restricted within the given limit.

The total longitude bias between the actual track and the nominal track is:

$$\Delta L_2 = \Delta L_1 + \Delta L_t$$

The relationship between the variant Δp of the period and the variant Δa of the semi-major axis is:

$$\Delta p = 3\pi\sqrt{\frac{a}{\mu}}\,\Delta \text{a}$$

The relationship between the decay rate $\dot{a}$ (in meter/day) of the semi-major axis and the decay rate $\dot{p}$ (in second/day) of the period is:

$$\dot{a} = \frac{1}{3\pi}\sqrt{\frac{\mu}{a}}\,\dot{p}$$

$\dot{a}$ and $\dot{p}$ are both negative.

Track Positioning and Track Maintenance

The maneuver sketch of the track positioning and track maintenance is shown in Figure 1.

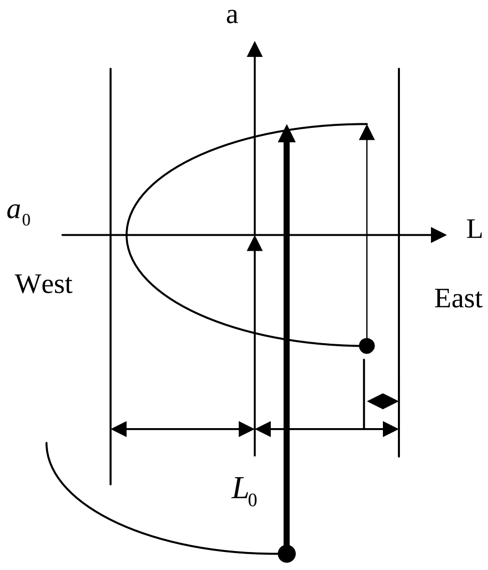

Figure 1: Orbit maneuver for track positioning and track maintenance

In terms of the determined orbit and the control errors, the control of the tracks is performed in two steps: in the first step, a great part of the theoretic adjustment size is used for control; in the second step, after the first-step maneuver calibration, the remaining adjustment, which is calculated by putting the target on the track drift parabola, is used for control.

The in-plane track maintenance maneuver is to change the parameters (a, e and ω) by means of providing the velocity increment in order to maintain the recurrent and frozen features of the orbit, and to control the bias between the actual track and the nominal track within the given limit in order to guarantee the coverage of the track grids. The main steps for the track maintenance maneuvers are as following:

- If the bias between the actual track and the nominal track is ΔL_1 (in degrees) and the left boundary is ΔL_L^* , the calculation formula of the theoretic adjustment size Δa is as follows:

$$\Delta a = (a_0 - a_1) + \sqrt{\frac{(\Delta L_L^* - \Delta L_1) a_0 \dot{a}}{270}}$$

- In terms of the determined orbit and the control errors, the target semi-major axis deviates $\Delta(\Delta a)$ (in meter) downwards, the actual adjustment size of the semi-major axis is taken as $\Delta a_c = \Delta a - \Delta(\Delta a)$.

- In terms of the formula given by the dynamics model for orbit maneuver, the velocity increments and their acting phases are calculated.

- In terms of the velocity increments and their acting phases, the time to turn on/off the thruster is calculated.

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