OPTIMAL MOTION AND POSITION CONTROL OF NONHOLONOMIC FLEXIBLE ARM

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Abstract

A robot arm operating in such environments as spacecraft and space vehicles, attached to a space platform, station or satellite, has to show considerable ligthness. This property can be achieved by an underactuated manipulator with links possessing some flexibility. In this paper, a two-flexible-link manipulator with the second joint unactuated and therefore a nonholonomic system , is modeled and a control method is proposed to attain a desired position from an initial position. As far as this system is categorized into underactuated manipulators, an exact solution for the optimal trajectory poses considerable difficulties. In this case, to find the optimal trajectory and control input for the cost function to be minimized, we make use of the Ritz method. Through the application of Fourier Basis Algorithm, the near optimal control parameters are acquired, and the solution is approximated by solutions of some finite-dimensional systems and then those solutions converge to the optimal solution.

Key words : Nonholonomic system, underactuated manipulator, flexible link, Fourier Basis Algorithm.

Introduction

Nonholonomic mechanical systems ,i.e. systems with non integrable differential constraints on the generalized constraints, are a growing field of research, since such applications as space robot arms could show increased ligthness by reducing the number of actuators at joints. Even if nonholonomy is a mechanical property of the system, it has definite effects on the control problem, in this case the configuration space dimension exceeds that of the control space, because of the free joint not equipped with an actuator. The additional property of flexibility, contributing to reduce weight, cost and energy consumption while still maintaining an adequate degree of dexterity, also adds extra difficulties to the control problem by increasing the generalized coordinates and turning the equations of motion more complex.

Two-flexible-link model

The manipulator consists of two links, with the second joint free. Hence, the control is performed only by the torque applied to the first joint .The system's general configuration is shown in Fig. 1. The modeling of the manipulator with flexible links is derived by using Lagrange's equation.

Referring to the figure, the position of any point on

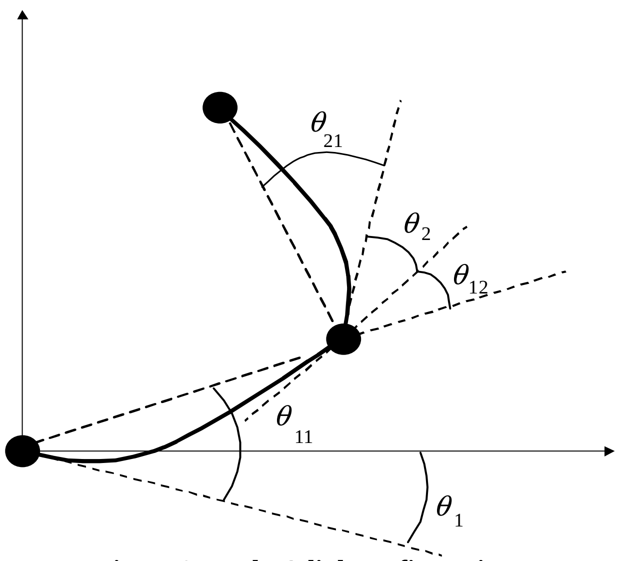

Figure 1 : The 2-link configuration

The first link is given by

$$x_1 = l_1 \cos(\theta_1 + \theta_{11}) \quad (1)$$

$$y_1 = l_1 \sin(\theta_1 + \theta_{11}). \quad (2)$$

The position of any point on the second link is described by the following relation :

$$x_2 = x_1 + l_2 \cos(\theta_1 + \theta_{11} + \theta_{12} + \theta_2 + \theta_{21}) \quad (3)$$

$$y_2 = y_1 + l_2 \sin(\theta_1 + \theta_{11} + \theta_{12} + \theta_2 + \theta_{21}). \quad (4)$$

The parameters of the two-link flexible arm are shown in Table 1.

Table 1 : Parameters of the 2-link system

l_1	length of arm 1	0.5 [m]
l_2	length of arm 2	0.5 [m]
m_1	mass of elbow	4.0 [kg]
m_2	mass of payload	4.0 [kg]
J_1	inertia moment of arm1	0.0008 [kg.m^2]
J_2	inertia moment of arm 2	0.00023 [kg.m^2]
J_{11}	inertia moment of elbow	0.0088 [kg.m^2]
J_{21}	inertia moment of payload	0.023 [kg.m^2]
J_{m1}	inertia moment of motor 1	0.068 [kg.m^2]
J_{m2}	inertia moment of motor 2	0.013 [kg.m^2]
E_1I_1	flexural rigidity of arm 1	3.28 [N/m^2]
E_2I_2	flexural rigidity of arm 2	3.28 [N/m^2]
C_1	viscous rotational damping coefficient of arm 1	0.01 [N.s/m]
C_2	viscous rotational damping coefficient of arm 2	0.01 [N.s/m]
C_{11}	internal damping coefficient of arm 1	0.0375 [N.s/m]
C_{21}	internal damping coefficient of arm 2	0.0375 [N.s/m]

To analize the physical model of the system, some assumptions are performed as follow :

a) The deformation of the arm for a simply supported beam with moments at both ends is assumed static.
b) The arm is massless.
c) The shaft friction force is negligibly small to drive the arm.
d) The deformation of the arm is much smaller than the length of the arm.
e) The influence of gravity is ignored.
f) The control stick and the driving stick are rigid bodies and the difference of the angles between them is very small.
g) The motion is restrained to the horizontal plane.

The potential energy of the system can be expressed in the form

$$U = \frac{3E_1I_1}{2l_1}\theta^2{}_{12} + \frac{3E_2I_2}{2l_2}\theta^2{}_{22} \;, \quad (5)$$

and the kinetic energy of the system is as follows

$$\begin{aligned} T &= \frac{m_1}{2}\left(\dot{x}_1^2 + \dot{y}_1^2\right) + \frac{m_2}{2}\left(\dot{x}_2^2 + \dot{y}_2^2\right) + \frac{1}{2}J_1\dot{\theta}_1^2 \\ &+ \frac{1}{2}J_{m1}\dot{\theta}_1^2 + \frac{1}{2}J_{11}\left(\dot{\varphi}_1 + \dot{\theta}_{12}\right)^2 + \frac{1}{2}J_2\left(\dot{\varphi}_2 + \dot{\theta}_{21}\right)^2 \\ &+ \frac{1}{2}J_{m2}\left(\dot{\varphi}_2 + \dot{\theta}_{21}\right)^2 + \frac{1}{2}J_{21}\left(\dot{\varphi}_2 + \dot{\theta}_{22}\right)^2, \end{aligned} \quad (6)$$

where

$$\varphi_1 = \theta_1 + \theta_{11} \quad (7)$$

$$\varphi_2 = \varphi_1 + \theta_{12} + \theta_2 + \theta_{21}. \quad (8)$$

The dissipation of energy can be written in the form :

$$F = \frac{1}{2}C_1\dot{\theta}_1^2 + \frac{1}{2}C_2\dot{\theta}_2^2 + \frac{1}{2}C_{11}l_1^2\dot{\theta}_{11}^2 + \frac{1}{2}C_{21}l_2^2\dot{\theta}_{21}^2. \quad (9)$$

Applying the Lagrangian , L , and Lagrange's equation :

$$T = L - U \;, \quad (10)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial F}{\partial \dot{q}_i} = \tau, \quad (11)$$

where the set of identified generalized coordinates is

$$q_i = \{\theta_1, \theta_2, \theta_{11}, \theta_{21}\}. \quad (12)$$

with θ_1, the rotation angle of the first arm; θ_2, the rotation angle of the second arm; θ_{11}, the deflection angle of the first link on the shoulder; and θ_{21}, the deflection angle of the second link on the elbow. Then, the equations of motion are obtained in the form :

$$M(\theta)\ddot{\theta} + h(\theta,\dot{\theta}) + C(\theta) = B\tau \ , \qquad (13)$$

where, θ is the vector of generalized coordinates, M is the matrix of coefficients, τ the applied torques, and the vectors C, h, and B are as indicated below.

$$q = [\theta_1, \theta_2, \theta_{11}, \theta_{21}]^T \ , \qquad (14)$$

$$M(\theta) = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} , \qquad (15)$$

$$h(\theta,\dot{\theta}) = [h_1 \quad h_2 \quad h_3 \quad h_4]^T , \qquad (16)$$

$$C(\theta) = \begin{bmatrix} 0 & 0 & \frac{3E_1 I_1}{l_1}\theta_{11} & 0 \end{bmatrix}^T , \qquad (17)$$

$$B = [1 \quad 0 \quad 0 \quad 0]^T \ . \qquad (18)$$

From here, the State Equation form (19) is obtained as follows

$$\frac{d}{dt}\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_{11} \\ \theta_{21} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_{11} \\ \dot{\theta}_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ N_1(M_{ij}) \\ N_2(M_{ij}) \\ N_3(M_{ij}) \end{bmatrix} u + \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_{11} \\ \dot{\theta}_{21} \\ 0 \\ F_1(M_{ij}) \\ F_2(M_{ij}) \\ F_3(M_{ij}) \end{bmatrix} , \qquad (19)$$

where, “u” is the input, i.e. the torque applied to the first joint, expressed by the acceleration of the angle θ_1.

Conditions of nonholonomic nature

The holonomic or nonholonomic nature of the system has to be determined, and for this purpose, the conditions of integrability of the dynamic equation (20) relative to the free joint is examined. Since no input term explicitly appears in equation (20), this may be interpreted as a constraint involving generalized coordinates as well as their first and second-order time derivatives.

$$M_{21}\ddot{\theta}_1 + M_{22}\ddot{\theta}_2 + M_{23}\ddot{\theta}_{11} + M_{24}\ddot{\theta}_{21} + h_2 = 0 \ . \qquad (20)$$

For this equation, the property of partial integrability has to be examined. If partial integrability holds, possible further integrability to a constraint of the form f(q,t)=0 must be investigated. If such a constraint exists, the equation (20) is said to possess the complete integrability property, or in other words, to be holonomic[2]. The ability to discern between holonomic and nonholonomic constraints is crucial, since the former can be used to reduce the system dimension by eliminating some coordinates. In this case, the condition for partial integrability is not fulfilled , since

$$\frac{\partial}{\partial\theta_2}(\dot{\theta}^T M(\theta)\dot{\theta}) \neq 0 . \qquad (21)$$

Therefore, the system has proved to possess nonholonomic nature, i.e. to be a nonholonomic system.

The Fourier Basis Algorithm

In order to obtain a near optimal solution for the problem of positioning the arm system from an initial configuration to a final configuration, the Fourier basis algorithm is applied. This kind of algorithm has already been applied successfully for computing optimal solutions for a variety of systems. It is assumed that the system is controllable so that the problem is solvable. The nonlinear system can be described by a general expression of the form,

$$\dot{x}(t) = f(x(t), u(t)) , \qquad (22)$$

with $x(0)=x_0$, and the cost function to be minimized corresponding this system should be written in the form :

$$J = \Delta x_{t_f}^T M \Delta x_{t_f} + \int_0^{t_f} \Delta x^T Q \Delta x dt \ , \qquad (23)$$

with

$$\Delta x_{t_f} = x(t_f) - x_{des} \ , \qquad (24)$$

and

$$\Delta x = x(t) - x_{m,des} \ , \qquad (25)$$

where, $x(t_f)$ is the actual final position , x_{des} is the final desired position and M , Q are the weighting matrices. Rewriting the cost function in terms of the control input u , we have

$$J = (x_f - x_d)^T M (x_f - x_d) + \int_{t_0}^{t} u^T R u dt \ , \qquad (26)$$

where the weighting matrices M and R are adequate diagonal matrices. Since the system is controllable, there exists a solution u ∈ $L_2([0,T])$, here L_2 denotes the Hilbert space of measurable vector –valued functions of the form :

$$u(t) = \left(u_1(t), \cdots u_m(t)\right)^T, \qquad t \in [0, T]. \qquad (27)$$

If e_i is an orthonormal basis, e.g., the Fourier basis, a function u , in basis terms, can be expressed as

$$u = \sum_{i=1}^{\infty} \alpha_i e_i \quad , \qquad (28)$$

for some sequence α = (α_1,α_2, . . .). The idea of the Fourier basis algorithm is to approximate the solution by solutions of some finite-dimensional systems, introduced by restricting the control to the first n terms of the basis[1]. By applying the Fourier basis method, we obtain the following expressions :

$$u(t) = \sum_{i=0}^{n} \alpha_i e_i \qquad (29)$$

$$u(t) = \alpha_0 + \alpha_1 + \alpha_2 + \cdots + \alpha_n + \cdots \qquad (30)$$

$$u(t) = \frac{a_0}{2} + \sum_{i=1}^{n} \left(a_i \cos nt + b_i \sin nt\right). \qquad (31)$$

It is demonstrated that as n→∞, solutions of the finite dimensional systems converge to the optimal solution. Finally, the cost function can be expressed in a general form as

$$J(u(t)) = J(a_0, a_1, a_2, \cdots\cdots, a_n, \cdots). \qquad (32)$$

This variational approach to obtain a direct way to solve the problem is the Ritz method.

Now, the algorithm to solve for optimal α , that is, α ∈ l_2 of minimum cost linking initial and final configurations will be constructed. By making R=1, with a control time t=2π , and considering the Fourier basis in

$$\int_{t_0}^{t} e_i(t)^T e_j(t) dt = \begin{cases} 0 & (i \neq j) \\ 1 & (i = j) \end{cases} . \qquad (33)$$

Replacing the expression for u in equation (26), the rewritten expression is :

$$J[u] = \left(x(t_f) - x_d\right)^T M \left(x(t_f) - x_d\right) + \sum_{i=1}^{\infty} \alpha_i^2$$
$$= \left(x(t_f) - x_d\right)^T M \left(x(t_f) - x_d\right) + \|\alpha\|^2. \qquad (34)$$

At this point, an efficient approach to minimizing J(α) is quadratic programming. For this effect, the computation of the Hessian about a point α_n is developed using its Taylor expansion :

$$J[\alpha_n + \delta] = J[\alpha_n] + \left\langle \left.\frac{\partial J}{\partial \alpha}\right|_{\alpha_n}, \delta \right\rangle$$
$$+ \frac{1}{2} \left\langle \left.\frac{\partial^2 J}{\partial \alpha^2}\right|_{\alpha_n} \delta, \delta \right\rangle + 0\left(\|\delta\|^3\right), \qquad (35)$$

where

$$\left.\frac{\partial J}{\partial \alpha}\right|_{\alpha_n} = 2\left(\alpha_n + Y_f^T M \left(x(t_f) - x_d\right)\right), \qquad (36)$$

and

$$\left.\frac{\partial^2 J}{\partial \alpha^2}\right|_{\alpha_n} = 2\left[I + Y_f^T M Y_f + \sum_{i=1}^{n} Z_{if} \left(M\left(x(t_f) - x_d\right)\right)_i\right]. \qquad (37)$$

Then, the Jacobian Y(t) and the Hessian $Z_i(t)$ can be expressed by eqs. (38) and (39) respectively

$$Y_f = Y(t) = \frac{\partial x(t)}{\partial \alpha} \qquad (38)$$

$$Z_{if} = Z_i(t) = \frac{\partial^2 x_i(t)}{\partial \alpha^2} . \qquad (39)$$

Since an expression for the differential equation for Y is needed, it can be obtained from the expressions (29), (38), and the general form of the state equation :

$$\dot{x} = h(x)u + g(x)$$
$$= h(x)E(t)\alpha + g(x) \ , \qquad (40)$$

with E = (e_1(t),e_2(t),. . .,e_n(t)); then

$$
\begin{aligned}
\frac{\partial Y}{\partial t} = \dot{Y} &= \frac{\partial}{\partial t}\frac{\partial x}{\partial \alpha} = \frac{\partial}{\partial \alpha}\frac{\partial x}{\partial t} \\
&= \frac{\partial}{\partial \alpha}\dot{x} = \frac{\partial}{\partial \alpha}\left(h(x)u + g(x)\right) \\
&= \frac{\partial h(x)}{\partial \alpha}u + h(x)\frac{\partial u}{\partial \alpha} + \frac{\partial g(x)}{\partial \alpha} \\
&= \sum_{i=1}^{r}\left[\frac{\partial h(x)}{\partial \alpha}u_i\right] + h(x)\frac{\partial (\Theta\alpha)}{\partial \alpha} + \frac{\partial g(x)}{\partial \alpha} \\
&= \sum_{i=1}^{r}\left[\frac{\partial h(x)}{\partial x}\frac{\partial x}{\partial \alpha}u_i\right] + h(x)E + \frac{\partial g(x)}{\partial x}\frac{\partial x}{\partial \alpha} \\
&= \sum_{i=1}^{r}\left[\frac{\partial h(x)}{\partial x}Yu_i\right] + h(x)E + \frac{\partial g(x)}{\partial x}Y \\
&= \left(\sum_{i=1}^{r}\left[\frac{\partial h(x)}{\partial x}u_i\right] + \frac{\partial g(x)}{\partial x}\right)Y + h(x)E \ .
\end{aligned} \qquad (41)
$$

Finally, in order to obtain an expression to update α, the modified Newton's method is used

$$\alpha_{n+1} = \alpha_n - \mu\left(\left.\frac{\partial J}{\partial \alpha}\right|_{\alpha_n} \Big/ \left.\frac{\partial^2 J}{\partial \alpha^2}\right|_{\alpha_n}\right) . \qquad (42)$$

Since the Hessians Z_{if} of the component functions are difficult to compute, by applying Newton these terms can be ignored. Then the expression for the updated α becomes

$$\alpha_{n+1} = \alpha_n - \mu\frac{\left[\alpha_n + Y_f^T M\left(x(t_f) - x_d\right)\right]}{\left[I + Y_f^T M Y_f\right]} \quad , \qquad (43)$$

where $\mu \in (0,1)$ is a parameter.

Summarizing the steps to follow to construct the basis algorithm :

a) Input : The initial value x_0 , the final desired value x_d, and the system's equation.
b) Output : The control input linking x_0 and x_d ; in this case, the Fourier parameter α .
c) Choose an orthonormal basis , in this case the Fourier basis, and retain the first n elements, to set the order of the series.
d) Choose the parameter of the Newton's method μ ,and solve the equations of the system as well as the calculation of the Jacobian. Examine the obtained data.
e) If the last $x(\alpha_n)$ satisfies the expected objective x_d, exit; otherwise repeat from step (d).

Positioning the Flexible Arm

The proposed method to attain an optimal control of the motion of the 2-flexible-link nonholonomic manipulator is applied for four cases. The initial and final configurations are listed in Table 2. The state variable vector is

$$x = \left[\theta_1, \theta_2, \theta_{11}, \theta_{21}, \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_{11}, \dot{\theta}_{21}\right]^T \ . \qquad (44)$$

Table 2 : Initial and Final Position

	Initial Position x_0	Final Position x_d
I	(0,0,0,0,0,0,0,0)	(20,0,0,0,0,0,0,0)
II	(0,0,0,0,0,0,0,0)	(20,20,0,0,0,0,0,0)
III	(25,0,0,0,0,0,0,0)	(50,20,0,0,0,0,0,0)
IV	(30,0,0,0,0,0,0,0)	(75,0,0,0,0,0,0,0)

Then the Fourier basis became :

$$E = \{1/2, Sin[1t], Cos[1t], Sin[2t], Cos[2t], \cdots, Sin[30t], Cos[30t]\} \ . \qquad (45)$$

The weighting matrices used for each case are shown in the Table 3. The time parameters are : t_0=0, and $t_f = 2\pi$.

Table 3 : Weighting matrices

	Weighting Matrix
I	M=diag[3000,3000,3000,3000,0,3000,3000,3000]
II	M=diag[3000,3000,3000,3000,0,3000,3000,3000]
III	M=diag[2000,2000,2000,500,0,500,500,500]
IV	M=diag[2000,500,500,500,0,2000,2000200]

Simulation Results

The results history plots corresponding History of the a view of the plots for the Figures 2 to 5

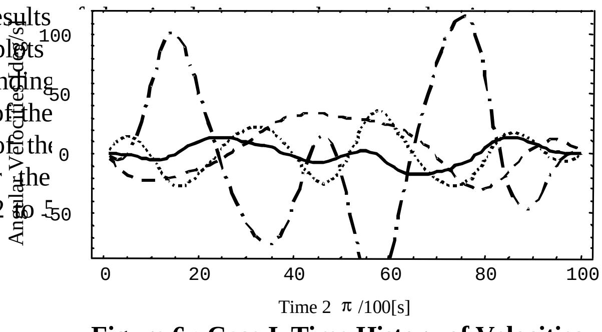

Figure 6 : Case I, Time History of Velocities

the angle θ_1; the dashed-and-dotted line represents the angle θ_2 ; the dotted line is deflection angle θ_{11} ; and the solid line is the deflection angle θ_{21}.

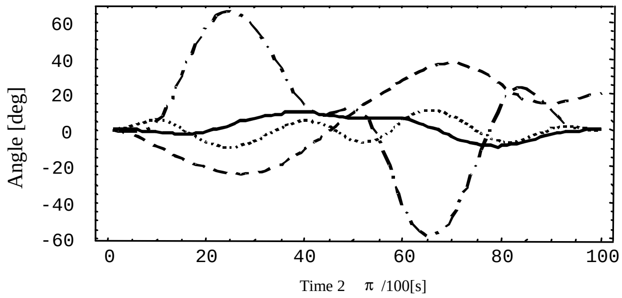

Figure 2 :Case I, Time History of Angles

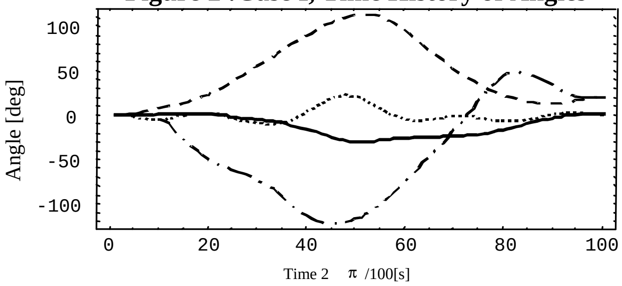

Figure 3 : Case II, Time History of Angles

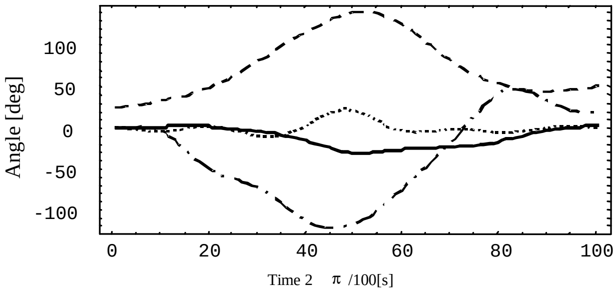

Figure 4 : Case III, Time History of Angles

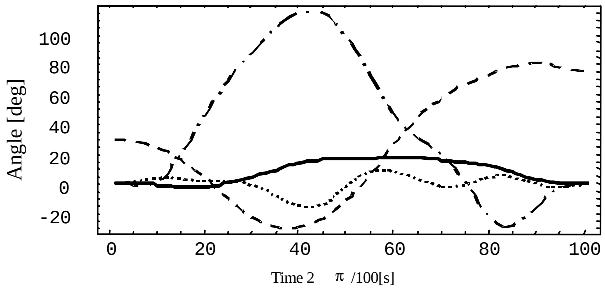

Figure 5 : Case IV, Time History of Angles

Next, the plots for the velocities of the angles are shown from Figure 6 to 9. As for the angles' plots, the dashed line represents the velocity of θ_1, the dashed-dotted the velocity of θ_2, the dotted line the velocity of θ_{11} and the solid line the velocity of θ_{21} .

The input control, the torque applied to the first joint, obtained through the algorithm is represented in Figures 10 to 13, for each case. The abscissa indicates the division of time and the ordinate indicates the torque.

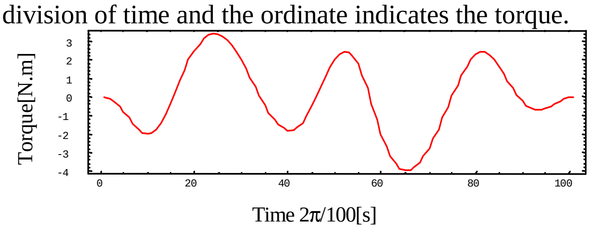

Figure 10 : Case 1, Time History of Torque

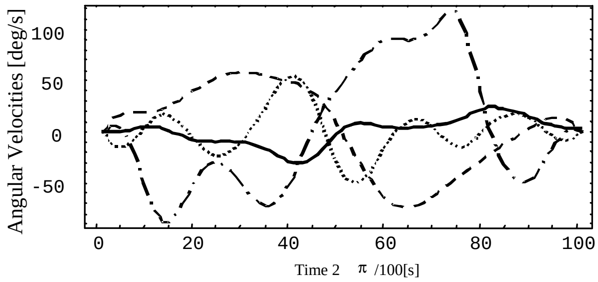

Figure 7 : Case II, Time History of Velocities

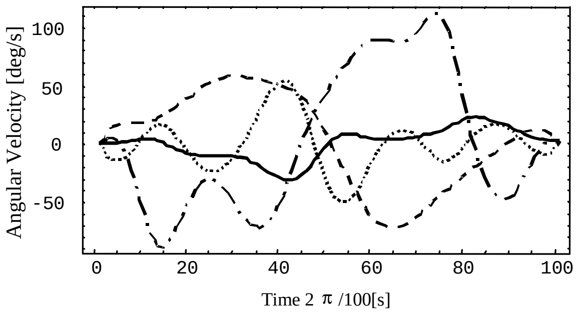

Figure 8 : Case III, Time History of Velocities

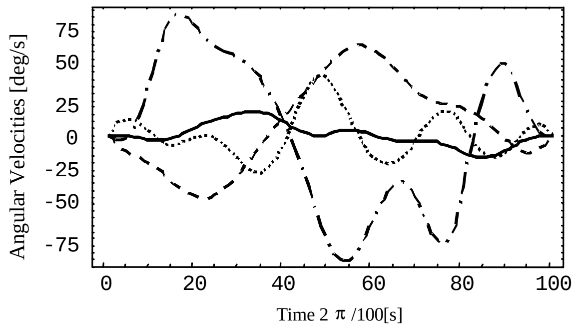

Figure 9 : Case IV, Time History of Velocities

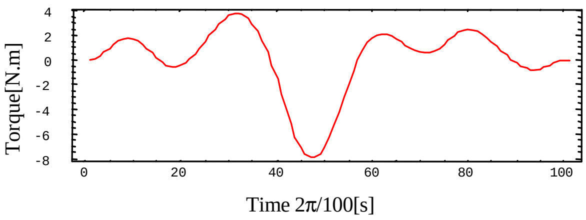

Figure 11 : Case II, Time History of Torque

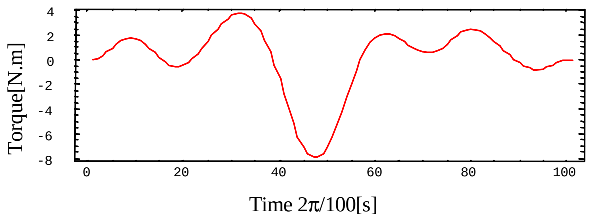

Figure 12 : Case III, Time History of Torque

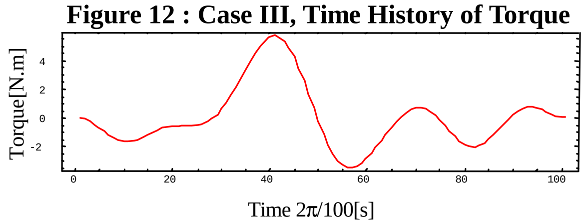

Figure 13 : Case IV, Time History of Torque

From the figures,we can see a similar pattern for cases II and III; the applied torque and the angular velocities have very close values. This could indicate that within certain range the behavior is almost the same despite the different targets. Except the first case, the curves for torque show only one peak value about the point where

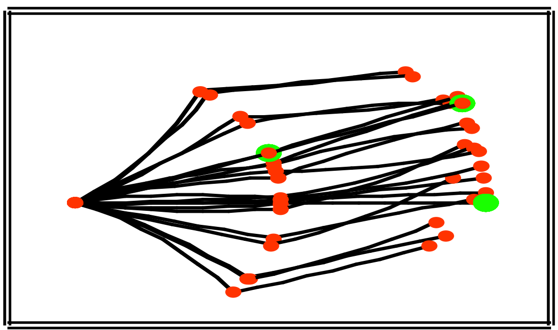

Figure14 :(0,0,0,0,0,0,0,0,0)⟶(20,0,0,0,0,0,0,0,0)

each link reaches its maximum angle. Finally, the animation of the controlled motion from the initial position to the desired target is shown in the following views, Figures 14 to 17.

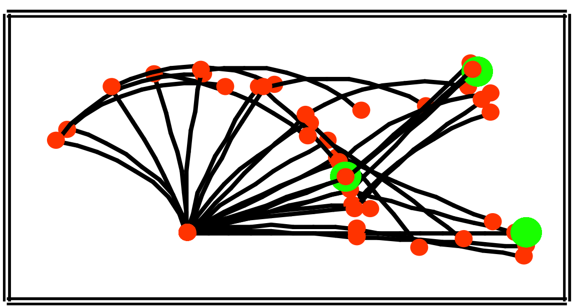

Figure15: (0,0,0,0,0,0,0,0,0) ⟶ (20,20,0,0,0,0,0,0,0)

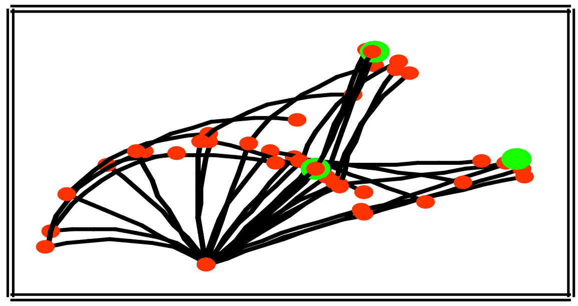

Figure 16:(25,0,0,0,0,0,0,0,0)⟶(50,20,0,0,0,0,0,0,0)

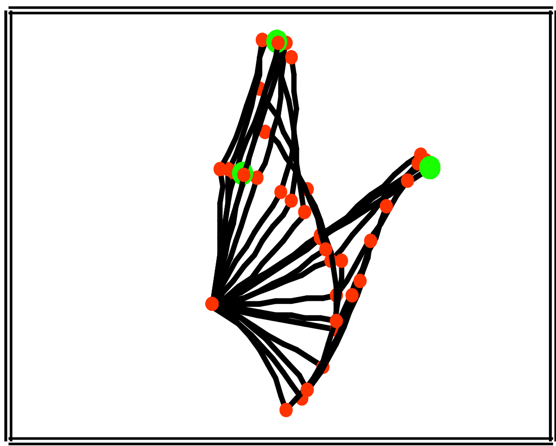

Figure 17: (30,0,0,0,0,0,0,0,0)⟶(75,0,0,0,0,0,0,0,0)

The position of the second joint (solid line) and the position of the tip of the second link (dotted line) obtained in each case, are also expressed in polar coordiantes (r,θ) and shown in Figures 18 to 21. From these graphs we can see more clearly the trajectories followed by the points, and notice the similarity of patterns among some of the cases. The left side graph represents the radius r and the right side graph represents the angle θ.

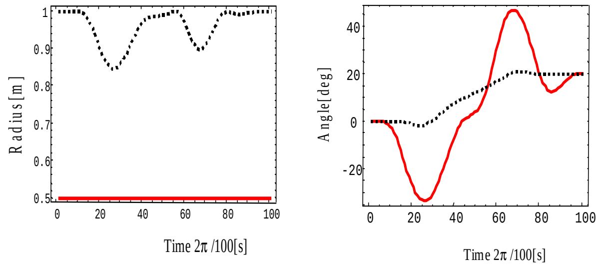

Figure 18 : Case I

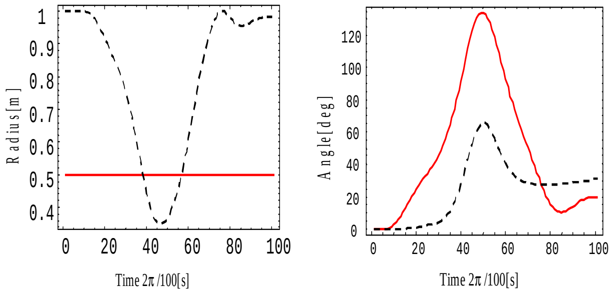

Figure 19 : Case II

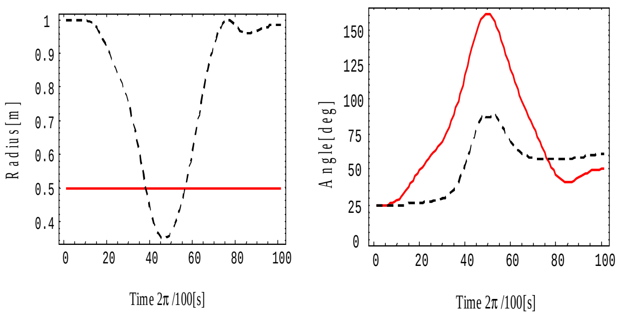

Figure 20 : Case III

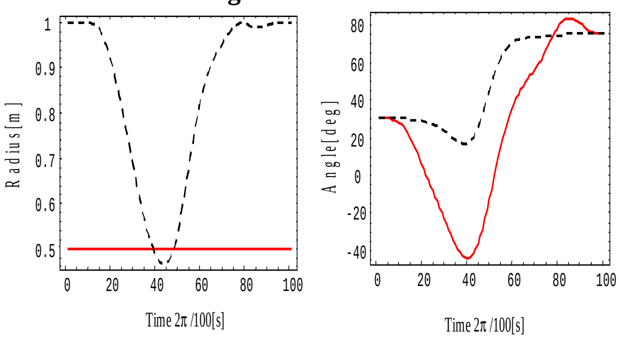

Figure 21 : Case IV

Conclusions

The proposed Fourier basis algorithm has been applied to solve the problem of finding the near optimal control input necessary for the accurate positioning of a two-flexible-link manipulator with a free joint. The nonholonomic nature of the system is one of the conditions to take into account in order to attain a reliable response. The modeling of the manipulator was derived using Lagrange principle, in addition to this, the flexibility of the links has been introduced considering the deformation of a beam. Besides the construction of the algorithm itself, where the parameters μ and the choice of the order of the Fourier series are important steps to reach the closest possible solution to the optimal solution, the choice of the weighting matrix M is also an important step in order to acquire significant and valid results. In this case, the results obtained through this method showed the validity of the procedure, since they are attained taking into account the conditions of nonholonomy and flexibility, two characteristis that are usually studied separately. Also by minimizing the cost function involving the energy and the position error, the method achieves a near optimal solution for the problem of controlling the motion and position of this kind of underactuated mechanism. The possibility of applying the algorithm for a multiple-link manipulator, in other words, to generalize the method for more than two links is being considered due to the effectiveness of its achievements.

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