Characterizing Orbit Uncertainty Due to Atmospheric Drag Uncertainty

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Abstract

The International Space Station (ISS) will perform maneuvers to avoid potential collisions with other space objects whenever the probability of collision, P_C, exceeds a specified value. P_C is a function of the uncertainty, i.e. the covariance, of the orbits of each of the two objects. To avoid unnecessary maneuvers that waste fuel and to ensure that necessary maneuvers are performed it is imperative that the covariance be accurate. The primary contributor to the covariance inaccuracy is the uncertainty in the atmospheric density. In this paper this uncertainty is modeled as the sum of three Markov processes. The effect of not including this atmospheric uncertainty in the dynamic model is presented.

Key Words: Covariance, Collision Avoidance, Orbit Determination

Introduction

The Space Shuttle (SS) currently performs maneuvers to avoid potential collisions with cataloged space objects whenever the estimated conjunction with an object falls within a box, centered on the estimated SS position, of dimensions ± 5 km in the in-track direction and ± 2 km in the radial and out of plane directions. The disadvantage of this criterion is that it does not take into consideration the uncertainty, or accuracy, of the ephemerides of the two objects or the geometry of the conjunction. If the ephemerides are well known then there is no need to perform a collision avoidance maneuver if the estimated miss distance is >1-2 km. Since a maneuver will disrupt microgravity experiments, using this criterion for the International Space Station (ISS) will result in too many maneuvers. In addition, unnecessary maneuvers will waste fuel, a precious commodity for the ISS. Therefore, the ISS has switched from the deterministic SS criterion to a probability based criterion for collision avoidance. In this approach the basis for the collision avoidance maneuver is the probability of collision, P_C, of the two objects. The calculation of P_C [1] requires the uncertainty (covariance) of the ephemerides of the two objects at conjunction. Currently, US Space Command calculates a covariance at epoch, t_0,

$$P(t_0) = \left(A^T W A\right)^{-1}, \tag{1}$$

where A is the matrix of the partial derivatives of the measurements with respect to the state at epoch, and W is a weighting matrix, which typically is a diagonal matrix with the elements being the inverse of the measurement variances. The covariance is propagated by

$$P(t) = \Phi(t,t_0) P(t_0) \Phi^T(t,t_0), \tag{2}$$

where Φ is the state transition matrix. The position covariance at epoch is reasonably accurate, but the velocity covariance is very optimistic (too small) because only measurement errors are considered in the computation of the covariance, the dynamic model is assumed to be perfect. Since the decision to maneuver must be made several hours before conjunction the covariance has to be propagated for 4-24 hours. A result of the perfect dynamic model assumption is that the estimated position error at conjunction can be quite optimistic, possibly by an order of magnitude. This

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incorrect (optimistic) covariance[2] can cause a significant error of several orders of magnitude in P_C. The primary error in the dynamic model is the uncertainty in the atmospheric density estimation. In order to have an accurate estimate of P_C a method for accurately including the atmospheric density uncertainty in the computation of the covariance is needed. The atmospheric density uncertainty is generally at least 15-20%, and it has both a temporal and spatial variation. The modeling of the uncertainty must capture both of these variations. An initial approach using a first order stationary Gauss-Markov process to represent the uncertainty is presented in Ref. 3. More accurate values of the sensor measurement errors have been obtained[4], and these have resulted in some improvement of the covariances.

In this paper the uncertainty in the atmospheric density is included by modeling it as the sum of the output of three first-order Markov processes. The effect of not including this uncertainty in the covariance and the probability of collision for a specific scenario is then presented.

Probability of Collision

The ISS is represented as a sphere of radius R. The probability of collision between the ISS during a close approach is defined as the probability that the debris object will intercept the sphere of radius R during the encounter. Let t = 0 at the estimated point of closest approach, i.e., at conjunction. Referring to Fig. 1, consider a set of perturbed trajectories for the ISS and debris given by $\tilde{\vec{r}}_{so}$ and $\tilde{\vec{r}}_{do}$ respectively. Mathematically, we can state this as

$$\tilde{\vec{r}}_{so} = \bar{\vec{r}}_{so} + \vec{e}_s \,,\; \tilde{\vec{r}}_{do} = \bar{\vec{r}}_{do} + \vec{e}_d \qquad (3)$$

where $\vec{e}_s$ and $\vec{e}_d$ are the uncertainty vectors for the ISS and debris. For these trajectories conjunction is not at t = 0.

The following assumptions are made:

- The ISS and debris object motion can be represented by rectinlinear motion (straight lines) with constant velocities during the encounter. This is justified because the time duration under consideration is no more than a couple of seconds.
- There is no uncertainty in the velocity during the encounter. This is justified because the velocity uncertainty is usually no more than several meters/second, and the time duration of the encounter is small.
- The position uncertainty during the encounter is constant, and equal to the value at the estimated conjunction.
- The position uncertainties can be represented by a Gaussian distribution.
- The ISS is much larger than the intercepting (debris) object so that the intercepting object can be considered a point mass.

The nominal trajectories near the estimated point of closest approach for both objects are given by

$$\bar{\vec{r}}_{so} = \bar{\vec{r}}_{so} + \vec{v}_s t \;, \qquad \bar{\vec{r}}_{do} = \bar{\vec{r}}_{do} + \vec{v}_d t \qquad (4)$$

Including the position uncertainties of the debris and ISS, the actual (perturbed) positions are

$$\tilde{\vec{r}}_s(t) = \tilde{\vec{r}}_{so} + \vec{v}_s t \,,\; \tilde{\vec{r}}_d(t) = \tilde{\vec{r}}_{do} + \vec{v}_d t \qquad (5)$$

The miss vector between the ISS and debris is

$$\begin{aligned} \tilde{\vec{r}}(t) &= \tilde{\vec{r}}_d(t) - \tilde{\vec{r}}_s(t) \\ &= \bar{\vec{r}}_{do} - \bar{\vec{r}}_{so} + (\vec{v}_d - \vec{v}_s)t + \vec{e}_d - \vec{e}_s \\ &= \bar{\vec{r}}_o + \vec{e}_d - \vec{e}_s + \vec{v}_r t \\ &= \tilde{\vec{r}}_o + \vec{v}_r t \end{aligned} \qquad (6)$$

Assuming a Gaussian distribution of the errors the probability distribution for $\tilde{\vec{\rho}}$ is given by

$$p\left(\tilde{\vec{\rho}}\right) = \frac{1}{(2\pi)^{3/2} (\det P)^{1/2}} * \exp\left[-\left(\tilde{\vec{\rho}} - \bar{\vec{\rho}}\right)^T P^{-1} \left(\tilde{\vec{\rho}} - \bar{\vec{\rho}}\right) 2 \right] \qquad (7)$$

where

$$P = P_s + P_d \qquad (8)$$

The probability of collision at this instant of time is

$$P_C = \frac{1}{(2\pi)^{3/2}(\det P)^{1/2}} * \int_V \exp\left[-\left(\vec{\tilde{\rho}}-\bar{\vec{\rho}}\right)^T P^{-1}\left(\vec{\tilde{\rho}}-\bar{\vec{\rho}}\right)/2\right]dV \quad (9)$$

where the integral is over the sphere of radius *R*.

Define the (*x*,*y*,*z*) coordinate system with unit vectors $(\vec{i},\vec{j},\vec{k})$ according to

$$\vec{j}=\frac{\vec{v}_r}{v_r},\ \vec{i}=\frac{\bar{\vec{\rho}}_o}{\bar{\rho}_o},\ \vec{k}=\vec{i}\times\vec{j} \quad (10)$$

The geometry of this system is shown in Fig. 2. In this coordinate system the y-component of the nominal miss vector at conjunction is zero. Foster[1] has shown that the probability of collision for the encounter reduces to

$$P_C = \frac{1}{2\pi\left(\det P^*\right)^{1/2}} * \int_{-R}^{R}\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \exp[-(\vec{s}-\vec{s}_o)^T P^{*-1}(\vec{s}-\vec{s}_o)]dxdz \quad (11)$$

where P^* is the 2x2 covariance in the $(\vec{i},\vec{j},\vec{k})$ frame and

$$\vec{s} = x\vec{i} + z\vec{k},\ \vec{s}_o = x_o\vec{i} + z_o\vec{k} \quad (12)$$

Thus, the collision sphere of radius *R* is now a collision circle of radius *R.* An alternative derivation using a different, but equivalent, definition of P_C, that leads to the same result is provided in Ref. 5.

The effect of an optimistic covariance is shown in the following analysis of the close encounter between the Mir and a US satellite, Object No. 23101, on September 1, 1997. During the encounter the crew went into the escape module. The estimated miss distance was about 800 meters and the angle between the orbital planes was approximately 104 degrees. Thus, the two trajectories were almost orthogonal. To analyze this encounter US Space Command provided orbital data. These data were the state of the two objects at the estimated conjunction and the covariance. Post processed, as well as 2, 8 and 24 hour predict data were provided. The estimated miss distances at conjunction for these cases are given in Table 1.

For the provided covariance data $P_C < 10^{-18}$. Thus, no maneuver would have been necessary. Figure 10 shows P_C as a function of *K*, where K^2 multiplies each row of the covariance. The larger estimated conjunction distance for the 24 hour predict is evident. For this encounter for $P_C > 10^{-4}$, $K > 12$ for the 8 hour predict and $K > 28$ for the others. Note again, particularly for the 8 hour predict, the large change in P_C for small changes in the covariance size.

Stochastic Drag Model

The equations of motion of a satellite in low Earth orbit (LEO) are

$$\dot{\vec{r}} = \vec{v}$$
$$\dot{\vec{v}} = -\frac{\mu}{r^3}\vec{r} + \vec{a}_g + \vec{a}_d, \quad (13)$$

where $\vec{a}_g$ is the acceleration due to the non-spherical Earth and third body gravitational perturbing accelerations and $\vec{a}_d$ is the atmospheric drag. The instantaneous acceleration is assumed to be opposed to the direction of motion and proportional to the atmospheric density ρ and the velocity squared as

$$\vec{a}_d = -\frac{1}{2B^*}\rho v\vec{v} \quad (14)$$

where B^* is the ballistic coefficient. The atmospheric density ρ is assumed to be the sum of the standard exponential atmosphere plus a stochastic component,

Table 1
Estimated Miss Distance

Predict time	Radial (m)	Horizontal (m)	Miss distance (m)
Post Processed	674.97	450.32	811.40
2 hour	646.07	506.99	821.25
8 hour	671.88	496.68	835.53
24 hour	629.09	861.48	1066.72

that is

$$\rho = \rho_p \exp\left[-k\left(r - r_p\right)\right] + p \tag{15}$$

where ρ_p and r_p are the density and radius at perigee and p is the stochastic component. The stochastic component is assumed to be the sum of the output of three first-order stationary Gauss-Markov processes. That is,

$$\begin{aligned} p &= p_1 + p_2 + p_3 \\ \dot{p}_i &= -a_i p_i + a_i w, a_i > 0 \end{aligned} \tag{16}$$

where w is zero-mean white noise with a constant covariance P_{ww}. The p_i have the following properties

$$\begin{aligned} &E\{p_i(t)\} = 0 \\ &E\{p_i(t+\tau)p_i(t)\} = \sigma_i^2(t)\exp(-a_i\tau) \text{for } \tau \geq 0 \end{aligned} \tag{17}$$

where $E\{\bullet\}$ is the expectation operator. σ^2 and $(1/a)$ are the variance and correlation of each of the stochastic processes. Strictly speaking, the quantity $(1/a)$ is the distance or time that it takes the autocorrelation function to decay $1/e$ times its initial value. In this paper it will be referred to as the correlation time. It can be shown that[6]

$$\begin{aligned} &\frac{d}{dt}\sigma^2(t) = -2a\sigma^2(t_0) + a^2 P_{ww}, \sigma^2(t_0) = \sigma_0^2 \\ &\sigma^2(t) = \left(\sigma_0^2 - 0.5aP_{ww}\right)\exp\left[-2a(t - t_0)\right] + 0.5aP_{ww} \end{aligned} \tag{18}$$

If we choose $\sigma_0^2 = aP_{ww}/2$, then

$$\sigma^2(t) = \frac{1}{2}aP_{ww} = \text{constant}, \forall t \tag{19}$$

Figures 4-7 show the density uncertainty for three correlation times and equal $\sigma_0, \sigma_0 = 0.1\rho_p/\sqrt{3}$

Numerical Results

Since we are concerned with potential collisions with the ISS simulations were performed for an object in a near circular orbit at the ISS altitude. The specific orbital parameters were a = 6748 km and i =70 deg. The correlation times were 1 orbit, 10 hours and 24 hours. The total atmospheric density uncertainty was selected to be 10% of the nominal density, i.e., $0.1\rho_p$. The ballistic coefficient was selected to give an in-track error of 30 meters due to the atmospheric uncertainty after four hours. This value was approximately equal to the ISS ballistic coefficient. Even though the fit span used by USSPACECOM for catalog maintenance of objects in LEO is 10 days current plans are to use a three day fit span for the orbit determination for objects that pose a threat to the ISS. The batch least squares orbit determination process estimates the position and velocity at epoch and the ballistic coefficient. Since many of the objects are small and are often only tracked by the FPS-85 at Eglin AFB our simulation used only this sensor. Each pass through the radar generated observations for a track with a maximum length of two minutes. It was assumed that the sensor measurements errors were zero-mean Gaussian and the noise standard deviations were known. With these assumptions the covariance, Eq. (1) at the end of three days was obtained. Since decisions on collision avoidance may have to be made as much as 24 hours before conjunction the covariance at epoch (end of the three day track) was propagated for 24 hours. Figures 8 and 9 show as a function of time the volume of the 1-σ error ellipsoid. with and without the atmospheric uncertainty. As expected, there was very little difference in the values since the least squares process used assumes a perfect dynamic model. Also note that the volume size does not increase much in 24 hours. This is a result of the estimated velocity error being too small.

Ignoring the atmospheric uncertainty in the dynamic model creates two errors sources. The first is that the velocity portion of the covariance at epoch is too small. Underestimating the velocity error at epoch causes the position error to be too small later. This is seen in Figures 8 and 9 with the slow growth of the position error. Propagation of the covariance without the uncertainty term causes another error. The covariance at epoch was then taken as the initial condition for the propagation of the covariance including the uncertainty effects. The covariance was propagated by integration of the Lyapunov equation. Figure 10 shows the growth of the 1-σ in-track error as a function of the ratio of the density uncertainty to the density, in this case an exponential model. For reasonable values of the uncertainty the error is much bigger than that resulting from ignoring the atmospheric density uncertainty.

Conclusions

Preliminary results show that ignoring the effect of the atmospheric density uncertainty in the determination of the covariance can create a significant error in the actual uncertainty. This can then cause a significant error in the probability of collision.

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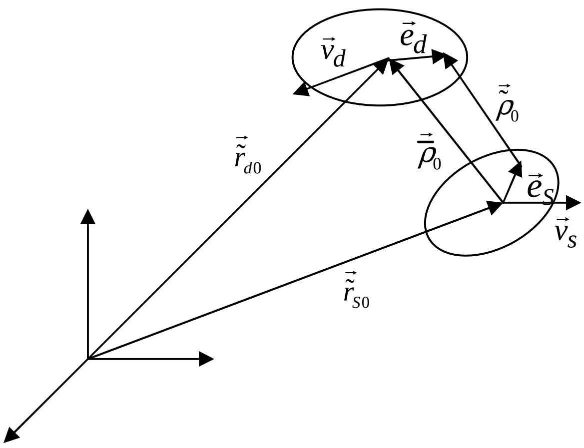


Figure 1 Encounter Variable Definition

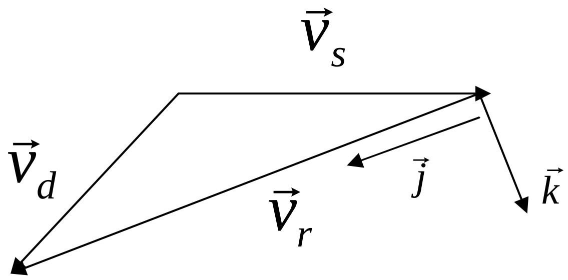


Figure 2 Encounter Coordinate System

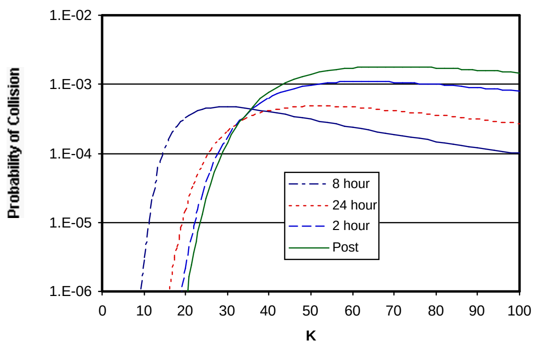


Figure 3 Mir US Sat Near Collision – P_C vs Covariance Size

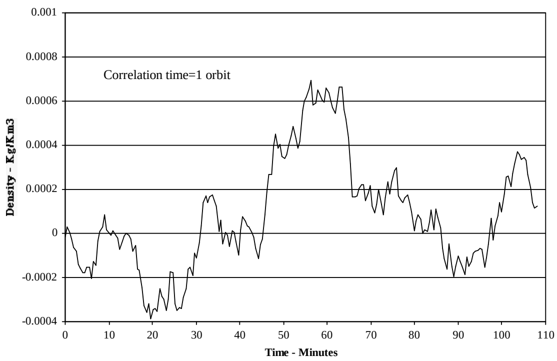


Figure 4 Atmospheric Density Uncertainty
Correlation Time = 1 orbit

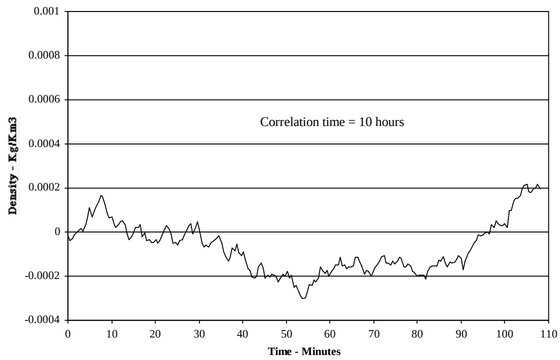
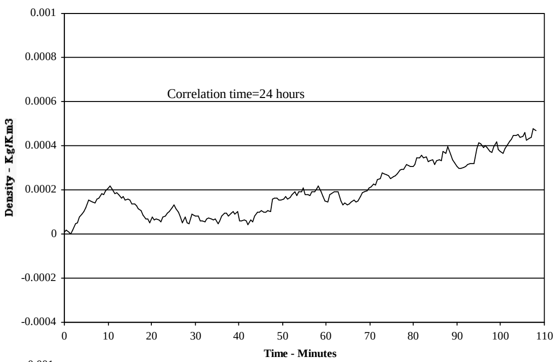


Figure 5 Atmospheric Density Uncertainty
Correlation Time = 10 hours



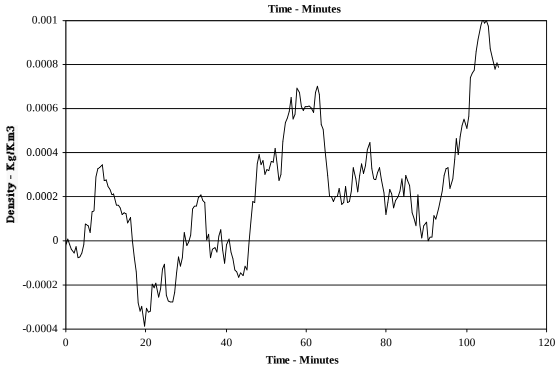


Figure 6 Atmospheric Density Uncertainty
Correlation Time = 24 hours

Figure 7 Atmospheric Density Uncertainty – Sum

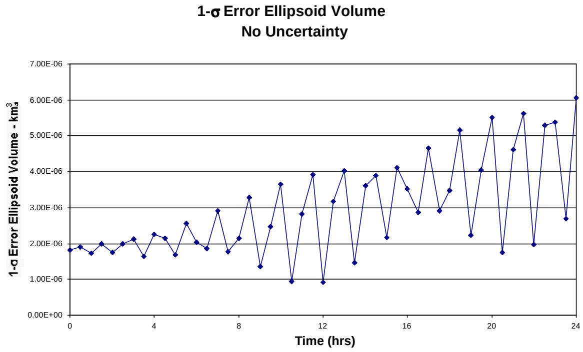


Figure 8 1-σ Error Ellipsoid Volume

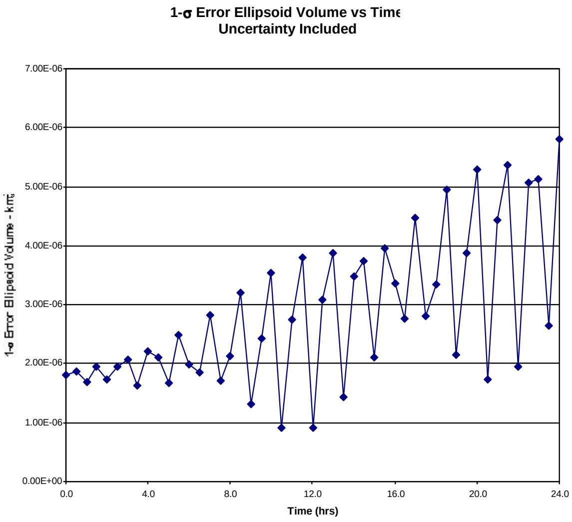


Figure 9 1-σ Error Ellipsoid Volume

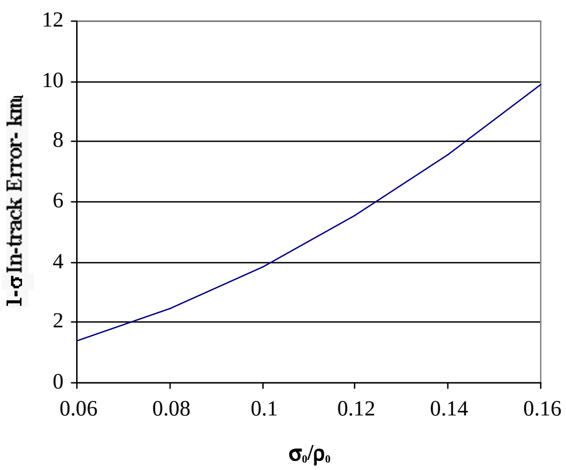


Figure 10 In-Track error Growth