INVESTIGATION OF SIMPLIFIED MODELS FOR ORBIT DETERMINATION USING SINGLE FREQUENCY GPS MEASUREMENTS

Ana Paula Marins Chiaradia*
Hélio Koiti Kuga
Antonio Fernando Bertachini de Almeida Prado
INPE – Instituto Nacional de Pesquisas Espaciais
Av. dos Astronautas, 1758 - Jardim da Granja
São José dos Campos - SP - Brazil - CEP: 12 227-010
*chiara@dem.inpe.br

Abstract

The main goal of this work is to investigate simplified models to determine in real time the orbit of an artificial satellite, using single frequency GPS measurements. This model should be compact providing standard precision at low cost. Cowell's method has been used to propagate the orbit state vector. The modeled forces are geopotential up to 23rd order and degree of the spherical harmonic coefficients. To propagate the state covariance matrix, it has been considered a more simplified model than the one used in dynamical model. For computing the state transition matrix, it is considered only keplerian motion. To estimate an orbit in real time the extended Kalman filter has been used throughout this paper. Many tests are carried out starting from the simplest two body model and varying the contribution sources of the errors to be considered. In order to assess the results, the estimated orbit for several cases is compared with a full reference Kalman filter, for the Topex/Poseidon satellite.

Key words: Kalman Filtering, GPS, Real time orbit determination.

Introduction

The GPS system has been used to determine artificial satellite orbits because it may provide orbit determination precision as good as or better than methods using conventional tracking stations. The later provides standard precision around hundred meters and the former can provide precision as tight as some centimeters. The attractiveness of GPS is magnified because of lower costs and autonomous navigation resources. With the advances of technology, the single frequency GPS receivers provide a good basis to achieve enough precision at relatively low cost still attaining the accuracy requirements of the mission.

The GPS system provides at a given instant a set of many redundant measurements which makes the orbit position observable geometrically. If the measurements are accurate, a sequentially dynamical orbit determination may not need such a precise force model as the whole information is locally provided by the measurements. This approach was applied in the work of Wu *et al.*[11], which uses the so-called reduced dynamic technique.

The state estimation method adopted was the Kalman filter which is used to estimate the spacecraft's orbit on board, excluding the need of iterating the data collected previously and being able to provide the current orbit in real time.

Estimation Method

The orbit determination problem, which has a non-linear dynamical system and a non-linear measurement system, can be formulated in a way that it is possible to apply one of the best known methods of sequential linear estimation, the Kalman Filter.

The extended Kalman filter is a Kalman filter version applicable to problems like this one, constituted by a time-update and a measurement-update cycles. The time-update phase updates the state and the covariance matrix along the time using the dynamical equations.

In this work, a simple reduced state vector is chosen to be estimated:

$$\boldsymbol{x} = \left(\boldsymbol{r}^T, \boldsymbol{v}^T, b\right), \tag{1}$$

where $\boldsymbol{r}^T = (x, y, z)$ and $\boldsymbol{v}^T = (\dot{x}, \dot{y}, \dot{z})$ are the spacecraft's position and velocity vector, and b is the receiver clock offset.

Then, the differential dynamic equations of motion to be integrated are given by:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t), \tag{2}$$

and

$$\dot{\Phi} = F\Phi , \quad (3)$$

where $f(x,t)$ is the vector-valued function of time and the state, Φ is the state transition matrix which relates the state between t_k and t_{k+1}, and $F = \partial f(x,t)/\partial x$.

Both equations should be numerically integrated simultaneously, so that F is evaluated always along the most current state x. Next, one updates the covariance matrix P by means of the discrete Riccatti equation:

$$\bar{P}_{k+1} = \Phi_{k+1}\hat{P}_k\Phi_{k+1}^T + Q_k , \quad (4)$$

where $\hat{P}_k$ is the covariance matrix after processing all measurements at time t_k, Φ_k is the state transition matrix obtained by the previous integration and Q_k is the discrete state-noise covariance which is a measure of the error between the reference state and true state arising from imperfect modeling. The P matrix is a measure of how well the errors are known.

At the end of this process $\bar{x}_{k+1}$ and $\bar{P}_{k+1}$, are obtained and are called time-updated state and covariance, respectively.

The measurement residual and sensitivity matrix are found by forming the computed observation equation. The model for a GPS pseudorange measurement is given by:

$$y_c(x_k,t_k) = \rho_k + \beta_k , \quad (5)$$

where

$$\rho = \sqrt{(x_{GPS}-x)^2 + (y_{GPS}-y)^2 + (z_{GPS}-z)^2} \quad (6)$$

is the geometric range, x, y, and z are the positional states of the user satellite at reception time, x_{GPS}, y_{GPS}, and z_{GPS} are the positional states of the GPS satellite at transmission time (corrected for light time delay), and β_k are errors steaming from ionosphere path delay, GPS satellite and receiver clock offsets, GPS ephemeris error, multipath, and other unmodeled errors.

Using the above equation the sensitivity matrix is given by:

$$H_k = \left[-\frac{(x_{GPS}-x)}{\rho}, -\frac{(y_{GPS}-y)}{\rho}, -\frac{(z_{GPS}-z)}{\rho}, 0,0,0,1\right] . \quad (7)$$

The measurement residual, or innovation sequence is:

$$y_k = Y_k - y_c(x_k,t_k) , \quad (8)$$

where Y_k is the observed measurement.

The measurement update phase uses the Kalman equations to incorporate the information given by the measurements themselves, and obtains improved estimates of the state and of the covariance:

$$K_k = \bar{P}_k H_k^T \left(H_k \bar{P}_k H_k^T + R_k\right)^{-1} , \quad (9)$$

$$\hat{x}_k = \bar{x}_k + K_k y_k , \quad (10)$$

$$\hat{P}_k = \left(I - K_k H_k\right)\bar{P}_k , \quad (11)$$

where R_k is the discrete measurement noise covariance which is basically a measurement weight matrix. These equations can be used to process measurements sequentially so that the matrix inversion in (9) is a scalar. To be precise, the measurements should be uncorrelated, in which case the measurement covariance noise R_k is diagonal.

Dynamical Model

The choice of the propagation method depends on the requirements of the mission. As the goal of this work is to have relatively standard accuracy along with minimum computational cost, the Cowell's method has been used to propagate the state vector.

The modeled forces in this work are due to geopotential taking into account the spherical harmonic coefficients up to 23rd order and degree of GEM10B model. The integration is carried out by using the simple fourth order Runge-Kutta algorithm without any mechanism of step adjustment or error control. The fourth order Runge Kutta is considered an adequate numerical integrator due to its simplicity, fair accuracy, and low computational burden.

The dynamic equation of motion is given by:

$$\ddot{r} = -\frac{\mu}{r^3} + a_{GEO} , \quad (12)$$

where μ is the geo-gravitational constant and a_{GEO} is the acceleration due to perturbing geopotential, computed according to Pines[8].

The user satellite coordinates are in true equator and equinox of date frame (ToD) and the computation of the acceleration of the satellite due to the spherical geopotential requires the coordinates of the satellite in the rotating Earth-fixed equator and prime meridian frame (EF). Then, it is necessary to provide the software with the coordinate transformations considering the effects of precession, nutation, polar motion, and sidereal rotation. Polar motion and difference between UTC and UT1 was disregarded as it requires uploads of these parameters to the satellite computer, increasing the ground task load and decreasing the satellite autonomy. To decrease the computational burden further, precession and nutation parameters and matrices may be considered constant for a span of one day.

Measurement Model

The equation of the pseudorange in L1 frequency is given by:

$$P_k^1 = \rho_k + I_k + c\left[dt_{GPS}(t_k) - dt_U(t_k)\right] - \varepsilon_k\,, \qquad (13)$$

where P_k^1 is the pseudorange in L1, ρ_k is the geometric range given by Eq. (6), I_k is the ionospheric delay, c is the vacuum speed of light, $dt_{GPS}(t_k)$ is the GPS satellite clock offset, $dt_U(t_k)$ is the receiver clock offset, t_k is the observation instant in GPS time, and ε_k is a remnant error supposed random gaussian.

Clock Error Terms

The third term of the right-side is the clock bias which represents the combined clock offsets of the satellite and of the receiver with respect to GPS time. Each GPS satellite contributes with one unknown clock bias. The information for the GPS satellite clocks is known and transmitted via the broadcast navigation message in the form of three polynomial coefficients with a reference time t_{oc}. The clock correction of the GPS satellite for the epoch t_{GPS} is[6]:

$$\Delta t_{SV} = a_{f0} + a_{f1}(t_{sv} - t_{oc}) + a_{f2}(t_{sv} - t_{oc})^2 + \Delta t_R\,, \qquad (14)$$

with

$$t_{sv} = t_{GPS} - \Delta t_{sv} \qquad (15)$$

and

$$\Delta t_R = -\frac{2}{c^2}\sqrt{a\mu}\,e\sin E = -\frac{2}{c^2}\,\boldsymbol{x} \bullet \dot{\boldsymbol{x}}\,, \qquad (16)$$

where Δt_R is a small relativistic clock correction caused by the orbital eccentricity e, a is the semimajor axis of the orbit, and E is the eccentric anomaly. The polynomial coefficients a_{f0}, a_{f1}, and a_{f2} are transmitted in units of sec, sec/sec, and sec/sec^2, respectively. The clock data reference time t_{oc} is also broadcast. The value of t_{sv} must account for the beginning or end-of-week crossovers.

The user clock offset is part of the estimated state vector.

Ionospheric Correction

The GPS transmitted signals pass through the ionospheric layer causing errors in the measurements, being the ionospheric effects one of the hardest effects to model.

As the user is a satellite, tropospheric effects are considered irrelevant, and a simple elevation mask will reject measurements reflecting into the Earth's troposphere.

The ionospheric effects depend on the frequency. It is possible to remove them easily by using dual frequency. However, in single frequency measurements, they cause many real troubles in the data processing, where the degradation of precision can achieve reasonable values depending on the solar activity and the spatial (geomagnetic) environment. The maximum effect in single frequency receivers is around 20m and for the case of dual frequency it is around 4.5cm[9].

Despite assuming single frequency in this work, the dual frequency model has been used to correct the ionospheric effects in the pseudo-range measurements. As such it allows one to analyze the impact of either neglecting or not the ionosphere effect on the orbit estimates. In this case, this is possible because our satellite test case, TOPEX/Poseidon, has a dual frequency receiver on board. The equation to correct the ionospheric effects of the pseudo-range measurement in L1 is given by[6]:

$$I_k^P = \frac{f_2^2}{f_2^2 - f_1^2}\left(P_k^1 - P_k^2\right), \qquad (17)$$

where I_k^P is the ionospheric correction to range measurements in L1 frequency, P_k^1 and P_k^2 are the pseudorange measurements in L1 and L2, respectively, f_1 is the frequency in L1 (1.575 GHz), and f_2 is the frequency in L2 (1.227 GHz).

Transition Matrix

The calculation of the state transition matrix presents one of the highest computational costs because it needs the evaluation of the Jacobian matrix (partial derivatives) and integration of the current variational equations. This matrix can pose cumbersome analytical expressions when using a complex force model[3].

A simple force model to state transition matrix considered the keplerian motion. The state transition's differential equation is defined as:

$$\dot{\Phi}(t_k, t_0) = F(t_k)\Phi(t_k, t_0), \tag{18}$$

where $\Phi(t_0, t_0) \equiv \boldsymbol{I}$ is the initial condition,

$$\Phi(\boldsymbol{t_k}, t_0) = \begin{bmatrix} \dfrac{\partial \boldsymbol{r}}{\partial \boldsymbol{r}_0}_{3x3} & \dfrac{\partial \boldsymbol{r}}{\partial \boldsymbol{v}_0}_{3x3} & \boldsymbol{0}_{3x1} \\ \dfrac{\partial \boldsymbol{v}}{\partial \boldsymbol{r}_0}_{3x3} & \dfrac{\partial \boldsymbol{v}}{\partial \boldsymbol{v}_0}_{3x3} & \boldsymbol{0}_{3x1} \\ \boldsymbol{0}_{1x3} & \boldsymbol{0}_{1x3} & \boldsymbol{1}_{1x1} \end{bmatrix} \tag{19}$$

and

$$F(t_k) = \begin{bmatrix} \boldsymbol{0}_{3x3} & \boldsymbol{I}_{3x3} & \boldsymbol{0}_{3x1} \\ G(t_k)_{3x3} & \boldsymbol{0}_{3x3} & \boldsymbol{0}_{3x1} \\ \boldsymbol{0}_{1x3} & \boldsymbol{0}_{1x3} & \boldsymbol{0}_{1x1} \end{bmatrix}. \tag{20}$$

$G(t) = \partial \boldsymbol{f}(\boldsymbol{r}, t)/\partial \boldsymbol{r}$ is the gradient matrix of the geogravitational force and $\boldsymbol{f}(\boldsymbol{r},t)$ is the acceleration of the satellite that, in this case, considered the Keplerian motion.

Data Set

As observation data TOPEX/Poseidon (T/P) GPS data set is used, because this satellite has dual frequency GPS receiver onboard and such data are easily found in Internet.

As suggested by Binning[2], one uses the T/P data set of November 18th, 1993, because Selective Availability (SA) was also not in operation for 17 of the 25 available GPS satellites allowing civilian users access to the most precise GPS measurements. At that time the GPS constellation was not yet considered fully operational, and therefore, Anti-Spoofing was also off. This allowed all users to receive clean data in both L1 and L2 frequencies.

The T/P receiver can track up to 6 GPS satellites on both frequencies if Anti-Spoofing is inactive[2]. This raw data was recorded 10 second interval of GPS time.

The observation data in Rinex format were obtained via Internet[10]. The navigation data in Rinex format were sent by Binning[2], although they also could be obtained via Internet[7].

The precise GPS satellite coordinates are obtained through Internet and interpolated[4] to appropriate times, providing the coordinates in EF frame.

The initial conditions to t_0 = 1sec in UTC time as of November, 18th, 1993 are given in the Table 1.

Table 1: Initial conditions to user satellite coordinates.

Initial Conditions		Nov 18, 1993
ToD	***x* (m)**	7617202.243009592
ToD	***y* (m)**	1235354.688733236
ToD	***z* (m)**	-135607.5368155133
ToD	**$\dot{x}$ (m/sec)**	-353.5738692980746
ToD	**$\dot{y}$ (m/sec)**	2898.599146009871
ToD	**$\dot{z}$ (m/sec)**	6568.36541232146

The initial conditions are obtained through the TOPEX/Poseidon Precise Orbit Ephemeris (POE)[10] generated by the Jet Propulsion Laboratory (JPL) in UTC time. The JPL POE is claimed to estimate Topex's position to an accuracy of better than 15 cm (See Ref. 1 for details). The states in the POE are provided in one minute increments in Inertial True of Date coordinates in UTC time. But, the TOPEX GPS measurements are in 10 second increments in GPS time. Accordingly to IERS, the difference between the UTC and GPS times is 9 seconds to this date. Therefore, it was necessary to interpolate the states in one second steps through our ODEM Orbit Determination software[4,5].

Table 2: Extended Kalman filter input parameters.

Filter Input Parameter	Initial values
P_r (m^2)	1.0×10^4
P_v (m^2/sec^2)	1.0×10^{-2}
P_b (sec^2)	1.0
Q_r (m^2)	1.0×10^1
Q_v (m^2/sec^2)	1.0×10^{-5}
Q_b (sec^2)	1.0×10^{-3}
R_k (m^2)	2.5×10^1

The Kalman Filter input parameters are given in the Table 2, where P_r is the position error, P_v is the velocity error, P_b is the user clock error, Q is the corresponding dynamical noise, and R_k is the covariance of measurement error.

The considered constants are given in Table 3.

Table 3: Constants in Topex case study.

WGS 84 gravitational parameter	3.986005×10^{14} m^3/sec^2
WGS 84 rotation rate	$7.292115167 \times 10^{-5}$ rad/sec
WGS 84 Earth's radius	6378137 m
speed of light	299792458 m
J_2	1.08263×10^{-3}

Results

All tests are accomplished considering the initial conditions cited in Tables 1 and 2.

The first test is aimed at checking the accuracy of the integrator of fourth order Runge-Kutta. The filter is turned off and several fixed step sizes are attempted. Also the geopotential truncation order is changed to verify the different level of errors.

In the cases 1 to 4, it is considered the Keplerian motion. In the cases 5 to 8, the geopotential includes the term in J_2. In the cases 9 to 12, up to 23rd order and degree of spherical harmonic coefficients are considered. The step size (Δt) varies from 1 second to 1 minute. In Table 4, Δr and Δv are the errors on the position and velocity states, respectively, compared with the POE.

Table 4 shows the maximum accumulated errors for each case after both one minute and two hours of integration with different step sizes. One can conclude that better accuracies are obtained in the cases 9 and 10. As the difference between them is very small, the 10 second step size is adopted in all foregoing cases with a goal of minimizing the computational cost.

Therefore, in the remaining cases, the TOPEX's orbit state is generated every 10 second step size maximum.

The next results are achieved with the filter turned on, considering around 2 hours of data. The reference trajectory is accomplished considering the full model Kalman filter, which shows good agreement with the JPL/POE ephemeris. It considered up to 23rd order and degree of the spherical harmonic coefficients and the effects of precession and nutation. The relativistic, ionospheric, GPS and user receiver clock offsets are considered in the measurement model as shown previously.

Table 4: Accuracy of the fixed step integrator RK4.

Case	After	Δt (sec)	Δr (m)	Δv (m/sec)
1	60 sec 2 hours	1	13.248 43594.4	0.442 40.801
2	60 sec 2 hours	10	13.248 43594.4	0.442 40.801
3	60 sec 2 hours	30	13.248 43593.7	0.442 40.800
4	60 sec 2 hours	60	13.246 43581.3	0.442 40.787
5	60 sec 2 hours	1	0.121 641.805	0.004 0.585
6	60 sec 2 hours	10	0.121 641.814	0.004 0.585
7	60 sec 2 hours	30	0.121 642.579	0.004 0.586
8	60 sec 2 hours	60	0.121 656.153	0.004 0.598
9	60 sec 2 hours	1	0.012 87.601	0. 0.084
10	60 sec 2 hours	10	0.012 87.609	0. 0.084
11	60 sec 2 hours	30	0.013 88.387	0. 0.085
12	60 sec 2 hours	60	0.042 102.123	0. 0.098

The full model Kalman filter results are then used to compare the several contribution sources to the errors. Fig. 1 shows the pseudorange residuals in L1 during the first 15 minutes of the filtering. Also, one can note that the filtering is converging. It is impossible to show all residuals because of scaling. Fig. 2 shows the number of GPS satellites at each epoch of the filtering. The average number of available GPS satellites is 5.536 and the average number of used GPS satellites (not rejected) is 5.515.

For cases 13 and 14, the tests are related to the dynamical model, i.e., only the perturbation of the dynamical model changed in each case. In case 13, the pure Keplerian motion was considered, while in the case 14, the effect of J_2 is considered. Table 5 shows the results obtained compared with the reference case.

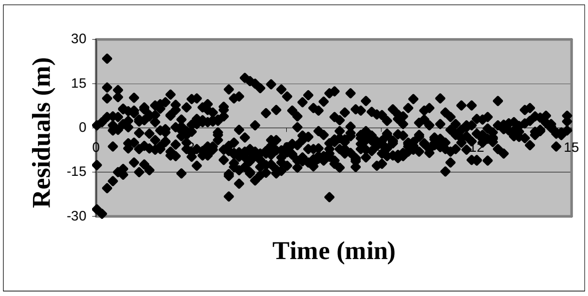

Figure 1: Sample of Residuals for the full model.

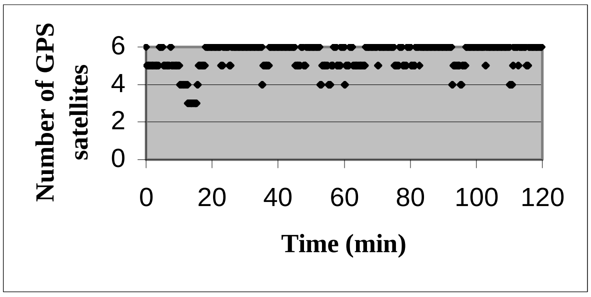

Figure 2: Number of processed GPS satellites in two hours run.

Table 5: Errors due to variation of the dynamical model.

Case	RMS_r (m)	RMS_v (m/sec)
13	2.87×10^{1}	4.21
14	0.33	0.05

In cases 15 to 18, one of the following effects are not considered in each case: GPS clock offset, receiver (satellite) clock offset, relativistic effect, ionosphere effect, and signal travel time. To depict the relative importance of each one to the accuracy of the orbit estimates, in case 15 the GPS clock offset is not considered; in case 16 without considering the receiver clock offset; in case 17, without accounting for the relativistic effect; in case 18, without the ionospheric correction; and, in case 19, without the signal travel time correction. Table 6 shows the obtained results compared with the reference case.

Finally, in case 20, the effects of precession and nutation in the coordinates are disregarded (See Table 7), when transforming the coordinates from J2000.0 to True of Date (See Table 7).

Conclusions

This paper considers several aspects of modeling when using the GPS for real time orbit determination. It is observed that for sampling rates of 10 seconds and using a simple fourth order Runge-Kutta numerical integrator the higher order geopotential model can yield already reasonable results. At least J_2 shall be included in the geopotential model because a single Keplerian model can not account for the short period oscillations, even for 10-second step size.

The measurement model shall be carefully modeled as some effects can be very pronounced. The GPS clock offset shall be taken into account, secondly the signal travel time correction. After comes the user clock offset, the ionosphere, and to a lesser extent the relativistic effect. As a last remark, the precession and nutation of coordinates shall be taken into account as they considerably change the actual coordinates. This leads to an additional computational burden which can be minimized if one considers the precession and nutation parameters constant over a given period, say one day.

Table 6: Errors due to variation of the measurement model.

Case	RMS_r (m)	RMS_v (m/sec)
15	2.03×10^{2}	0.15
16	3.71×10^{1}	3.03
17	6.27	0.008
18	3.86	0.005
19	6.89×10^{1}	0.09

Table 7: Errors due to neglecting of the effects of precession and nutation.

Case	RMS_r (m)	RMS_v (m/sec)
20	8.8×10^{3}	7.84

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