ACCURATE AND QUICK ACCOUNT OF THE TIDAL EFFECTS ON A SATELLITE'S MOTION BY THE NEW ANALYTICAL METHOD

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Abstract

Influence of the solid Earth, ocean and pole tides on satellite motion is not negligible. Specially dedicated missions like TOPEX/POSEIDON are launched to improve our knowledge of these effects. As a result, the models of the Earth's tides have dramatically been expanded last years.

To predict satellite's motion both numerical and analytical integration methods are being used in satellite dynamics. However, when calculating orbital perturbations due to the tidal forces numerical procedures are rather time-consuming, because at each integration step it is necessary to evaluate a great number (more than 2.000, presently) trigonometric series representing tidal variations of the Earth gravity coefficients.

Here we present the new analytical method for accounting both solid Earth, ocean and pole tides in satellite motion. It calculates these effects precisely (up to one-centimeter accuracy) and quickly (more than ten times faster as compared with numerical procedures).

Introduction

In precise orbit calculations the influence of the solid Earth and ocean tides on satellite motion is taken into account. As a result of specially dedicated missions like TOPEX/POSEIDON the accuracy of the models for the tidal effects has greatly been improved last years. For example, now the ocean tides model includes some 2000 terms; the models of solid Earth tides and the planet's rotational deformations due to polar motion (pole tide) account the effects of Earth's anelasticity. The Table 1 shows how the tidal models have been expanded only over several last years. The comparison is done by analysis of two IERS (International Earth Rotation Service) Standards issued in 1993[1] and 1996[2], respectively. (Let us remind these Standards contain the most precise and up-to-date models of the forces affecting Earth's satellite during its flight.)

Table 1: Recent improvement in Earth tides models

Standard	Ocean tides	Solid tides	Pole Tide
IERS Standards, 1992, /Technical Note 13/	*55* terms in gravity coefficients up to degree *6* and order *2*	*Elastic* Earth, *7* extra terms due to frequency – dependent Love numbers	*Elastic* Earth, *Real* value for Love number k_2
IERS Conventions, 1996, /Technical Note 21/	*1931* terms in gravity coefficients up to degree *30* and order *28*	*Anelastic* Earth, *49* extra terms due to frequency – dependent Love numbers	*Anelastic* Earth, *Complex* value for Love number k_2

The IERS recommendations[1,2] on the procedures of calculating the tidal effects lead to representation of the Earth's gravity coefficients as time-dependent functions. If a numerical method is employed for predicting a satellite's motion the tidal variations of harmonics of the geopotential expansion have to be calculated at each integration step. This is a rather time-consuming procedure as the corrections are given in the form of trigonometric series for a large number of the harmonics.

In the present work an analytical method has been used to account for the tidal variations of the gravity coefficients. It was initially developed by the author[3] for analytical calculation of satellite perturbations due to the Earth's irregular rotations (such as precession, nutation, polar motion, etc.), where the gravity coefficients were represented as trigonometric functions of time in an inertial coordinate system after their rotational transformation from the Earth-fixed system. The method is well adapted to handle time-dependent gravity coefficients given in the form of trigonometric series, and it has been applied to account for the effects of the solid Earth, ocean and pole tides on satellite motion.

The author's fifth-order analytical theory[4] of satellite motion uses this method and calculates relevant orbital perturbations to the high accuracy compatible with that of numerical procedures, but many times faster.

Earth's tides affecting a satellite

Several types of Earth tides affect a satellite during its flight. Here we present their short description and formulation.

Ocean tides

This type of tides is generated by re-distribution by water mass over the Earth surface under the Moon and Sun attraction. As a result, Earth gravity coefficients get periodic variations of amplitude to order 10^{-10}. The most current and complete model of the ocean tides[5], CRS 3.0, was determined from TOPEX/POSEIDON altimeter data and includes about 2.000 terms for a large number of the harmonics of the Earth potential expansion up to degree 30 and order 28, inclusive. All the relevant corrections are represented there in the form of trigonometric series with numerical coefficients where a moment of time is the argument of the series:

$$\Delta\overline{C}^{O}_{nm} = F_{nm}\sum_{s(n,m)}\left[\left(C^{+}_{snm}+C^{-}_{snm}\right)\cos\Theta_s+\left(S^{+}_{snm}+S^{-}_{snm}\right)\sin\Theta_s\right], \quad (1)$$

$$\Delta\overline{S}^{O}_{nm} = F_{nm}\sum_{s(n,m)}\left[\left(S^{+}_{snm}-S^{-}_{snm}\right)\cos\Theta_s-\left(C^{+}_{snm}-C^{-}_{snm}\right)\sin\Theta_s\right], \quad (2)$$

where

$\Delta\overline{C}^{O}_{nm}, \Delta\overline{S}^{O}_{nm}$ - ocean tidal corrections to normalized Earth's gravity coefficients of degree *n* and order *m*,

$C^{+}_{snm}, C^{-}_{snm}, S^{+}_{snm}, S^{-}_{snm}$ - numerical ocean tide coefficients determined from observations,

F_{nm} - parameter depending on the values of seawater density and Earth's load deformation coefficients,

Θ_s - linear function of the fundamental arguments of the nutation series calculated at an epoch.

Other details can be found, e.g., in the IERS Standards[1, 2].

Solid Earth tides

Solid Earth tides is the response of the non-rigid Earth to attraction of the Moon and Sun. Numerically they are described as follows[6, 7]

$$\Delta\overline{C}^{T}_{nm} - i\Delta\overline{S}^{T}_{nm} = \frac{k_{nm}}{2n+1}\sum_{j=2}^{3}\frac{GM_j}{GM_{\oplus}}\left(\frac{R_{\oplus}}{r_j}\right)^{n+1}\overline{P}_{nm}(\sin\Phi_j)e^{-im\lambda_j} \quad (3)$$

where

$\Delta\overline{C}^{T}_{nm}, \Delta\overline{S}^{T}_{nm}$ - solid Earth tidal corrections to normalized gravity coefficients of degree *n* and order *m*,

k_{nm} - complex Love number of degree *n* and order *m*,

$R_{\oplus}, GM_{\oplus}$ - Earth's equatorial radius and gravitational parameter,

GM_j, r_j, Φ_j and λ_j are respectively, gravitational parameter, geocentric distance, latitude and longitude (from Greenwich) of the Moon ($j = 2$) and Sun ($j = 3$),

$\overline{P}_{nm}$ - normalized associated Legendre functions, and $i \equiv \sqrt{-1}$.

Only for *n* equal to 2 and 3 as well as for several coefficients of degree 4 solid Earth tides have to be accounted when calculating satellites' orbits. Obtained according to (3) variations of the planet's gravity coefficients are to the order of 10^{-8}. The 2nd degree harmonics are also complemented by some dozens extra terms due to frequency-dependent corrections to Love numbers. Since recently the Earth's anelasticity is accounted in the most precise theories of satellite motion. It is numerically described by adopting the complex values for Love numbers.

Pole tide

Pole tide is the effect of the non-rigid Earth's rotational deformation caused by polar motion. It results in an external centrifugal potential that can be equivalently described by additional periodic changes in the Earth's gravity coefficients of the 2nd degree[8,9]. Although the changes are produced in all of such coefficients, only variations of the harmonics of the 1st order, $\overline{C}_{21}$ and $\overline{S}_{21}$, are significant enough to be accounted in satellite orbit determination. To the first order of values of the polar coordinates the relevant variations can be written as follows[8,9]:

$$\Delta\overline{C}^{P}_{21} = \frac{k_2}{k_s}\sqrt{3}\left(x_p-\overline{x}_p\right)\overline{C}_{20}, \quad (4)$$

$$\Delta\overline{S}^{P}_{21} = -\frac{k_2}{k_s}\sqrt{3}\left(y_p-\overline{y}_p\right)\overline{C}_{20}, \quad (5)$$

where

$\Delta\overline{C}^{P}_{21}, \Delta\overline{S}^{P}_{21}$ - changes in normalized gravity coefficients of the 2nd degree, 1st order due to the pole tide,

$\overline{C}_{20}$ - normalized Earth zonal harmonic of the 2nd degree,

k_2 - 2nd degree Love number,

k_s - secular Love number,

x_p, y_p - IERS coordinates of the pole,

$\bar{x}_p, \bar{y}_p$ - mean values of x_p, y_p over at least one Chandler circle.

Variations $\Delta\bar{C}^p_{21}$, $\Delta\bar{S}^p_{21}$ are to the order of 10^{-9}.

To account for the Earth anelasticity complex values for k_2 are used.

It is worth to note that variations of the gravity coefficients C_{21}, S_{21} due to the solid Earth, ocean and pole tides describe changes of position of the Earth's figure axis (the principal axis of maximum inertia). Both solid Earth and ocean tides cause daily motion of the axis with maximum amplitude of about 60 meters[10]. Pole tide leads to long-term motion of the mean (averaged over a day) position of the axis with amplitude of about 0.2 meters[10] and of Chandler period (Fig.1).

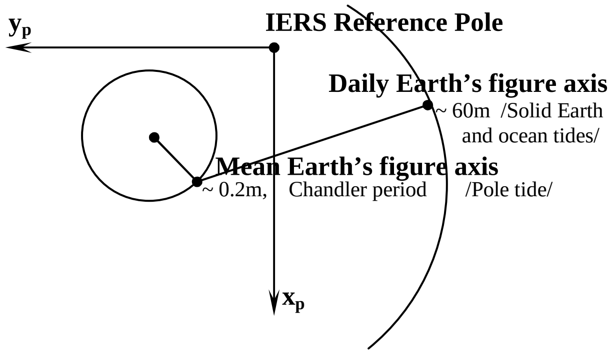

Figure 1: Earth tides and the planet's figure position

Thus, all the Earth tides lead to time-dependent values for harmonics of the geopotential expansion that are described by the relevant adopted models. Account by the new analytical method of the effect of variability of the Earth's gravity coefficients on satellite motion is presented in the next chapter.

Analytical account of the tidal effects on satellite motion

Description of the method

A new fifth-order analytical theory of satellite motion is being developed by the author[3,4]. All secular and short-periodic perturbations proportional up to and including the fifth order of the small parameter of a perturbing force are calculated. Long-periodic perturbations are derived with accuracy of up to the fourth-order, inclusive.

The feature of the theory is combination of the classical Poincare small parameter method with possibilities of current computers of large memory. In our theory the orbital perturbations of satellite are calculated not explicitly, but in the form of trigonometric series with numerical coefficients where a moment of time is the argument. Such form is kept for any order of the perturbations arising from the forces affecting a satellite during its flight. To get perturbations of high orders it is necessary to multiply several series representing lower orders perturbations. If the latter series are trigonometric, the resulted series will be trigonometric as well. As the coefficients are numerical, such a procedure does not request much computer time, and is to be performed just once for specific orbital elements of a satellite. At each step of the analytical integration an analysis is done to leave in the obtained expressions only the perturbations of amplitudes which are large enough as compared with the accuracy of processed tracking measurements of satellite.

The numerical coefficients of the final trigonometric series representing the satellite osculating elements are stored in the computer memory, and to get the satellite position at any epoch one needs just to input the moment of time as the argument to these series.

Thus, the theory is quite accurate as this approach allows to calculate high order perturbations, and simultaneously it is very effective as the amount of computation does not depend on the time interval for which one needs to predict the satellite motion.

Such a feature is especially important when it is necessary to get accurate satellite positions or to process its tracking measurements collected over a long time span (e.g., several years and more). There is a special kind of geodetic satellites, like LAGEOS or ETALON, which orbits are not corrected; these scientific spacecraft are specially designed to study geodetic and geophysical effects by processing their very precise laser tracking measurements which can be collected for many years.

Another field of the theory possible application is use of its compact, precise and long-standing analytical series aboard satellites of perspective navigational systems, like GNSS-2. Currently, to calculate coordinates of a satellite of navigational constellations GPS and GLONASS the onboard software uses short-time polynomials uploaded from the Earth every 12-24 hours. Onboard predicting the satellites motion by the analytical series of our theory would allow to expand this time to a few months.

The work[4] shows that motion of ETALON (which orbit is similar to that of GPS, GLONASS or GNSS-2) in the static Earth's gravitational field by the analytical theory can be predicted to accuracy of one centimeter over a few years. In the follow-on work[3] the similar accuracy of the analytical method is reached when accounting precession and nutation of the geoequator, polar motion and irregularities of the Earth's rotation. To do that the coefficients of the Earth gravitational field were assumed to be static in the Terrestrial Reference Frame. Then they have been transformed to an inertial Celestial Reference Frame and represented there as time-dependent functions. A new method is developed[3] to handle by the analytical motion theory the variable gravity coefficients. It leads to input of additional trigonometric series (that represent the time-dependent coefficients) to the right-hand sides of Lagrange differential equations of satellite motion. The new series for every coefficient is multiplied by other series of the equations, and then the integration scheme of the analytical theory is applied.

At the new step of the work, presented here, the Earth's gravity coefficients are not already considered as static ones even in the Terrestrial Reference Frame, because of the Earth tides. The method described above and used when accounting the Earth irregular rotations has been extended to handle the tidal effects. Each variation of the gravity coefficients due to the Earth tides has been represented by a relevant trigonometric series.

The current model for ocean tides (1)-(2) provides corrections to the coefficients that are already in the trigonometric form. They are directly input to the analytical theory software.

The model for solid Earth tides (3) is written in the form adapted for numerical integration procedure. It includes the angular coordinates of the Moon and Sun that are obtained from the bodies' ephemerides. To calculate the latter, analytical theories usually approximate motion of the Moon, Sun (and of big planets, if necessary) by precessing Keplerian ellipses. Such an assumption often leads to quite good results, but to predict a satellite motion to centimeter accuracy it is not enough. So, we employ another approach. By using the most current planet ephemerides, DE403/LE403[11], the values for variations of the gravity coefficients due to solid Earth tides are firstly computed strictly according to the expression (3) at many epochs equidistributed along the interval of time covering satellite measurements. Then the arrays of obtained tidal corrections are approximated by Fourier series. It is done for every gravity coefficient included to the model of solid Earth tides (those of degree 2 and 3, and several coefficients of degree 4; total number is 17). The time span between the epochs at which the variations are calculated depends on the order *m* of the coefficient and is usually chosen so minimal to see diurnal variations of the harmonics of order 1, semi-diurnal ones of the coefficients of order 2, etc.

These Fourier series are trigonometric by definition and can be further handled by the analytical theory.

The last effect to be considered - pole tide, is accounted in a similar way. The values for polar coordinates x_p, y_p by IERS are published with minimal step of one day. We take these values over at least one Chandler period covering the interval of satellite measurements, then perform Fourier analysis of the data. The obtained series exactly represent the polar coordinates for the chosen time span. The mean values for polar coordinates - $\bar{x}_p, \bar{y}_p$, required by (4) are just the zeroth terms of the Fourier series for x_p, y_p.

Thus, all the perturbing functions due to the Earth tides can be represented by trigonometric series and consequently accounted by the analytical motion theory. The next section describes the results of testing the analytical method of calculating the tidal perturbations in a satellite's orbit by comparison of it with a numerical procedure.

Testing the method

Some hundreds coordinates of ETALON-1 geodetic satellite distributed over one year interval have been calculated by Everhart, 15th-order, numerical integration method. The complete model JGM-3 of the geopotential was employed and all of the tidal effects were strictly accounted.

Then these positions were assumed as simulated observations, and processed with use of the analytical integration method. Account of the perturbations due to the geopotential and to the tidal effects has been included to the analytical theory as well. The parameters of the latter, six mean Keplerian elements at the initial epoch, were improved by the least-square procedure. The r.m.s. difference between the two sets of the satellite positions provided by the numerical and analytical methods was found to be not more than one-two centimeters over the total one year's interval when considering any of the tide effects. It is an advantage of the analytical theory that its time of computing the tide effects on satellite motion was at least ten times less than that of the numerical method.

To evaluate the level of measurement residuals in the case of not accounting the Earth's tides in ETALON motion an experiment has been performed. The same set of the fictitious observations was processed, but the force model in the analytical theory did not include the tidal effects. Even after final adjustment of the theory's parameters the errors in calculated positions of the satellite reach ten meters (c.f., accuracy of ETALON laser range measurements is about 1-2 centimeters).

This work completes the author's studies[3,4] on precise analytical account of satellite motion perturbations caused by the Earth’s body (its non-central potential, irregular rotations and different tides).

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