ON NEW INVESTIGATIONS OF GEOSTATIONARY SATELLITE MOTION

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Abstract

The discovery of more than 50 cases of occasional rate changes in the motion of geostationary satellites has stimulated investigations in some directions. For taking into account the influence of interaction between perturbing forces of the Earth non-sphericity and luni-solar attraction the new intermediate orbit has been constructed. Its perturbing function includes beyond the secular terms two largest long-period terms of the development of the Moon perturbation function and the principal term of the solar light pressure. Its solution based on quasi-integrals of the adopted perturbation function. The accuracy of the new theory motion of nonfunctional geostationary satellites have been studied over a more long time interval and compared with NASA Two-Line Elements. The influence of the geopotential model errors on the results of calculations has been also investigated.

Key words: Geostationary Satellite, Orbit, Collision, Explosion.

Introduction

The founded phenomenon of occasional velocity changes in the motion of geostationary satellites (GS) has been described in the paper[1]. It is turned out, that the reality of this phenomenon existence is beyond any doubt. The possible causes are collisions of GS with cosmic debris or micro explosions of GS.

Because occasional rate changes happen more often in the vicinity of exploded satellites, it seems that a large part of dangerous debris are due to breakups of GS. The preliminary model of distribution of fragments of these GS has shown, that there are a wide diversity of motion types and some orbits of fragments can cross both the parent orbit and their neighbour ones.

According to the study of evolution of the orbital eccentricities of many GS it can be founded that the exploded GS as a rule have a rather large ratio of their cross sections to mass. Therefore the necessity of more precise theory motion of GS arose, which would include the interactions between the all significant perturbations.

With this aim the accuracy of the used NASA Two-Line-Elements (TLE) were also estimated[2]. At last in this paper the influence of the geopotential model errors on the results of calculations has been investigated. It is especially important for the objects which achieve the region of the unstable librating points (160° E and 12° W).

Basic Equations, Intermediate Orbit

For the purpose of more careful investigation of GS motion the intermediate orbit for following four orbital elements has been constructed:

$$\begin{aligned}
&\frac{de}{dt} = -\frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega}\ ,\\
&\frac{di}{dt} = \frac{\operatorname{ctg} i}{na^2\sqrt{1-e^2}}\frac{\partial R}{\partial \omega} - \frac{1}{na^2\sqrt{1-e^2}\,\sin i}\frac{\partial R}{\partial \Omega}\ ,\\
&\frac{d\omega}{dt} + \frac{d\Omega}{dt} = \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} + \operatorname{tg}\frac{i}{2}\frac{1}{\sqrt{1-e^2}}\frac{\partial R}{\partial i}\ ,\\
&\frac{d\Omega}{dt} = \frac{1}{na^2\sqrt{1-e^2}\,\sin i}\frac{\partial R}{\partial i}\ ,
\end{aligned} \tag{1}$$

where for taking into account the interaction of perturbations from the geopotential, luni-solar attraction and sun light pressure the following function U according to[2] has been introduced:

$$\begin{aligned}
U &= \frac{J_{20}}{4}n^2a^2\left(\frac{a_e}{a}\right)^2\frac{(1-3/2\sin^2\Lambda)(3\cos^2 i-1)}{(1-e^2)^{3/2}} + \\
&+\frac{\kappa}{8}a^2\left(1+3/2\,e^2\right)\left(1-3/2\sin^2(\varepsilon-\Lambda)\right)\left(3\cos^2 i-1\right) - \\
&-\mu_L n_L^2 a^2\Big[\,1/5\left(1+3/2\,e^2\right)\left(1-e_L^2\right)^{-3/2}E_{211}(\varepsilon-\Lambda)\times \\
&\times D_{2-11}(i_L)D_{211}(i)\cos(\Omega-\Omega_L) + \frac{1}{7}\left(\frac{a}{a_L}\right)\times \\
&\times\left(5/2\,e+15/8\,e^3\right)e_L\left(1-e_L^2\right)^{-5/2}E_{311}(\varepsilon-\Lambda)D_{3-12}(i)\times \\
&\times D_{311}(i)\cos(\omega+\Omega-\omega_L-\Omega_L)\,\Big] + 3/2\,Qea\times \\
&\times\cos^2\frac{\varepsilon-\Lambda}{2}\cos^2\frac{i}{2}\cos(\omega+\Omega-\lambda_S)\ ,
\end{aligned} \tag{2}$$

where

$$\kappa = n_L^2 \mu (1-e_L^2)\left(1-\frac{3}{2}\sin^2 i_L\right) + n_S^2 (1-e_S^2)^{-3/2}\ ,$$
$$\mu = m_L/(m_E + m_L),\qquad (3)$$
$$Q = P_0\, A/m\ ,$$

m_L and m_E are masses of the Earth and the Moon, $P_0 = 4.65\times 10^{-5} dyne/cm^3$, J_{20} – parameter of geopotential, a_e – mean equatorial radius, A/m – a surface-to-mass ratio expressed in cm^2/g; a, n, i, e, Ω, ω (orbital elements of GS) are referred to the Laplacian plane inclined to Equator on the angle $\Lambda = 7.°34$ and lies through a mean equinox of date; the orbital elements of the Moon are referred to Ecliptic and have index L, the orbital elements of the Sun have index S, ε is the inclination of Ecliptic to Equator.

The functions of inclination E and D are expressed by following way

$$E_{211}(\varepsilon-\Lambda) = \frac{1}{2}(1+\cos(\varepsilon-\Lambda))(2\cos(\varepsilon-\Lambda)-1),$$
$$E_{311}(\varepsilon-\Lambda) = 6\cos^2\left(\frac{(\varepsilon-\Lambda)}{2}\right)\sin^4\left(\frac{(\varepsilon-\Lambda)}{2}\right) -$$
$$-8\cos^4\left(\frac{(\varepsilon-\Lambda)}{2}\right)\sin^2\left(\frac{(\varepsilon-\Lambda)}{2}\right) + 6\cos^6\left(\frac{(\varepsilon-\Lambda)}{2}\right)\ ,$$
$$D_{2-11}(i_L) = \frac{\sqrt{15}}{2}\sin i_L \cos i_L\ ,$$
$$D_{3-12}(i_L) = \frac{\sqrt{7}}{6}\left(9\cos^2\frac{i_L}{2}\sin^4\frac{i_L}{2} - 12\cos^4\frac{i_L}{2}\times\right.$$
$$\left.\times\sin^2\frac{i_L}{2} + \frac{3}{2}\cos^6\frac{i_L}{2}\right),\qquad (4)$$
$$D_{211}(i) = \frac{\sqrt{15}}{2}\sin i \cos i\ ,$$
$$D_{311}(i) = \frac{\sqrt{7}}{6}\left(9\cos^2\frac{i}{2}\sin^4\frac{i}{2} - 12\cos^4\frac{i}{2}\times\right.$$
$$\left.\times\sin^2\frac{i}{2} + \frac{3}{2}\cos^6\frac{i}{2}\right)\ .$$

Substituting (2) in the Lagrange's equations (1) and introducing new variables:

$$z = \Omega - \Omega_L,\ R = \Omega + \omega - \Omega_L - \omega_L,\ x = \Omega + \omega_L - \lambda_S, \qquad (5)$$

equations (1) can be rewritten as

$$\frac{dz}{dt} = \dot{z}\left(1 + B\frac{\cos 2i}{\sin i}\cos z\right), \qquad (6)$$

$$\frac{di}{dt} = B\dot{z}\cos i \sin z - \frac{A\dot{R}e}{\mathrm{tg}\, i}\sin R, \qquad (7)$$

$$\frac{dR}{dt} = \dot{R} + B\dot{z}\left(3\sin i\cos i + \cos 2i\,\mathrm{tg}\frac{i}{2}\right) +$$
$$+\frac{A\dot{R}}{e}\cos R - \frac{D\dot{x}}{e}\cos x, \qquad (8)$$

$$\frac{de}{dt} = A\dot{R}\sin R - D\dot{x}\sin x\,; \qquad (9)$$

where A, B, D, $\dot{x}$, $\dot{z}$ and $\dot{R}$ are adopted for constants.

Solution of Lagrange's Equations

The second term in right-hand side of (7) is too small (it changes the result no more than by 0."03) and may be neglected. In such case multiply (6) and (7) by –0.5Bsin2$i$sin$z$ and sin$i$+Bcos2$i$cos$z$ accordingly and add them one can receive the first integral:

$$(1 - Bs)\cos i = C, \qquad (10)$$

where C is the constant of integration and

$$s = \sin i \cos z. \qquad (11)$$

Using (10), the resolution of system (6), (7) may be expressed in following form:

$$\cos z = s/\sin i; \qquad (12)$$

where

$$s = \frac{1}{B}\left(F - \frac{\gamma}{\beta + \sqrt{b^2 + B^2 ac}\cos(\dot{\Omega}_L\sqrt{\gamma}\,(t - C_1))}\right),$$
$$a = 1 + 3B^2C^2 - 8B^4C^2,$$
$$b = B^2C^2(1 + 2B^2(1 - C^2)),$$
$$c = 1 - C^2 - 5B^4C^2(1 - C^2)^2,$$
$$F = 1 + KC/\dot{\Omega}_L, \qquad (13)$$
$$\beta = aF + b,$$
$$\gamma = aF^2 + 2BF - B^2c,$$
$$C_1 = \frac{1}{\dot{\Omega}_L\sqrt{\gamma}}\arccos\left(\left(\frac{\gamma}{(F - Bs_0)} - \beta\right)\frac{1}{\sqrt{b^2 + B^2ac}}\right);$$

s_o denotes the initial value of s.

Substituting into (8) and (9) the new variables introduced by the expressions

$$p = e\cos R + A,\quad q = e\sin R, \tag{14}$$

equations (8) and (9) may be rewritten as

$$\frac{dp}{dt} = -(\dot{R}+\delta)q + D\dot{x}\sin(R-x),\quad \frac{dq}{dt} = (\dot{R}+\delta)p - D\dot{x}\cos(R-x) - A\delta; \tag{15}$$

where

$$\delta = \frac{B\dot{z}}{2}\cos z\left(3\sin 2i + \cos 2i\,\mathrm{tg}i\right). \tag{16}$$

Using (12), the linear system of Equations (15) may be resolved without any mathematical difficulties. The resolution can be written as

$$e = \sqrt{(p-A)^2 + q^2},\quad R = \mathrm{arctg}\left(\frac{q}{p-A}\right), \tag{17}$$

where

$$\begin{aligned}
&p = (e_{10} + \Delta e_1)\cos(G - C_{20} - \Delta C_2),\\
&q = (e_{10} + \Delta e_1)\sin(G - C_{20} - \Delta C_2);\\
&\Delta e_1 = D[\dot{x}/(\dot{x}+\delta)\cos(G - C_{20} + x - R) - \delta_1/(\dot{x}+\delta)^2 \times\\
&\times \sin(G - C_{20} + x - R)] + A[\delta/(\dot{R}+\delta)\cos(G - C_{20}) +\\
&+ \delta_1/(\dot{R}+\delta)^2 \sin(G - C_{20})] - \Delta e_{10},\\
&\Delta C_2 = (D[\dot{x}/(\dot{x}+\delta)\sin(G - C_{20} + x - R) +\\
&+ \delta_1/(\dot{x}+\delta)^2\cos(G - C_{20} + x - R)] + A[\delta/(\dot{R}+\delta)\times\\
&\times \sin(G - C_{20}) - \delta_1/(\dot{R}+\delta)^2\cos(G - C_{20})])/e_{10} - \Delta C_{20},\\
&G = \left(K(5C^2 - 2C - 1)/2 - \dot{\omega}_L - \dot{\Omega}_L\right)t + (ms+n)r,\\
&m = B^2((1-5C)/F_1^2 + (12.5C - 2)/F_1 - 10C + 2 -\\
&\quad -1/(C+1)^2)/2,\\
&n = B((5C-1)/F_1 - 5C + 2 - 1/(C+1)),\\
&F_1 = 1 + \frac{\Omega_L}{KC},\\
&e_{10} = \sqrt{p_0^2 + q_0^2},\\
&\delta_1 = B\sin z(KC + \dot{\Omega}_L)^2(3/2\sin 2i + \cos 2i\,\mathrm{tg(i/2)}),\\
&C_{20} = (ms_0 + n)r_0 - \mathrm{arctg(q_0/p_0)};
\end{aligned} \tag{18}$$

Δe_{10} and ΔC_{20} denote the initial values of Δe_1 and ΔC_2, respectively.

Investigations of Accuracy of Orbital Elements

Application of the intermediate orbits gives us the possibility of investigation of accuracy of different systems of orbital elements of nonfunctional GS and particularly of NASA TLE orbits.

In[1] some examples of oscillations in GS longitude residuals $(O - C)_\lambda$ has been given, with amplitudes about 0.05° and periods nearly an year. These oscillations restrict the accuracy of ephemerides and may create the false effect of collision. this effect can reach 0.001°/day or 3 mm/s when the length of the observational period consists roughly one year; for a longer period it must be less, accordingly.

On Fig. 1 the residuals of longitude for one of GS (86027A Cosmos 1738) is shown.

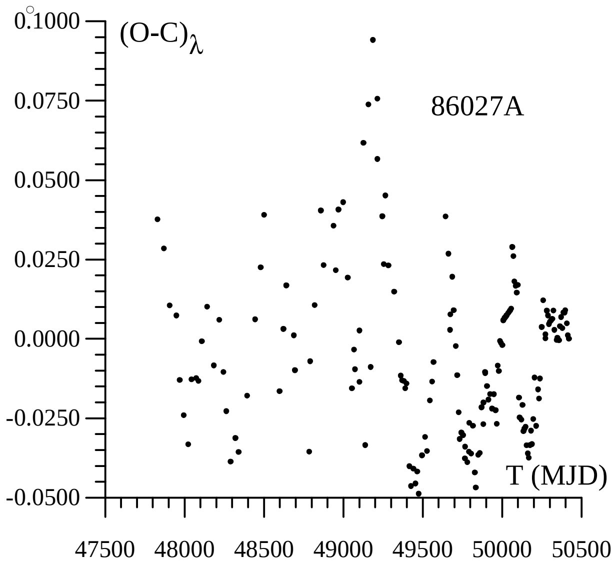

Figure 1: The longitude residuals for GS 86027A

An investigation of TLE orbits demonstrates that these oscillations are due to the systematic errors in the perigee longitudes (ω) and eccentricities (e) of observed GS. On Fig. 2 and 3 the curves of experimental and theoretical values of ω and e as a function of time for some GS are illustrated. The oscillations of curves on Fig. 2 and 3 are due to the solar light pressure. The systematic differences in phases is seen almost in every cases; as a rule the experimental curves on 60 days drop behind the theoretical ones. These discrepancies provoke errors in longitudes of order of $e\sin\omega\Delta\omega$ or $\cos\omega\Delta e$.

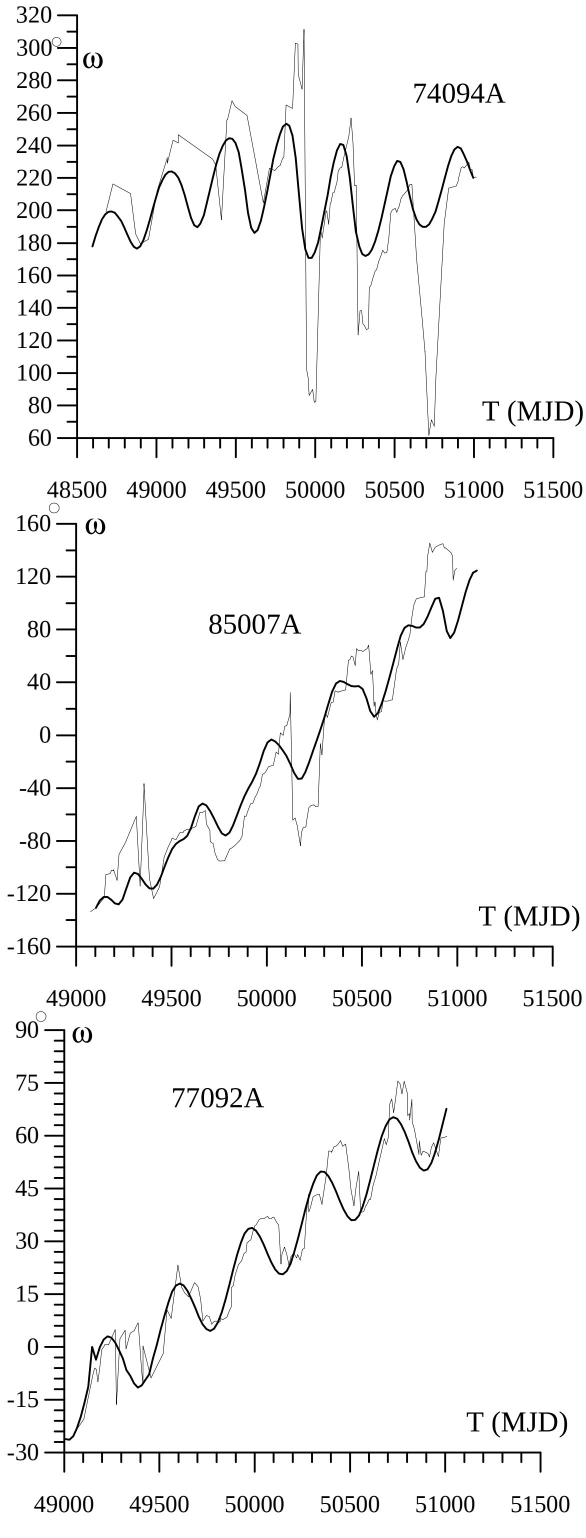

Figure 2: The experimental (thin line) and theoretical (bold line) curves of arguments of perigee as function of time

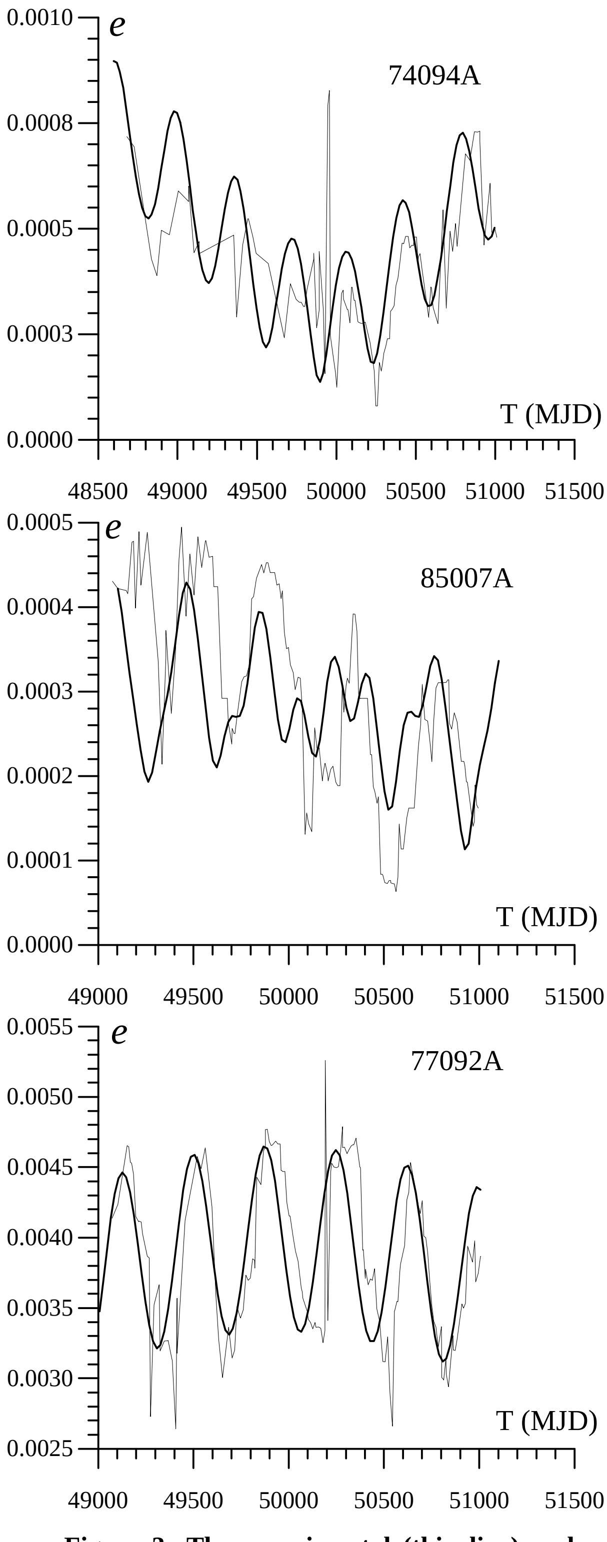

Figure 3: The experimental (thin line) and theoretical (bold lines) curves of eccentricity as function of time

The main part of these residuals are due to great accidental errors in the TLE.

Fig. 4 illustrates the difference between the accuracy of TLE and photographic observations in the case of GS 84016A (Statsionar-raduga 14).

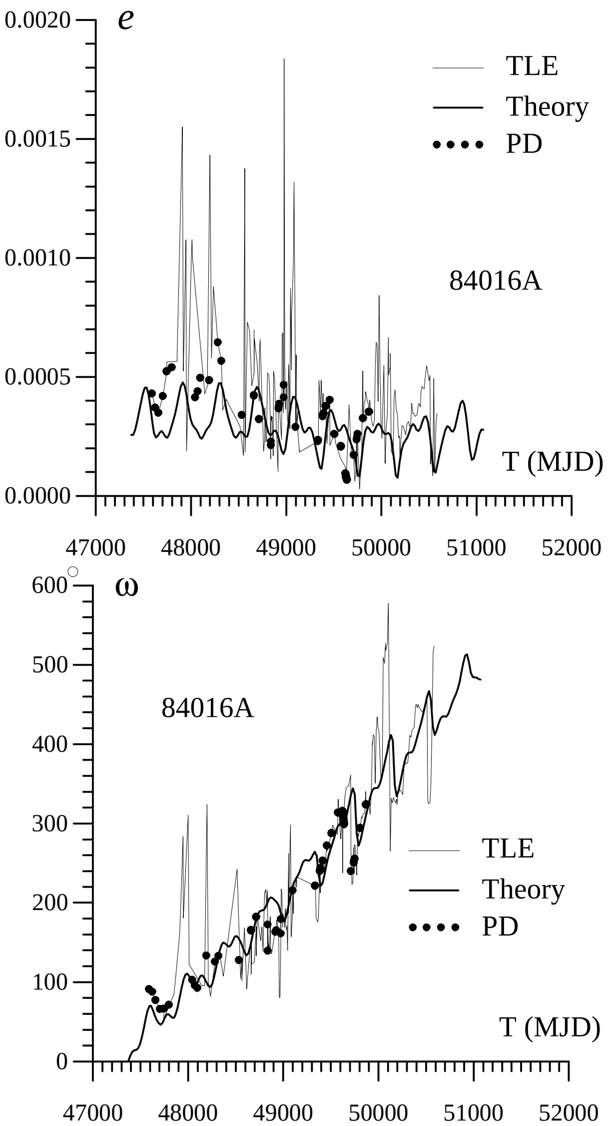

Figure 4: Comparison of TLE and photographic data (PD) of GS 84016A (Statsionar-raduga 14) with theory

The Influence of Geopotential model on Results

The accuracy of improving GS orbital elements and hence precision of the ephemerides depends on used model of geopotential. In the most cases this dependence is weak and may be neglected. But in the cases of GS moving in vicinity of the unstable librating points (160° E and 12° W longitudes), the selection of model is a decisive factor.

In[3] 14 objets of this kind are listed. Using these GS it is possible to improve the values of some low harmonics of geopotential. Such investigations became complicated by collisions of GS with space debris, slightly changing their motion.

On Fig. 5 the influence of chosen geopotential model on longitude residuals for such GS 85007A (Gorizont 11) is shown. There are (O - C)$_\lambda$ curves calculated by means of Joint Gravity Model-2 (JGM-2) and with changed parameters of this model – RKM:

$$\begin{aligned} C_{22} &= 2.4392837, \quad & S_{22} &= -1.4001493, \\ C_{31} &= 2.0337997, \quad & S_{31} &= 0.2476866, \\ C_{33} &= 0.7217939, \quad & S_{33} &= 1.4135969. \end{aligned} \tag{19}$$

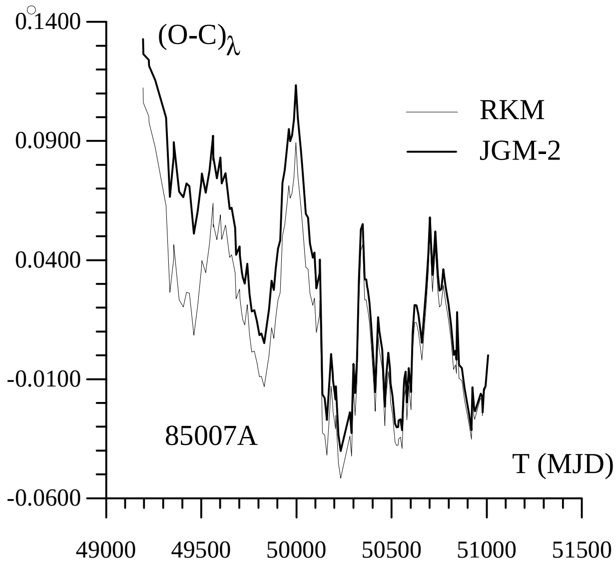

Figure 5: The longitude residuals of GS 85007A (Gorizont 11) for different geopotential models

Another example of this kind is shown on Fig. 6, where (O - C)$_\lambda$ curves for several geopotential models for GS 67026A (Intelsat 2 f-3) have been shown.

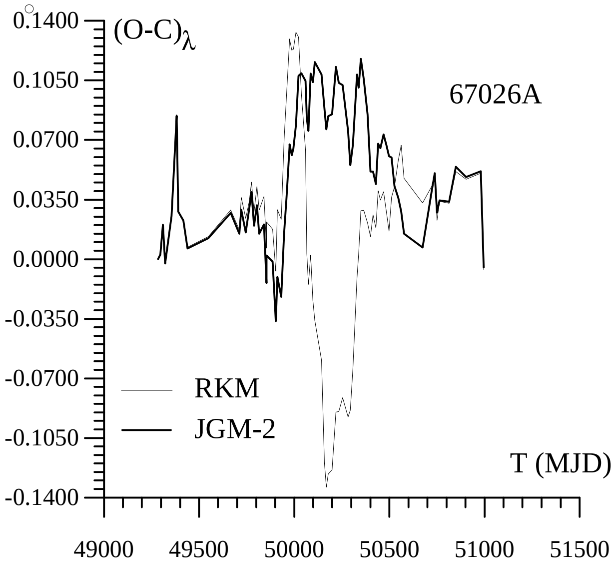

Figure 6: The longitude residuals of GS 67026A (Intelsat 2 f-3) for different geopotential models

At last, the most impressive example is the GS 87091D (Cosmos 1894 f.s.), depicted on Fig. 7.

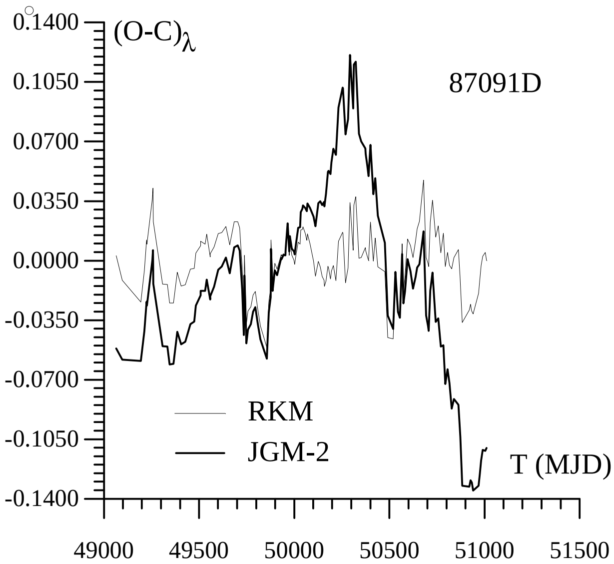

Figure 7: The longitude residuals of GS 87091D (Cosmos 1894 f.s.) for different geopotential models

It must be noted that in Fig. 5-7 the longitude residuals has been given over the time intervals which include the region of the passage of the unstable librating points 12° W and 160° E by these GS.

The New Version of New Catalog of GS Orbital Elements

Basing on new data spread by Internet and also the ones gathered from some observatories, the new version of the electronic catalog of GS orbits has been prepared. Catalog-98 has been added by the new satellites determined from optical observations. The method of identification of observations has been described. These results could be useful for organization of the survey observations for a search of the lost and a new objects.

By the statistical investigation of Catalog-98 one can come to the conclusion that more than 60% of GS endure a sudden changes in motion (by a few mm/s) because of their explosions or of collisions with a space debris. These results have permitted to conclude, that the occasional rate changes are taken place more often in the vicinity of explode satellites.

Conclusions

The resent investigations of GS motion shows that the great part of them suddenly changes speed because of micro explosions or collisions with space debris. For the several GS, moving on unstable orbits, the selection of geopotential model is very important. Unfitness of existing geopotential models for these GS seems obvious.

References

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