MULTI-MODEL FILTERING FOR ORBIT DETERMINATION DURING MANOEUVRE

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Abstract

Satellite orbit determination is often interrupted during manoeuvres and their effects are solely taken into account by increasing the standard deviation of the estimated orbital parameters. If the manoeuvre lasts a long time, this increase can be important. A more accurate estimation of the orbital parameters can be provided by using measurements collected during manoeuvre, if the dynamic model of the satellite includes the manoeuvre. In order to do so, two filtering approaches have been tested. The first one is based on a classical Kalman filter with change on the dynamic when the manoeuvre is detected. The second one is based on Generalised Pseudo Bayesian Algorithm which is a multi-model filtering. This algorithm aims at automatically detecting occurrences of dynamic changes The performances of these algorithms have been compared for a north-south geostationnary manoeuvre with electrical thrusters. The simulations have shown the abilities of the multi model filter algorithm to detect the beginning and the end of manoeuvre and also to perform orbit determination during the manoeuvre.

Key words: Multi-model filtering, orbit determination, manoeuvre

Introduction

Applications requiring quasi-real time orbit determination with good accuracy make it necessary to take into account manoeuvres. Satellite orbit determination is often interrupted during manoeuvres and their effects are solely taken into account by increasing the standard deviation of the estimated orbital parameters. If the manoeuvre lasts a long time, this increase can be important. A more accurate estimation of the orbital parameters can be provided by using measurements collected during manoeuvre, if the dynamics model of the satellite includes the manoeuvre. In order to do so, two filtering approaches are tested. The first one is based on a classical Kalman filter with change on the dynamic when the manoeuvre is detected. The second one is based on Generalised Pseudo Bayesian Algorithm which is a multi-model filtering. We tested the abilities of these algorithms during a geostationary North-South manoeuvre with electrical thrusters.

Acronyms

m^i : ith model ;
$M = \{m^1, ..., m^n\}$: set of the models;
m_k : model at time t_k;
t_k : discrete time of the simulation;
X_k : state at time t_k;
y_k : measurement at time t_k;
Y_k : the set of measurement up to time t_k;
V_k and W_k : gaussian noises;
$S_k= \{m_1, ..., m_k\}$: sequence of models from t_1 to t_k;
$\Pi =[p(m^i/m^j), i,j=1 \text{ to } n]$: transition matrix;
$p(m^i/m^j)$: probability to jump from models m^j to m^i.

Multi-model techniques

Markovian approaches

The basic principle of multi-model techniques[1] is to manage several Kalman filters, each based on different dynamic and observation models. Each filter estimates the state of its model with measurements. The true state of the system is a weighted sum of the filtered states. The weightings are computed as the a posteriori probabilities of the different models.

In this study, we only consider the discrete time approach. Among the various multi-model techniques[2], Markovian methods present the advantages of dealing easily with jumps between model modelled as Markov process.

The set of models, $M=\{m^1, m^2, ..., m^n\}$, is considered as the set of the states of a discrete Markov chain. The jumps between models are modelled by a discrete Markovian process, characterised by:

- the matrix Π of probabilities of transition between models ($\Pi=[p(m^i/m^j), i,j=1 \text{ to } n]$);
- the initial distribution of the different models.

Each state model is described as :

$$X_k = F(m_k,m_{k-1}).X_{k-1} + G(m_k,m_{k-1}).V_{k-1} \quad (1)$$
$$y_k = C(m_k) + D(m_k).W_k, \quad (2)$$

where X_k is the state at time t_k,
y_k is the measurement at time t_k,
V_k and W_k are gaussian noises
F,G,C,D are matrix.

The aim of a multi-model algorithm is to estimate the hybrid state $[X_k,\ m_k]$ at time t_k. This hybrid state is defined by the probability density of X_k, $p(X_k/Y_k)$ and the probability of each model $p(m_k/Y_k)$, where Y_k is the set $\{y_1, ..., y_k\}$.

It exists an optimal solution to this problem.

Optimal solution

We are seeking for the best estimates, in the sense of the maximum a posteriori probability, of $p(m_k/Y_k)$ and $p(X_k/Y_k)$

We considered the sequence $S_k= \{m_1, ..., m_k\}$, which is a sequence of models since the time t_1 up to the time t_k. The solution of equations (1) and (2) is defined by:

$$p(X_k/Y_k) = \sum_{S_k} p(X_k/Y_k,S_k).p(S_k/Y_k) \quad (3)$$

$$p(m_k/Y_k) = \sum_{S_{k-1}} p(S_k/Y_k), \text{ with } S_k=\{S_{k-1},m_k\} \quad (4).$$

$p(X_k/Y_k,S_k)$ is given by a Kalman filter with the fixed sequence of models S_k.

$p(S_k/Y_k)$ is determined recursively, using n^k Kalman filters :

$$p(S_k/Y_k) = \frac{p(y_k/S_k,Y_{k-1}).p(m_k/m_{k-1}).p(S_{k-1}/Y_{k-1})}{p(y_k/Y_{k-1})} \quad (5)$$

where $p(y_k/S_k,Y_{k-1})$ is provided by the Kalman filter under the hypothesis S_k.

Sub-optimal approaches

In fact, this optimal solution needs n^k Kalman filters in parallel. In order to reduce the number of Kalman filters, different sub-optimal methods exist : pruning methods and merging methods.

The pruning methods keep a reduced number of sequences at each time. These sequences can be chosen for their high probability of occurrence (Detection and Estimation Algorithm[3-4]) or generated using a random process with known probability density function (Random Sampling Algorithm[5]).

The merging methods consist in forgetting the past beyond some horizon by merging several sequences into one. Contrary to the pruning techniques, there is no loss of information, but a compression of the information. We keep only limited sequences. The merging may be done after the estimation by the Kalman filters (Generalised Pseudo Bayes Algorithm : GPBA[1-6-7-8-9-10]) or before (Interacting Multiple Models[11]). The GPBA can be defined with different length of the merging sequences. The nth-order GPBA is defined with merging sequences of length n-1. Comparison of these algorithms[12] leads us to choose the 2nd-order GPBA.

2nd order Generalised Pseudo-Bayes Algorithm

The algorithm can be split into three steps :
- the extension step;
- the filter step;
- the merging step.

Figure 1 shows these three steps, in case of system with two models ($M=\{m^1, m^2\}$

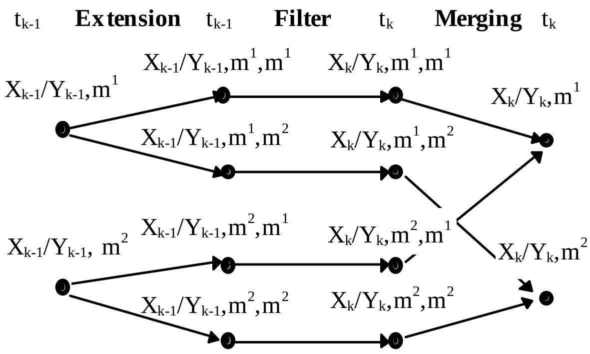

Figure 1 : scheme of 2nd-order GPBA

In this case, the algorithm uses 4 filters in parallel.

Extension step

The aim of extension step is to take into account the jump between models. After this step, the sequences are two models long : the previous model and the new model.

For the discrete part of the state m_k, this step uses the transition matrix Π:

$$p(m_{k-1},m_k/Y_{k-1}) = p(m_k/m_{k-1}).p(m_{k-1}/Y_{k-1}) \quad (6)$$

For the continuous part of the state X_k, the extension reflects possible changes in the dynamic model that are translated as modifications of the probability density function $p(X_{k-1}/Y_{k-1},m_{k-1})$, characterised by its mean and covariance matrix (we make the hypothesis that the state X_k follows a gaussian law). We obtain a new probability density function $p(X_{k-1}/Y_{k-1},m_{k-1},m_k)$.

Filter step :

During the filter step, we use, for each state provided by the extension step, a classical Kalman filter in order to predict the new state at the following time t_k and update its values according to the new measurements, both for continuous and discrete part of the state:

$$p(X_k/Y_{k-1},m_{k-1},m_k) = \int_{X_k} p(X_k/X_{k-1},m_k).p(X_{k-1}/Z_{k-1},m_{k-1},m_k)dX_k \quad (7)$$

$$p(X_k/Y_k,m_{k-1},m_k) = \frac{p(y_k/X_k).p(X_k/Y_{k-1},m_{k-1},m_k)}{\int_{X_k} p(y_k/X_k).p(X_k/Y_{k-1},m_{k-1},m_k).dX_k} \quad (8)$$

$$p(m_{k-1},m_k/Y_k) = \frac{p(y_k/Y_{k-1},m_{k-1},m_k).p(m_{k-1},m_k/Y_{k-1})}{\sum_{m_k}\sum_{m_{k-1}} p(y_k/Y_{k-1},m_{k-1},m_k).p(m_{k-1},m_k/Y_{k-1})} \quad (9)$$

Merging step :

Next, we merge all the states provided by sequences which end with the same model.

So, we have for the state :

$$p(m_k/Y_k)= \sum_{m_{k-1}} p(m_{k-1},m_k/Y_k) \quad (10)$$

$$p(X_k/Y_k,m_k)= \sum_{m_{k-1}} p(X_k/Y_k,m_{k-1},m_k).p(m_{k-1}/Y_k,m_k) \quad (11)$$

where $p(m_{k-1}/Y_k,m_k)$ is given by :

$$p(m_{k-1}/Y_k,m_k) = p(m_{k-1},m_k/Y_k)/p(m_k/Y_k) \quad (12)$$

This step provides also the true estimate of the continuous part of the state, which is a weighted sum of the state for each model :

$$p(X_k/Y_k) = \sum_{m_k \in M} p(X_k/Y_k,m_k).p(m_k/Y_k) \quad (13)$$

Orbit determination during north-south manoeuvre of a geostationary

Description of the simulation

We have studied the orbit determination of a geostationary satellite during a north-south manoeuvre with a ION thruster. The manoeuvre parameters are the following :

Duration = 2 hours

Thrust = 20 mN (± 10 %)

ΔV = 0,13 m/s on the cross-track axis (± 10 %)

The orbit determination is made by 4 ground stations with pseudo-range measurements. We simulate synchronisation error for the station clocks and for the geostationary clock. Moreover, we include the atmospheric degradation on the signal : ionospheric and tropospheric delay, multi-path. The global error is between 1 to 3 m, depending on the elevation angle. The cadence of measurement is 15 minutes.

For orbit determination, the manoeuvre acceleration is estimated using these initial values :

radial acceleration = 0 m/s ($\sigma = 10^{-6}$ m/s)

along-track acceleration = 0 m/s ($\sigma =10^{-6}$ m/s)

cross-track acceleration = 0 m/s ($\sigma = 2\ 10^{-5}$ m/s)

We want to compare the performances of classical filtering with sub-optimal Markov approach to perform orbit determination during manoeuvre, without knowledge of its date. We first perform a simulation with exact knowledge of the time of manoeuvre, which will be the reference.

Orbit determination with exact time of manoeuvre

The first simulation results are obtained using the Kalman filter when we know exactly the instant of beginning and end of the manoeuvre.

Figure 2 shows the error and standard deviation of the estimation of the position for radial axis, along-track axis and cross-track axis. The manoeuvre is represented as a step signal. We see that the main error occurs on the cross-track axis. After the manoeuvre, we find again the same level of standard deviation after 3 hours.

We can see a bias on the along-track error. This bias is due to synchronisation biases on the station clocks, which are not observable by the filter.

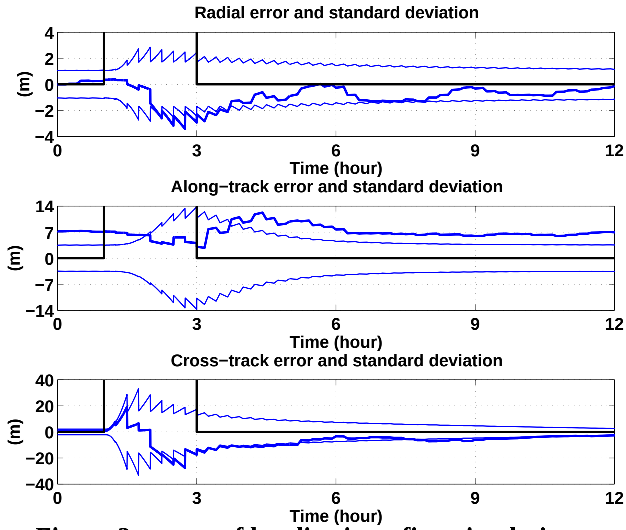

Figure 2 : error of localisation - first simulation

Detection of manoeuvre with Kalman filter

The second simulation consists in estimating the time of the manoeuvre. In order to detect the beginning of the manoeuvre, we use detector based on residuals. It is possible to survey directly each residual, or a combination of residual parameters (for example $\gamma^T.\Gamma^{-1}.\gamma$, where γ is the residual vector and Γ the covariance matrix of the residual). In order to limit the sensitivity to false alarm, it is also possible to consider sliding window as in tests proposed by Bar-Shalom[13] :

$$G_n = \alpha.G_{n-1} + \gamma^T.\Gamma^{-1}.\gamma > \lambda \qquad (14)$$

where α is a forgetting factor and λ is computed according to a false alarm probability. The mean value of the detector G_n is p/(1-α), where p is the number of measurements. So, p/(1-α) can be considered as the window width. We call G(1) the detector with a window width of 1 (α=0), G(2) the detector with a window width of 2 (α=0.5) ... The detector follows a $1/(1-\alpha).\chi^2$ law with p degrees of freedoms.

Figure 3 shows the detection of the manoeuvre for the G(1) detector and with a threshold corresponding to 99%. The slope of the curve at the detection time is used to estimate the instant of manoeuvre.

An optimistic estimation could be an error of 15 minutes for the start of the manoeuvre.

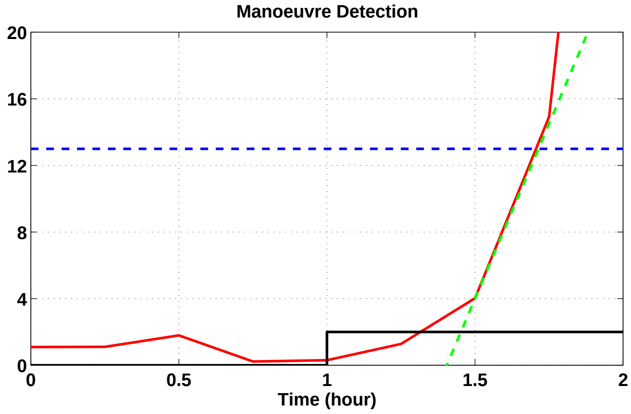

Figure 3 : detection of the beginning of manoeuvre

Now, we must determine the end of the manoeuvre. Figure 4 shows the results for the G(1) detector. We can consider we have a fairly good estimate of the end of manoeuvre.

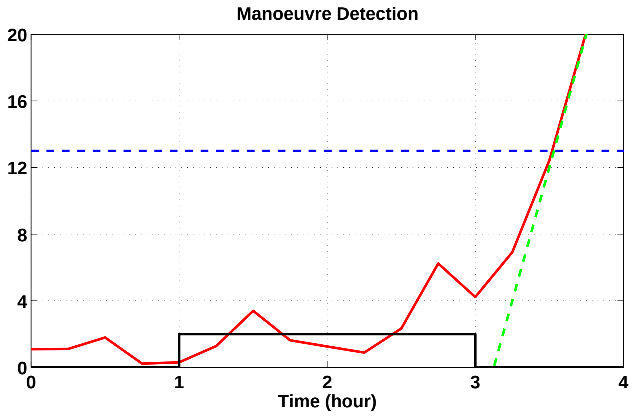

Figure 4 : detection of the end of manoeuvre

We can compare the obtained localisation accuracy with the one obtained through the first simulation.

Figure 5 shows the radial, along-track and cross track errors and standard deviation. After the manoeuvre, there is a bias on the estimation of radial and cross-track errors due to a bad estimation of the manoeuvre accelerations. Moreover, the ratio between cross-track error and its standard deviation shows a divergence of the filter.

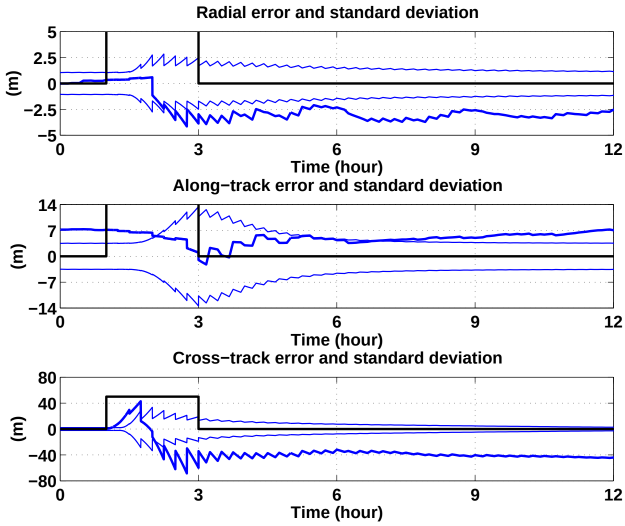

Figure 5 : localisation error with detection of manoeuvre

Simulations with a multi-model filter

Finally, we use the 2nd-order Generalised Pseudo-Bayesian Algorithm to perform orbit determination during the manoeuvre. We begin the simulation with the same estimate and covariance matrix as in the previous simulations. We have two models :

1) model without manoeuvre (m^1)

2) model with a manoeuvre (m^2).

We consider the same initial estimate and standard deviation as in the Kalman filter simulations for the manoeuvre accelerations. These values are used at each time in the extension step of the algorithm when we consider the jump between model m^1 and model m^2..

The initial probability of the models are :

$p(m^1) = 0.99$

$p(m^2) = 0.01$

The transition matrix is : $\begin{pmatrix} 0{,}99 & 0{,}01 \\ 0{,}01 & 0{,}99 \end{pmatrix}$

These values mean that we give an advantage to the phase with constant model.

Figure 6 presents the probability of the manoeuvre. The detection of the manoeuvre appears clearly in the figure. The filter estimates the time of manoeuvre with an error greater than in simulation 2 (about one hour).

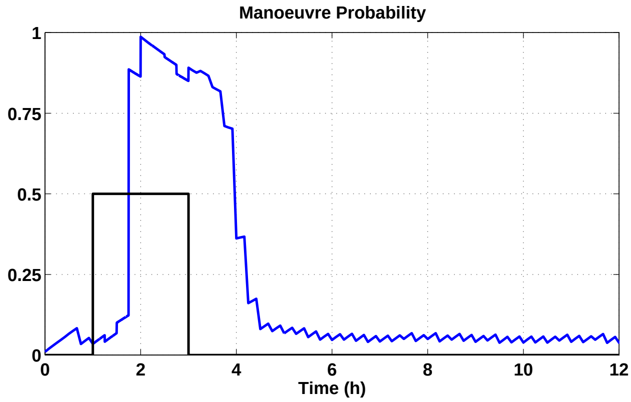

Figure 6 : probability of the manoeuvre

But this bad estimation doesn't degrade the performance of localisation of the satellite (see Figure 7). We find the same level of accuracy after 4 hours. This good behaviour is due to the fact that the multi-model filter uses the manoeuvre model, after the end of manoeuvre, in order to improve the localisation according to the measurements.

If we compare with Figure 2, we see only slight differences. In fact, as at each time all possible jump between models are considered, the increase of the error between two measurements is slightly greater than in the first simulation. This is particularly obvious on the cross-track error at the end of the simulation.

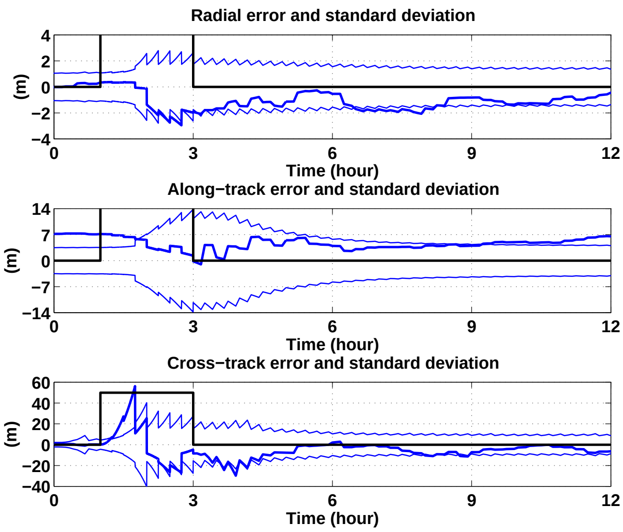

Figure 7 : localisation errors with multi-model filter

Conclusions

The multi-model filter shows good capability to perform orbit determination during manoeuvre. The tuning of the parameters is easy, and the performances are similar as those obtained by a Kalman filter with a perfect knowledge of the time of manoeuvre.

Nevertheless, a multi-model filter can not replace a Kalman filter for the nominal phase of the orbit determination. Indeed, as the two models are taken into account all the time, it is impossible to perform good prediction of the orbit. This technique could be used when a manoeuvre occurs and provide a good estimation after the manoeuvre for a Kalman filter.

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