STUDY OF TRIDIMENSIONAL IMPULSIVE MANEUVERS WITH THRUST ERRORS

Maria Matilde Netto dos Santos Paulo
CDT-ETEP - Av. Barão do Rio Branco 882 - São José dos Campos - SP - 12242-800
Antonio Fernando Bertachini de Almeida Prado
Marcelo Lopes Oliveira e Souza
Instituto Nacional de Pesquisas Espaciais - São José dos Campos - SP - 12227-010
Phone: 55-12-3456201 - Fax: 55-12-3456226
E-mail: prado@dem.inpe.br and marcelo@dem.inpe.br

Abstract

In this paper the problem of spacecraft orbit transfer with minimum fuel consumption is considered. The main goal is to study the effects of errors in the thrust vector in a tridimensional biimpulsive orbit transfer problem. After a search in the literature and analysis of the results available, we select and extend a method developed by Altman and Pistiner to be the base to the algorithm developed. The spacecraft is supposed to be in Keplerian motion controlled by the thrusts, that are assumed to be impulsive with errors in magnitude and direction. Results of simulations are also presented, to show the extra fuel and time required to complete the maneuver, as functions of those errors.

Key words: Orbital Transfer, Impulsive Maneuvers, Thrust Errors.

Introduction

The launching of a geostationary or an heliosynchronous satellite, the orbit corrections, the maintenance of space stations, the interplanetary trips and the interceptation of celestial bodies are examples of ordinary space missions very popular nowadays due to the great advance of the Space Sciences, and that require orbital maneuvers for their execution. Since it became necessary to use vehicles equipped with propulsion systems to perform such space missions, it became also necessary to study the optimal transfer problem of a spacecraft between two given orbits.

In this paper we summarize part of our study[2] on tridimensional orbital transfers with errors in the thrust vector. There, we give a conceptual definition of the orbital transfer problem and its options. We also study tridimensional optimal biimpulsive transfers solving the poin-to-point formulation of Altman and Pistiner[1]. We extended it to become an orbit-to-orbit formulation and to include maneuvers with errors in the thrust vector. Simulations were performed to test the developed program. Finally, we present the contributions of this research, as well as the conclusions obtained concerning the effects of such errors on the tridimensional biimpulsive maneuvers. They show that the method may give good "a priori" estimates of the extra time and consumption required by a transfer which considers the effects of such errors. They also show that in the transfers where the error in magnitude drives the thrust to a value superior to the nominal, there is a bigger increase in ΔV (requiring more time and a bigger number of maneuvers to reach an orbit closer to the final orbit desired) than when the error in magnitude results in a thrust smaller than the nominal. Yet, the extra expenditures in time and fuel can not be considered insignificant fractions of nominal time and fuel, and do not necessarily decrease with the proximity of the orbits. Thus, the impact of the errors in the thrust vector is an important factor to be considered in the study of transfers.

Revision of the Literature

The problem of optimal transfers (in the sense of minimum fuel consumption) between two Keplerian coplanar orbits has been investigated for a long time. In particular, many papers solve this problem for an impulsive thrust system with a fixed number of impulses. The literature is full of solutions for particular cases, like the Hohmann[3] and the Hoelker-Silber[4] transfers between two circular orbits and its variants for ellipses in particular positions. Different approaches can be found in Lawden[5,6] and Ting[7].

Goddard[8] was one of the first researchers to work on the problem of optimal transfers of a spacecraft between two points. He proposed optimal approximate solutions for the problem of sending a rocket to high altitudes with minimum fuel consumption.

After him comes the very important work done by Hohmann[3]. He solved the problem of minimum ΔV

transfers between two circular coplanar orbits. His results are largely used nowadays as a first approximation of more complex models and it was considered the final solution of this problem until 1959. A detailed study of this transfer can be found in Marec[9] and an analytical proof of its optimality can be found in Barrar[10].

The Hohmann transfer would be generalized to the elliptic case (transfer between two coaxial elliptic orbits) by Marchal[11]. Smith[12] shows results for some other special cases, like coaxial and quasi-coaxial elliptic orbits, circular-elliptic orbits, two quasi-circular orbits. A numerical scheme to solve the transfer between two generic coplanar elliptic orbits is presented by Bender[13].

Hohmann type transfers between noncoplanar orbits are discussed in several papers, like McCue[14], that study a transfer between two elliptic inclined orbits including the possibility of rendezvous; or like Eckel and Vinh[15], that solve the same problem with time or fuel fixed.

Another line of research studies the effects of the reality of finite thrust in the results obtained from the impulsive model. Zee[16] obtained analytical expressions for the extra fuel consumed to reach the same transfer and for the errors in the orbital elements and energy for a nominal maneuver (a real maneuver that uses the impulses calculated with the impulsive model).

More recently, the literature studied the problem of a two-impulse transfer where the magnitude of the two impulses are fixed, like in Jin and Melton[17], Jezewski and Mittleman[18].

The three-impulse concept was introduced in the literature by Hoelker and Silber[4]. They showed that a bi-elliptical transfer between two circular orbits has a lower ΔV than the Hohmann transfer, for some combinations of initial and final orbits. After that, Ting[7] showed that the use of more than three impulses does not lower the ΔV, for impulsive maneuvers. Roth[19] obtained the minimum ΔV solution for a bielliptical transfer between two inclined orbits.

Following the idea of more than two impulses, we have the work done by Prussing[20] that admits two or three impulses; Prussing[21] that admits four impulses; Eckel[22] that admits N impulses.

Another line of research that comes from the Hohmann transfer is the study of multi-revolutions transfer with N impulses applied during N successive passages by the apses. Spencer, Glickman and Bercaw[23] show equations and pictures to obtain the ΔV required for this transfer, as a function of the number of revolutions allowed for the transfer. After that, Redding[24] and Matogawa[25] would extend this concept of multi-revolution transfer to the non-impulsive case, by applying finite thrust around the apses.

Some other researchers worked on methods where the number of impulses was a free parameter, and not a value fixed in advance. It is the case of the papers made by Lion and Handelsman[26], Jezewski and Rosendaal[27], Gross and Prussing[28], Eckel[29] and Prussing and Chiu[30]. Most of the research done in this particular case is based on the"Primer-Vector" idea developed by Lawden[31, 32].

Orbital Transfers Without Errors

An orbital transfer consists of changing the state of a space vehicle. The state is defined as the position, velocity and mass of the vehicle at a given time. Fig. 1 shows an orbit transfer between two points marked by the subscripts “0” and “f”.

The most studied transfer is the biimpulsive coplanar transfer. Suppose that we have a spacecraft in a Keplerian orbit O_1. We desire to transfer this spacecraft to a final Keplerian orbit O_2, coplanar with O_1. Figure 2 shows a sketch of the transfer and defines some of the variables used. At the point P_1 (r_1, θ_1), we apply an impulse with magnitude ΔV_1 that has an angle ϕ_1 with the local transverse direction. The transfer orbit crosses the final orbit at the point P_2 (r_2, θ_2), where we apply an impulse with magnitude ΔV_2 making an angle ϕ_2 with the local transverse direction.

Figure 3 shows a noncoplanar orbital transfer. The spacecraft follows an initial orbit A until it reaches point P_1. At P_1 it receives an impulsive thrust that changes the velocity in zero time, putting the spacecraft into a transfer orbit $P_1 - P_2$. At P_2, a new impulsive thrust puts the spacecraft into a final orbit B (the orbits are defined by their orbital elements; points A and B are defined by their true anomalies).

Options For Dynamics, Actuators and Optimization Methods

There are several choices that we can make related to those aspects. For the dynamics we have the most usual possibilities:

i) Two-body Problem
ii) Two-body Perturbed Problem
iii) Three-body Problem (in particular, the restricted version of this problem)
iv) N-bodies Problem

For the control (engine to be used to complete the maneuver) we have two main possibilities:

i) Impulsive system, (ΔV), that changes the velocity in a time very short time, that can be considered zero;
ii) Continuous system, that applies a finite force during a certain time.

As far as the optimization method is concerned, the main possibilities are:
i) Direct methods (search of parameters that minimizes a certain objective function);
ii) Indirect method (first-order necessary conditions);
iii) Hybrid approach (first-order necessary conditions are written and transformed in a search of parameters).

The Method of Altman and Pistiner and Our Extension of It

The method developed by Altman and Pistiner[1] solves the problem of minimum biimpulsive transfer between two given points. It uses an analytical approach based in the hodograph orbital parameters that transform the solution of this problem in finding the root of a polynomial of order 8, that is then solved numerically. This method was extended by Santos-Paulo[2], to solve the problem of minimum transfer between two given orbits. This is done by varying the true anomaly of the initial and final points involved in the transfer and applying the method developed by Altman and Pistiner for each pair of points. Each solution is saved and compared with the previous results to find the global minimum.

Orbital Transfers With Errors

Based on our extension[2] of the Altman and Pistiner[1] algorithm that solves the minimum fuel biimpulsive transfer, we developed a numerical scheme that incorporates errors in magnitude and direction of the impulsive thrust. The steps to be followed are (Fig. 4):
Step zero: Calculate the optimum biimpulsive transfer (TSE) between the two given orbits (ID, FD); not including the errors;
Step one: Calculate the actual transfer orbit, subject to errors (TCE). The values of the errors are assumed by hypothesis;
Step two: The actual transfer orbit (TCE) is assumed to be the initial orbit of a new maneuver. The calculation of the new minimum fuel maneuver (T2SE) is made assuming that there are no errors involved;
Step three: Step one is repeated, and we can find a new real transfer orbit, including errors (T2CE).
Step four: We repeat step two, now assuming that the real transfer orbit (T2CE) obtained in step three is the initial orbit. The steps above are repeated as many times as necessary to achieve a final orbit (TNCE $\cong$ FD) with the accuracy required. Fig. 5 shows a sequence of maneuvers used in a complete transfer.

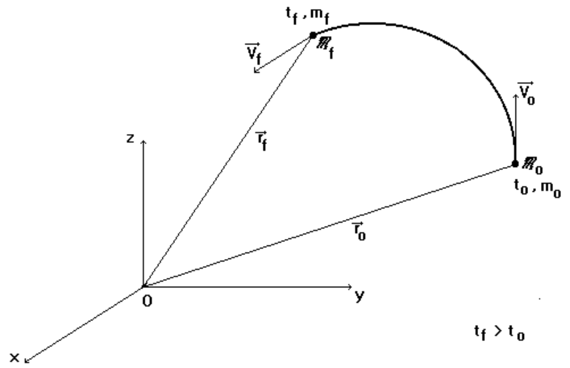


Figure 1 – Orbital Transfer

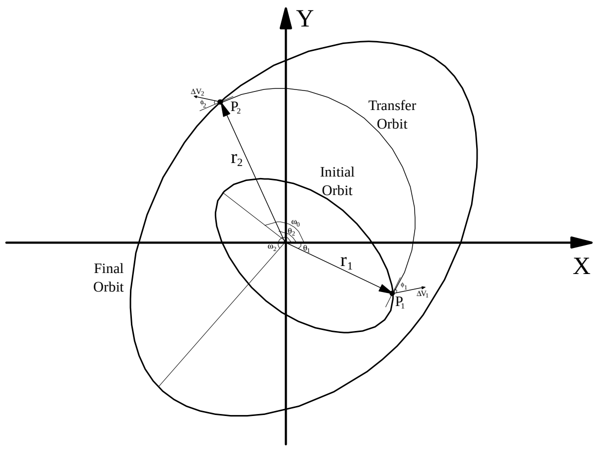


Figure 2 - A Biimpulsive Coplanar Transfer.

Results

Two test cases were calculated to show the importance of the algorithm developed:
1) The initial orbit ID has a = 7500 Km, e = 0, i = 10°, Ω = 0°; and the final orbit FD has a = 9000 Km, e = 0, i = 10°, Ω = 45°. Table 1 shows their results for several error values.
2) The initial orbit ID has a = 7500 Km, e = 0.1 , i = 0°, Ω = 0°, ϖ = 10°; and the final orbit FD has a = 800 km, e = 0,2, i = 5°, Ω = 0°, ϖ = 10°. Table 2 shows their results for several error hypotheses.
They show that: a)an increase in the ΔV ($\delta\Delta V > 0$) causes an increase in the time and in the number of iterations used to solve the problem; b) when the real ΔV is smaller than the nominal value ($\delta\Delta V < 0$), the corrections due to the errors are faster and cheaper;

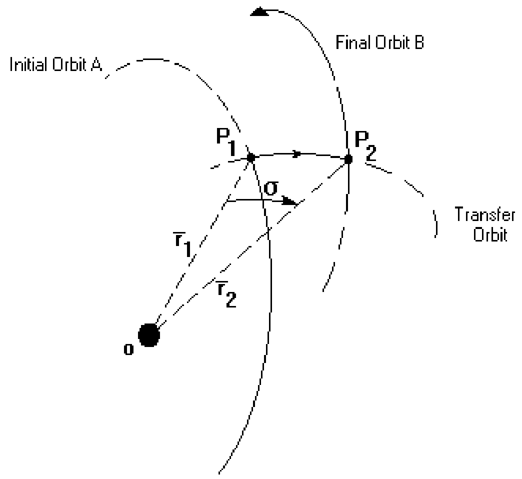
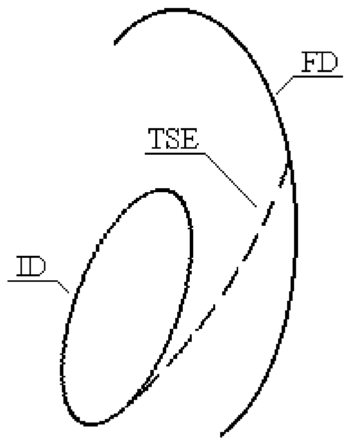
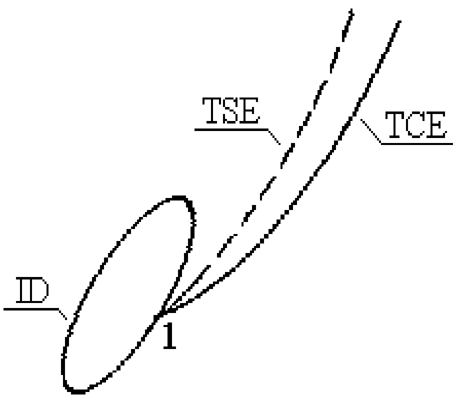


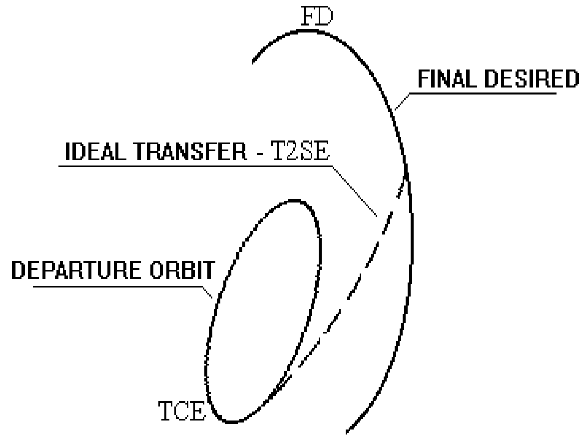
Figure 3 – Noncoplanar Transfer



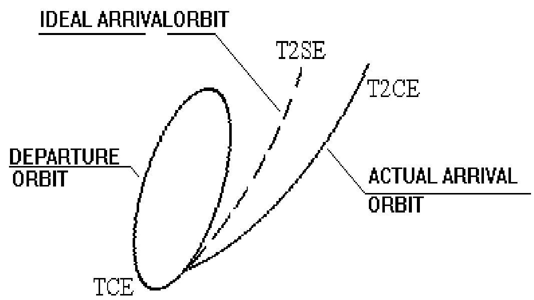
STEP ZERO



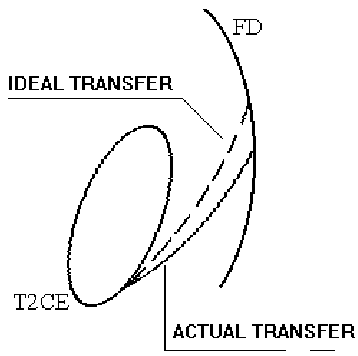
STEP 1
Figure 4 – Steps for the Orbital Transfer with Errors.



STEP 2



STEP 3



STEP 4

Figure 4 (cont.) – Steps for the Orbital Transfer with Errors.

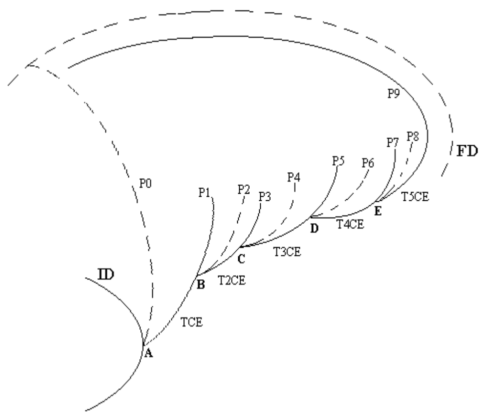


Figure 5 – Complete Orbital Transfer with Errors.

c) considering errors in direction only, when they make the angles smaller than the nominal values ($\delta\alpha < 0$, $\delta\beta < 0$), it is necessary to use more ΔV and time to complete the maneuvers, and the accuracy obtained is smaller.

Conclusions

We developed, implemented and tested a numerical algorithm that calculates minimum fuel maneuvers that use an impulsive propulsion system with errors in magnitude and direction. The tests showed the applicability of the method. We considered that the errors in the thrust vector already exist and that it is not possible to change them, since the spacecraft is already flying. Then, what we did was to estimate their effects. Several simulations were performed with this goal. We were able to see that, in some simulations, even increasing the number of iterations and so the fuel and time required for the transfer, the $N^{th.}$ transfer orbit with errors TNCE does not reach the final desired orbit FD. This will happen even for infinite number of iterations due to the thrust errors.

The results also showed that the longitude of the ascending node and the inclination changes very little, while the argument of the periapse has larger changes. This happens because, when we modify the shape of an orbit the periapse and apoapse may exchange places. This fact may cause larger changes in this parameter with little errors in the maneuvers.

The longitude of the ascending node Ω and the inclination i require more torque to be changed, because it is necessary to change the angular momentum of the orbit. This angular momentum is perpendicular to the orbital plane, so modifications in the direction of this plane (parameters "i" and "Ω") change the angular momentum. Since the magnitude of the angular momentum is large, to change it requires a large torque.

The application of this research is very large, including all the space missions that require orbital maneuvers and that can be modelled by an impulsive thrust, such as the Brazil-China Remote Sensing Satellite-CBERS. Interplanetary missions can also benefit from this method in parts of the mission.

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Table 1 - Results for the First Simulation

KEPLERIAN ELEMENTS OF THE DESIRED FINAL ORBIT	a = 9000 km			e = 0		i = 10°		Ω = 45°		ω = undefined		Δt_{IDEAL} 1 hour		ΔV_{IDEAL} 1,1261 km/s
$\delta\alpha = \delta\beta$ $\delta\Delta V$	**A 0° 3%**	**B 0° -3%**	**C 0° 5%**	**D 0° -5%**	**E 0° 10%**	**F 0° -10%**	**G -2,5° 0%**	**H 2,5° 0%**	**I - 5° 0%**	**J 5° 0%**	**L - 2,5° -5%**	**M 2,5° -5%**	**N -5° -5%**	**P 5° -5%**
KEPLERIAN ELEMENTS OFTHE REACHED FINAL ORBIT														
a (km)	**8993**	**8999**	**8794**	**8982**	**8933**	**8977**	**8951**	**8937**	**8988**	**8970**	**8952**	**8992**	**8959**	**8985**
e	**0,0005**	**0,0006**	**0,06**	**0,003**	**0,009**	**0,002**	**0,006**	**0,006**	**0,005**	**0,002**	**0,02**	**0,001**	**0,04**	**0,004**
i (degrees)	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**10**	**9**	**10**	**10**	**9**
Ω (degr.)	**45**	**45**	**34**	**43**	**42**	**45**	**45**	**45**	**45**	**45**	**28**	**45**	**34**	**42**
ω (degr.)	**312**	**211**	**125**	**37**	**295**	**350**	**55**	**37**	**260**	**341**	**14**	**39**	**24**	**320**
Δt_{TOTAL}	**5h32’**	**3h28’**	**9h09’**	**2h45’**	**5h40’**	**3h18’**	**4h30’**	**2h07’**	**5h29’**	**3h31’**	**5h36’**	**4h17’**	**4h33’**	**4h29’**
ΔV_{TOTAL} (km/s)	**2,9430**	**1,1537**	**2,1116**	**1,1214**	**1,9987**	**1,1668**	**1,8287**	**1,2610**	**1,9849**	**1,2255**	**1,4388**	**1,1753**	**1,6288**	**1,0946**
Increment in t	**4h32’**	**2h28’**	**8h09’**	**1h45’**	**4h40’**	**2h18’**	**3h30’**	**1h07’**	**4h29’**	**2h31’**	**4h36’**	**3h17’**	**3h33’**	**3h29’**
Increment in ΔV (km/s)	**1,8169**	**0,0276**	**0,9855**	**- 0,0047**	**0,8726**	**0,0407**	**0,7026**	**0,1349**	**0,8588**	**0,0994**	**0,3127**	**0,0492**	**0,5027**	**- 0,0315**
Number of iterations	**13 (13)**	**7 (7)**	**15 (19)**	**7 (7)**	**9 (9)**	**7 (7)**	**9 (9)**	**7 (7)**	**9 (9)**	**7 (7)**	**9 (9)**	**7 (7)**	**9 (9)**	**7 (7)**

Table 2 - Results for the Second Simulation

KEPLERIAN ELEMENTS OF THE DESIRED FINAL ORBIT	a (km)	e	i (degrees)	Ω (degrees)	ω (degrees)	Δt_{IDEAL}	ΔV_{IDEAL}
	8000	0,2	5	0	10	45 minutes	0,7545 km/s

ERROR HYPOTHESES	**A** $\delta\alpha = \delta\beta = 0°$ $\delta\Delta V = 3\%$	**B** $\delta\alpha = \delta\beta = 0°$ $\delta\Delta V = -3\%$	**G** $\delta\alpha=\delta\beta=-2,5°$ $\delta\Delta V = 0\%$	**H** $\delta\alpha=\delta\beta= 2,5°$ $\delta\Delta V = 0\%$
KEPLERIAN ELEMENTS OF THE REACHED FINAL ORBIT				
a (km)	**8045**	**7981**	**8072**	**7982**
e	**0,2**	**0,2**	**0,2**	**0,2**
i (degrees)	**5**	**5**	**5**	**5**
Ω (degrees)	**358**	**359**	**359**	**358**
ω (degrees)	**11**	**12**	**10**	**24**
Δt_{TOTAL}	**43’**	**45’**	**2h50’**	**60’**
ΔV_{TOTAL} (km/s)	**0,7352**	**0,7347**	**0,7609**	**1,0100**
Increment in t	**-2’**	**zero**	**2h05’**	**15’**
Increment in ΔV (km/s)	**-0,0194**	**-0,0198**	**0,0064**	**0,2555**
Number of iterations	**3 (7)**	**3 (7)**	**7 (7)**	**5 (7)**