

TRACKING OF THE LAUNCH VEHICLE DURING THE INSERTION TO EARTH ORBIT

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Abstract

The paper presents algorithms, software and experience results of the system for the trajectory monitoring during the injection to LEO. The system was used at the KIAM for monitoring the launch vehicle's (LV) trajectories of the spacecraft (SC) Progress and Soyuz directed to MIR orbital complex. The SC insertion to the LEO was monitored by range and angular measurements from the ground tracking stations located along the launch track. The system gives the state vector and others parameters of the SC motion as result of the measurement processing. Also the system allows to obtain the earliest orbital parameters of SC after engine operating is ended.

The paper contains the statistical results for tracking of the Soyuz and Progress SC during the insertion to LEO from 1988 to 1998.

Using GPS and GLONASS navigation systems at the SC can increase the accuracy of the state vector estimation. In the paper we present the results of system work simulating with processing both measurements and navigation system data.

Key words: Trajectory monitoring, Measurement processing, Launch vehicle.

Introduction

KIAM has been working in navigation and orbit control for the SC Soyuz and Progress, directed to the orbital stations "Salut-6", "Salut-7" and "MIR" since the launch of the orbital station "Salut-6". These works in particular include the real time monitoring of the LV trajectory during the insertion. The data from the tracing station are obtained by the Ballistic center of the Institute via connection lines. Receiving and processing of the information is made by a special soft- and hardware system, developed in KIAM. The delay from receiving a group of measurements to delivering the results of orbit estimation is about a few seconds (less than 10). The results are transmitted to the Mission Control Centre and displayed in every 10 seconds. The

display format allows to observe a deviation of the LV motion from the nominal trajectory during the flight.

Results of the launch monitoring for "Zarya" module of international station "Alpha" are presented at the Fig.1. as an example.

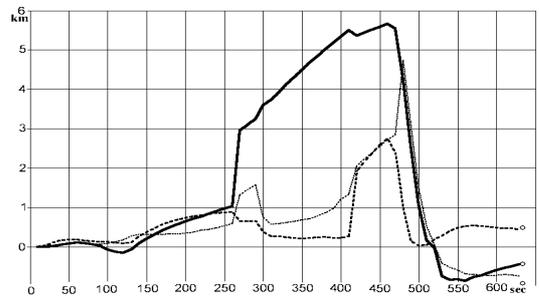


Figure 1: Launch vehicle of "Zarya" SC. Deviation of the estimated trajectory from the nominal one

Bold line represents the deviation along the target line as a function of time since the launch and the dotted line represents deviation from the plane of the nominal trajectory.

A few generations of computers were changed during the existence of KIAM ballistic center. The current system is realized by OS Unix and the corresponding software is written in the C language.

The system includes deciphering, primary processing of measurements, estimating the LV motion and displaying the information. Each part is realised as a task in OS Unix. The information exchange is performed by the Unix Domain Sockets mechanism. The tasks can obtain information from files during testing or researching.

The Kalman's filter algorithm is used to estimate the state vector of the LV. Efficiency of the estimator depends mainly on the two factors: on the correspondence of an accepted model to the actual motion of the LV and on the possibility of finding and rejecting abnormal measurements by the algorithms of estimation. The chosen model is based on the

assumption that the actual motion of the LV is close to the nominal one. The nominal trajectory is inputted for the estimator in the table form. The deviation of the launch vehicle's motion from this trajectory in a limited interval is assumed to be uniformly accelerated. The other deviations are considered as a noise. This simplified model revealed high efficiency in practice. It has sufficient accuracy for the small deviations and produces a satisfactory qualitative description of the LV motion for large deviations. In case of emergency (reverse maneuver etc.) the tables of LV nominal trajectory can be substituted by others better corresponding to the situation.

There are two stages of rejecting abnormal measurements. At the first stage during the preliminary processing of trajectory measurements the time correspondence of a measurement is checked. At the second stage one checks the correspondence of a account a current LV state vector, accuracy of a measurement and accuracy of the chosen motion model.

The accuracy of the state vector estimation could be increased by situating on the LV GPS or Glonass receiver and transmitting its estimates of the state vector along with the range and range rate measurements to the center of processing.

The purpose of the present paper is to analyze the efficiency of the system and to estimate its prospects. The results can be used for investigation of the quality of LV insertion to parking LEO.

Mathematical model of the LV motion

The trajectory of LV mass center is very complicated due to the influence of a great number of different forces¹. In order to calculate a nominal trajectory the organization, constructing a SC, uses the most comprehensive and reliable model. The equations, describing motion of the LV, include contributions of the thrust force, the Earth gravity field, aerodynamic forces, Coriolis forces, caused by the rotation of the LV around the mass center and by the motion of fuel and gases with respect to the LV body, control forces.

To solve the problem of orbit estimation in a real time the nominal trajectory is stored in the table form. At the interval from t_k to t_{k+1} the nominal equations of LV motion can be written in the form

$$\dot{\bar{x}}_n = \bar{F}(t, \dot{\bar{x}}_n), \quad (1)$$

while the real motion equations as

$$\dot{\bar{x}}_r = \bar{F}(t, \dot{\bar{x}}_r) + \delta\bar{W}(t) \quad (2)$$

where

$\bar{x}^T = \{\bar{r}^T \ \bar{v}^T\}$ is a 6-dimensional state-vector,

$\bar{F}^T = \{\bar{v}^T \ \bar{f}^T\}$ vector of motion equations RHS,

\bar{r} is a position, \bar{v} is a velocity, \bar{f} is an acceleration of the LV,

$\delta\bar{W}^T = \{0^T \ \delta\bar{w}^T\}$, $\delta\bar{w}$ is a deviation of an actual acceleration from the nominal one.

The LV motion equations relatively to the nominal trajectory are obtained by subtracting equation (1) from the equation (2)

$$\begin{aligned} \Delta\dot{\bar{x}} &= \bar{F}(t_k + \tau, \dot{\bar{x}}_n + \Delta\bar{x}) - \bar{F}(t_k + \tau, \dot{\bar{x}}_n) + \delta\bar{W}(t) \\ \tau &= t - t_k, \\ \Delta\bar{x} &= \bar{x}_r - \bar{x}_n \end{aligned} \quad (3)$$

For small time intervals τ and small deviations $\Delta\bar{x}$ terms of the second order can be omitted, so that the difference $\bar{F}(t, \bar{x}_r) - \bar{F}(t, \bar{x}_n)$ can be rewritten as

$$\begin{aligned} &\bar{F}(t_k + \tau, \dot{\bar{x}}_n + \Delta\bar{x}) - \bar{F}(t_k + \tau, \dot{\bar{x}}_n) = \\ &= [\bar{F}(t_k + \tau, \dot{\bar{x}}_n + \Delta\bar{x}) - \bar{F}(t_k, \dot{\bar{x}}_n + \Delta\bar{x})] + \bar{F}(t_k, \dot{\bar{x}}_n + \Delta\bar{x}) - \\ &\quad - [\bar{F}(t_k + \tau, \dot{\bar{x}}_n) - \bar{F}(t_k, \dot{\bar{x}}_n)] - \bar{F}(t_k, \dot{\bar{x}}_n) \approx \\ &\approx \frac{\partial \bar{F}}{\partial t}(t_k, \bar{x}_n + \Delta\bar{x}) \tau - \frac{\partial \bar{F}}{\partial t}(t_k, \bar{x}_n) \tau + \bar{F}(t_k, \dot{\bar{x}}_n + \Delta\bar{x}) - \bar{F}(t_k, \dot{\bar{x}}_n) \approx \\ &\approx \frac{\partial^2 \bar{F}}{\partial t \partial \bar{x}} \tau \Delta\bar{x} + \bar{F}(t_k, \dot{\bar{x}}_n + \Delta\bar{x}) - \bar{F}(t_k, \dot{\bar{x}}_n) \approx \\ &\approx \bar{F}(t_k, \dot{\bar{x}}_n + \Delta\bar{x}) - \bar{F}(t_k, \dot{\bar{x}}_n) \end{aligned} \quad (4)$$

This expression does not depend on time within $[t_k, t_{k+1}]$. The noise $\delta\bar{W}$, describing factors, that were not taken into account in the model, is also considered to be constant at $[t_k, t_{k+1}]$. Finally, the equations of the LV motion relatively to the nominal trajectory is written in the form

$$\begin{aligned} \Delta\dot{\bar{r}} &= \Delta\bar{v} \\ \Delta\dot{\bar{v}} &= \Delta\bar{w} \end{aligned} \quad (5)$$

where

$\Delta\bar{r}$ is a position deviation from the nominal trajectory, $\Delta\bar{v}$ is a relative velocity, $\Delta\bar{w} = \bar{f}(t_k, \dot{\bar{x}}_n + \Delta\bar{x}) - \bar{f}(t_k, \dot{\bar{x}}_n) + \delta\bar{w}$ is a constant acceleration relatively to the nominal trajectory.

The solution of (5) can be expressed as

$$\begin{aligned}
\Delta\bar{r}(t_k + \tau) &= \Delta\bar{r}(t_k) + \Delta\bar{v}(t_k)\tau + \frac{1}{2}\Delta\bar{w}(t_k)\tau^2 \\
\Delta\bar{v}(t_k + \tau) &= \Delta\bar{v}(t_k) + \Delta\bar{w}(t_k)\tau \\
\Delta\bar{w}(t_k + \tau) &= \Delta\bar{w}(t_k)
\end{aligned} \tag{6}$$

The state vector in the inertial coordinate system is calculated by using nominal trajectory tables as

$$\begin{aligned}
\bar{r}(t) &= \bar{r}_n(t) + \Delta\bar{r}(t_k + \tau) \\
\bar{v}(t) &= \bar{v}_n(t) + \Delta\bar{v}(t_k + \tau)
\end{aligned} \tag{7}$$

The values of $\bar{r}_n(t)$ and $\bar{v}_n(t)$ are calculated by the interpolation.

Coordinate systems

LV nominal trajectory is written in the mobile (noninertial) starting coordinate system (SCS). Its axes are attached to the rotating Earth and the beginning is placed in the start point. The axis X is directed along the target line, the axis Y is directed upward vertically and the axis Z supplements the system to the right one.

To define a model of mass centre motion, write the differential equations and algorithms of the state vector estimation the original starting coordinate system (OSCS) is used. It is an inertial coordinate system, its centre and axes being motionless. In the moment of start this system coincides with SCS.

The calculation of measurable parameters is performed in the North-East-Zenith (NEZ) coordinate system. Its centre is in a point of disposition of a tracking antenna. The axis X is directed to the north, the axis Z - upward vertically and the axis Y is chosen so that to obtain the right system.

Calculation of measurable orbit parameters

At the present time ballistic centre obtain from tracking station the range - D , azimuth - A and the elevation - γ of the LV. In the NEZ coordinate system the nominal values of these parameters are written as

$$D = \sqrt{(x')^2 + (y')^2 + (z')^2} \tag{8}$$

$$A = \begin{cases} \arccos \frac{x'}{\sqrt{(x')^2 + (y')^2}} & \text{if } y' \geq 0 \\ -\arccos \frac{x'}{\sqrt{(x')^2 + (y')^2}} & \text{if } y' < 0 \end{cases} \tag{9}$$

$$\gamma = \arcsin \frac{y'}{\sqrt{(x')^2 + (y')^2 + (z')^2}}, \tag{10}$$

where t is a moment of signal reception by a tracking station, τ is a time of signal coming from the LV to the tracking station, $\bar{r}\{x, y, z\}$ is a LV position in the NEZ coordinate system, $\bar{r}'(t) = \bar{r}(t - \tau)$ is a LV position at the moment of signal emitting.

Recursive algorithm of state vector estimation

To estimate the state vector a recursive algorithm is used²⁻⁴. The estimated parameter is a 9-dimensional vector $\Delta\bar{X} = \{\Delta\bar{r}, \Delta\bar{v}, \Delta\bar{w}\}^T$ of deviations from the nominal positions, velocities and accelerations respectively.

The dependence of a measurement at a moment t_i on a current LV state vector can be expressed as

$$\bar{y}_i^k = \bar{\psi}^k(\bar{x}(t_i)) + \bar{\eta}_i^k \tag{11}$$

where $k = k(i)$ denotes the number of a measurement system type. For example, $k = 1$ we use for measurements of range, $k = 2$ for measurements of azimuth etc. In the general case a k -th type measurement can be a scalar or a vector. In the latter case the correlation of a measurement vector components are essential for the estimation algorithm. The dimension of a measurement $\bar{\psi}^k$ is denoted as $m = m(i)$.

$\bar{\eta}_i^k$ denotes an error of k -type measurement, made at the moment t_i . The errors are assumed to be independent and have a normal distribution with a covariance R_k .

In the linear approximation the dependence (11) can be written as

$$\bar{y}_i^k = \bar{\psi}^k(\bar{x}_n(t_i)) + \bar{z}^k(t_i), \tag{12}$$

where $\bar{z}^k(t_i) = \bar{H}^k(t_i) \Delta\bar{x}(t_i) + \bar{\eta}_i^k$, $\bar{H}^k(t_i) = \frac{\partial \bar{\psi}^k}{\partial \bar{x}}(t_i)$ - is a matrix of partial derivatives of a k -th type measured function with respect to the 6-dimensional state vector. Prediction of a vector

$\Delta\bar{X}$ from the moment t_α on a time t_β is made by the equation

$$\Delta\bar{X}(t_\alpha) = \Phi(t_\alpha, t_\beta) \Delta\bar{X}(t_\beta) \quad (13)$$

For the accepted model of the LV motion the transition matrix Φ_i is written as

$$\Phi(t_\alpha, t_\beta) = \begin{cases} E(t_\beta - t_\alpha) E^{\frac{1}{2}}(t_\beta - t_\alpha)^2 E \\ O & E & (t_\beta - t_\alpha) E \\ O & O & E \end{cases} \quad (14)$$

where E is a unit 3x3 matrix and O is a zero 3x3 matrix.

The estimation of a current state vector $\Delta\bar{X}_i$, $i = 1, \dots, n$ is performed by the is performed by the Kalman's filter algorithm

$$\begin{aligned} \Delta\hat{X}_i &= \Phi_{i,i-1} \Delta\hat{X}_{i-1} + P_i H_i^T R_i^{-1} [\bar{z}_i - H_i \Phi_{i,i-1} \Delta\hat{X}_{i-1}], \\ P_i &= \hat{P}_i - \hat{P}_i H_i^T [H_i \hat{P}_i H_i^T + R_i]^{-1} H_i \hat{P}_i, \\ \hat{P}_i &= \Phi_{i,i-1} P_i \Phi_{i,i-1}^T + Q_{i-1}, \end{aligned} \quad (15)$$

where

$$\bar{z}_i = \bar{z}^{k(i)}(t_i), \quad \bar{H}_i = \bar{H}^{k(i)}(t_i), \quad R_i = R^{k(i)}(t_i),$$

Q_i is covariance caused by inaccuracy of the accepted motion model, $\Delta X(t_0) = 0$.

Algorithm of rejecting abnormal measurements

The analysis of trajectory measurements for a great number of SC shows, that errors of most measurement are normally distributed. It is usually assumed in estimating algorithms. Nevertheless, there are measurements with rough errors, caused by distortions during transmitting, defects of measurement devices or other reasons. These error measurements should be rejected from the processing.

Rejecting of error measurements is performed in two stages.

At the first stage the analysis is made by algorithms, independent of the estimation procedure. One checks the format of a measurement, agreement of its value with the possible range of values, its time correspondence an so on.

At the second stage a measurement is compared with its estimation, taking into account the accuracy of a current state vector, the accuracy of a measurement and the accuracy of the accepted motion model.

Let an estimation of the state vector $\Delta\hat{X}_{i-1}$ with respect to the nominal trajectory is obtained after processing of measurements $\bar{y}_1, \bar{y}_2, \dots, \bar{y}_{i-1}$, carried out at t_1, t_2, \dots, t_{i-1} . The difference between measured and estimated values of a parameter t_i can be written as

$$\delta\bar{z}_i = \bar{z}_i - H_i \Phi_{i,i-1} \Delta\hat{X}_{i-1} \quad (16)$$

A covariance of $\delta\bar{z}_i$ is a sum of covariance of a measurement \bar{y}_i and a covariance of its estimation:

$$K_{yi} = H_i \hat{P}_i H_i + R_i \quad (17)$$

If $\delta\bar{z}_i$ is in agreement with its covariance, the measurement \bar{y}_i will be processed. It means, that the condition

$$|\delta z_i^l| < 3\sqrt{k_{yi}^{ll}} \quad l = 1, \dots, m \quad (18)$$

Here δz_i^l is a component of $\delta\bar{z}_i$, k_{yi}^{ll} is a diagonal element of the matrix K_{yi} , and m is a dimension of \bar{y}_i . If any of component of the vector $\delta\bar{z}_i$ does not satisfy the above condition, the measurement \bar{y}_i is rejected.

Analysis of system work results

In this section we try to estimate the efficiency of system work studying the actual results of the LV orbit control. Available data also allow to investigate the errors of SC insertion to LEO. As a parameters, characterising the system work and the quality of insertion one considers

- * covariance estimation of orbit accuracy,
- * estimation of actual accuracy of the state vector in the end of active interval, obtained by comparing it with the SC state vector in the beginning of the passive trajectory.
- * statistical average deviations between actual and nominal trajectories.

Tables 1-5 contain a list of spacecrafts, inserted by the present system. Spacecrafts listed in each table had the same nominal trajectory. Each nominal trajectory has its own number. The total amount of the nominal trajectories is 5.

Table 1: Nominal trajectory 1.

Spacecraft's name	Date	Launch time (Moscow)
Progress - 39	1988.12.25	07.11.37
Progress - 41	1989.03.16	21.54.15
Progress - 42	1990.05.05	23.44.01
Progress - M4	1990.08.15	07.00.41
Progress - M5	1990.09.27	13.37.42
Progress - M6	1991.01.14	17.50.27
Progress - M7	1991.03.19	16.05.15
Progress - M8	1991.05.30	11.04.03
Progress - M9	1991.08.21	01.54.10
Progress - M10	1991.10.17	03.05.25
Progress - M11	1992.01.25	10.50.20
Progress - M12	1992.04.20	00.29.25
Progress - M13	1992.06.30	19.43.14
Progress - M14	1992.08.16	01.18.32
Progress - M15	1992.10.27	20.19.41
Progress - M17	1993.03.31	06.34.13

Table 2: Nominal trajectory 2.

Spacecraft's name	Date	Launch time (Moscow)
Progress - M19	1993.08.11	01.23.45
Progress - M21	1994.01.28	05.12.10
Progress - M22	1994.03.22	07.54.12
Progress - M23	1994.05.22	07.30.04
Progress - M24	1994.08.25	17.25.12
Progress - M27	1995.04.09	22.34.12
Progress - M28	1995.07.20	06.04.41
Progress - M29	1995.10.08	21.50.40
Progress - M31	1996.05.05	10.04.18
Progress - M32	1996.07.31	23.00.06
Progress - M35	1997.07.05	07.11.54

Table 3: Nominal trajectory 3.

Spacecraft's name	Date	Launch time (Moscow)
Progress - M33	1996.11.20	02.20.38
Progress - M34	1997.04.06	19.04.05

Progress - M36	1997.10.05	18.08.57
Progress - M37	1997.12.20	11.45.02
Progress - M38	1998.03.15	01.45.55
Progress - M39	1998.05.15	01.12.59

Table 4: Nominal trajectory 4.

Spacecraft's name	Date	Launch time (Moscow)
Soyuz - M9	1990.02.11	09.16.00
Soyuz - M11	1990.12.02	11.13.32
Soyuz - M12	1991.05.18	15.50.28
Soyuz - M13	1991.10.02	08.59.38
Soyuz - M14	1992.03.17	13.54.30
Soyuz - M15	1992.07.27	09.08.42
Soyuz - M16	1993.01.24	08.58.05
Soyuz - M17	1993.07.01	17.32.58
Soyuz - M18	1994.01.08	13.05.34
Soyuz - M19	1994.07.01	15.24.50
Soyuz - M20	1994.10.04	01.42.30

Table 5: Nominal trajectory 5.

Spacecraft's name	Date	Launch time (Moscow)
Soyuz - M9	1995.03.14	09.11.34
Soyuz - M11	1995.09.03	12.00.23
Soyuz - M12	1996.02.21	15.34.05
Soyuz - M13	1996.08.17	16.18.03
Soyuz - M14	1997.02.10	17.09.30
Soyuz - M15	1997.08.05	18.35.54
Soyuz - M16	1998.01.29	19.33.42

The data, defining the quality of orbit estimation and the quality of insertion to LEO by LV, are shown at the Fig. 2–16. Parameters, defining the deviations along the velocity vector (X axis of OSCS) are presented at the Fig. 2, 5, 8, 11, 14; vertical deviations (Y axis of OSCS) – at the Fig. 3, 6, 9, 12, 15; deviations, orthogonal to the orbit plane (Z axis of OSCS) – at the Fig. 4, 7, 10, 13, 16. Each of these figures contains

* average X, Y and Z deviations in OSCS (bold line),

* two envelopings, corresponding to maximal or minimum deviations (dashed line),

* two envelopings, corresponding to root-mean-square deviations from the, average value (dotted line),

* accuracies (in the bottom of figure).

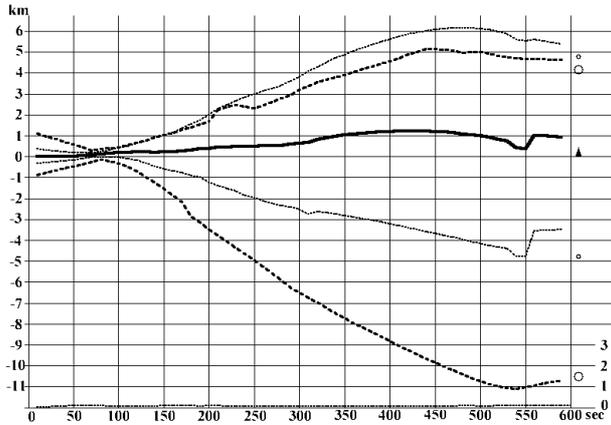


Figure 2: Nominal trajectory 1. X axis of OSCS

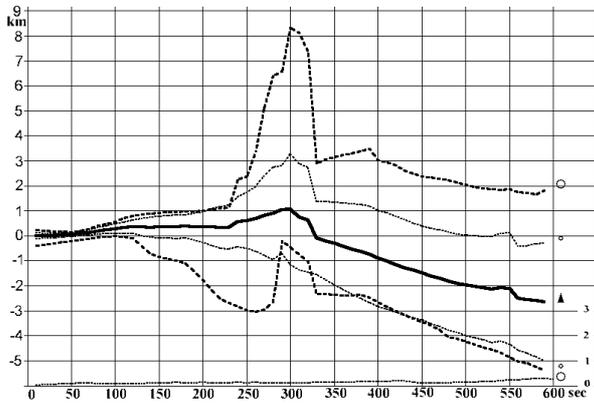


Figure 3: Nominal trajectory 1. Y axis of OSCS

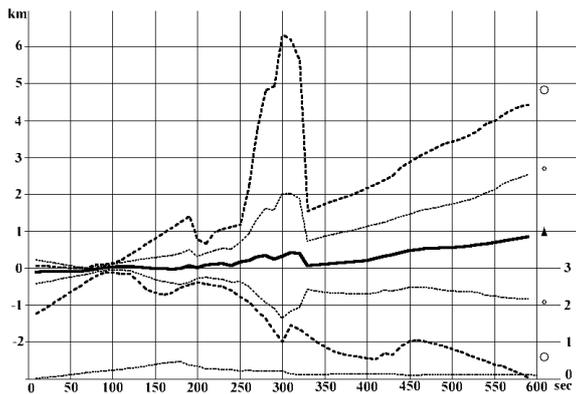


Figure 4: Nominal trajectory 1. Z axis of OSCS

The improved state vector of the SC in the end of active interval can be compared with the SC state vector in the beginning of passive motion. The latter can be obtained with much more accuracy, so that it can be considered as a standard. These standard values were found by trajectory measurements at two first revolutions of passive motion, transformed into OSCS system. The data, obtained by the help of standard state vectors are marked in right of a figure. They independently define an actual accuracy of orbit estimation in the end of active motion.

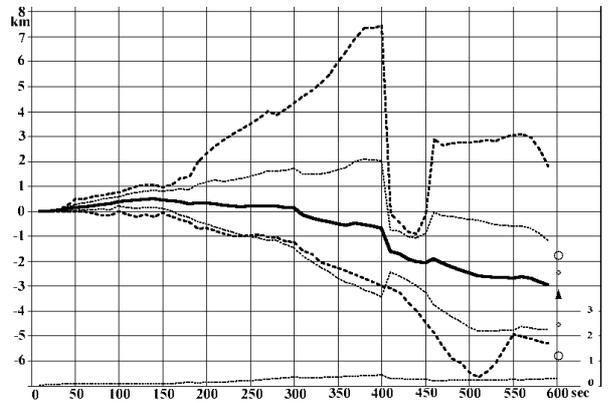
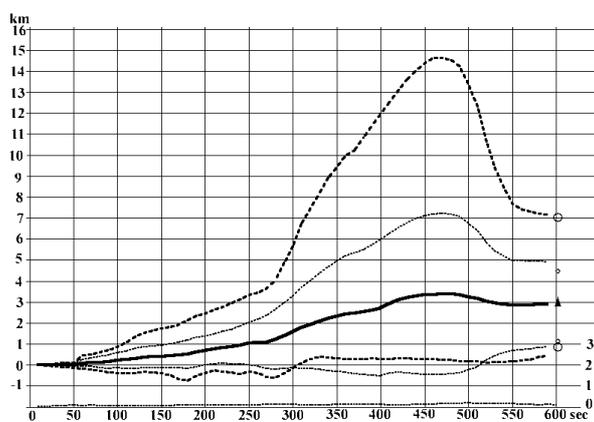


Figure 5: Nominal trajectory 2. X axis of OSCS

Figure 6: Nominal trajectory 2. Y axis of OSCS

The plot of average motion is marked by the black triangle, the envelopings, corresponding to maximal and minimum deviations – by large circles and the envelopings, corresponding to the root-mean-square deviations – by small circles.



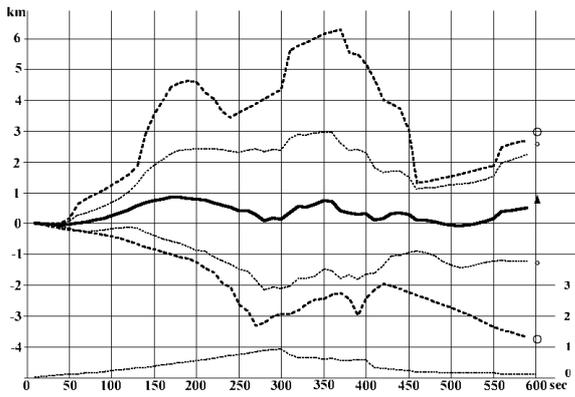


Figure 7: Nominal trajectory 2. Z axis of OSCS

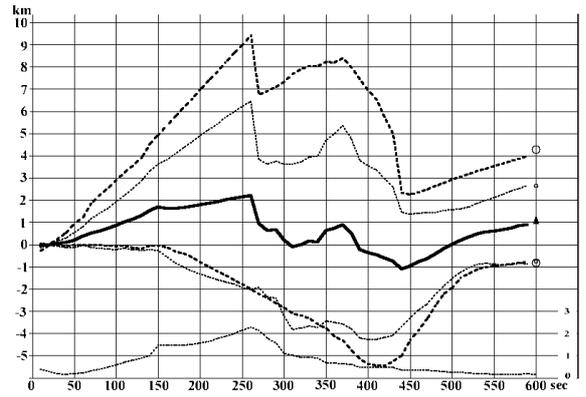


Figure 10: Nominal trajectory 3. Z axis of OSCS

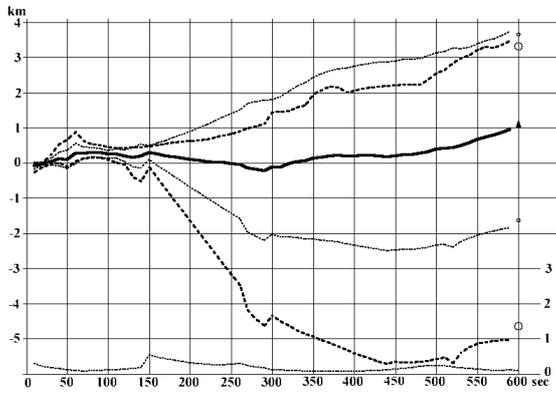


Figure 8: Nominal trajectory 3. X axis of OSCS

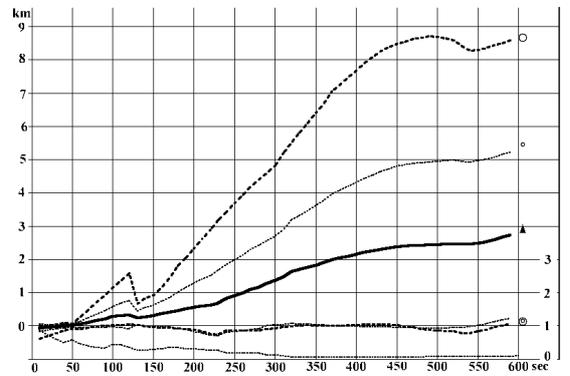


Figure 11: Nominal trajectory 4. X axis of OSCS

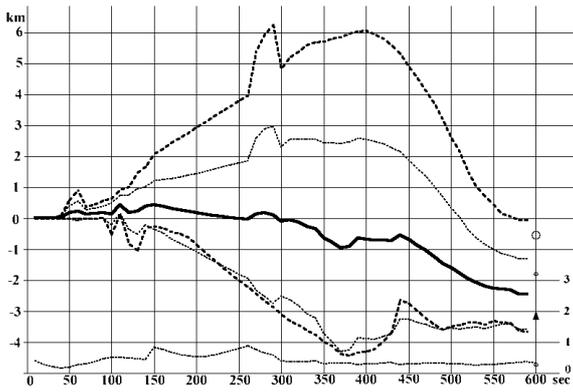


Figure 9: Nominal trajectory 3. Y axis of OSCS

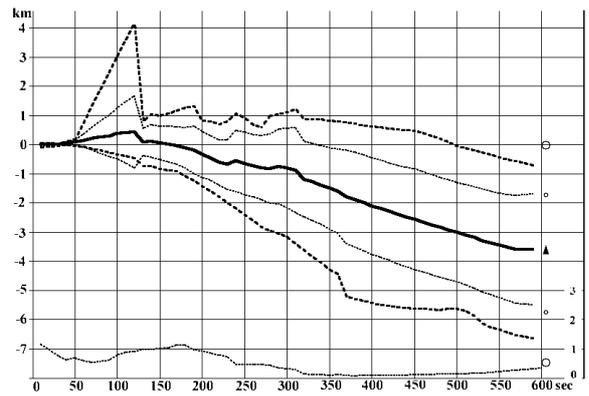


Figure 12: Nominal trajectory 4. Y axis of OSCS

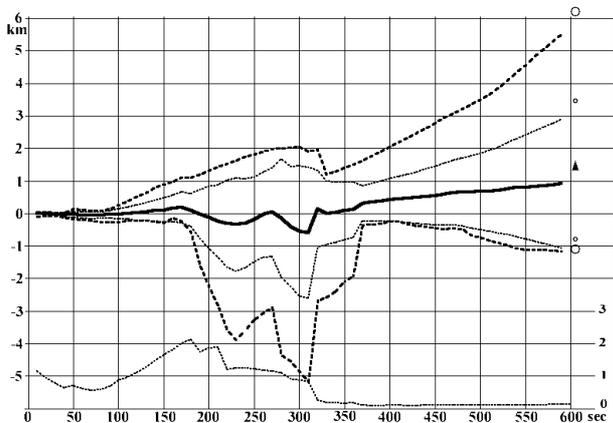


Figure 13: Nominal trajectory 4. Z axis of OSCS

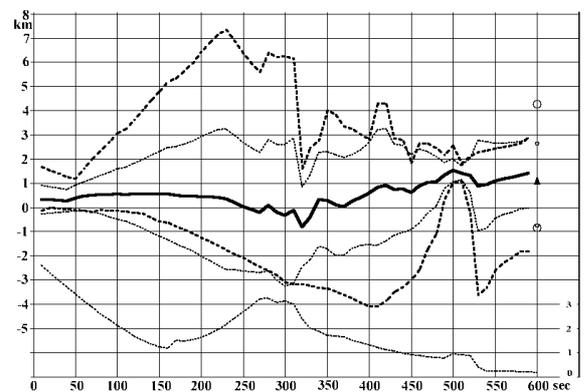


Figure 16: Nominal trajectory 5. Z axis of OSCS

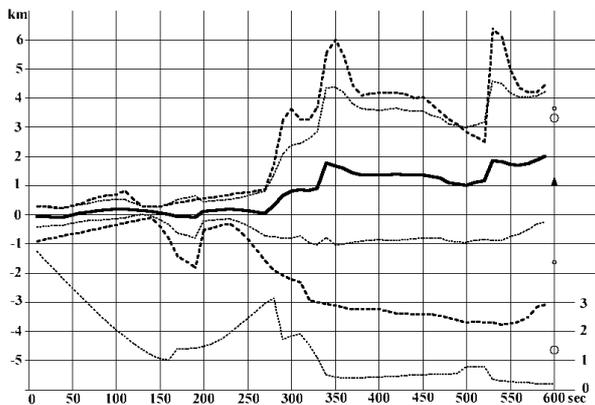


Figure 14: Nominal trajectory 5. X axis of OSCS

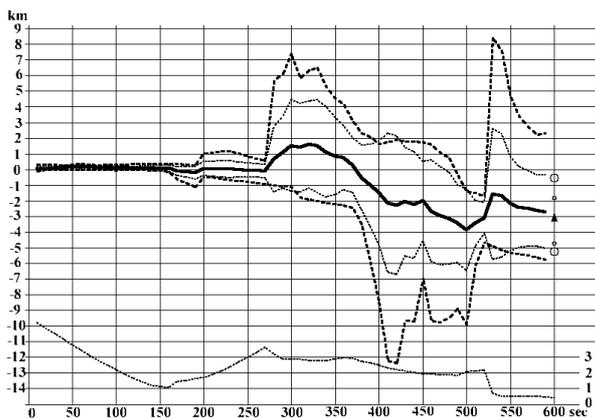


Figure 15: Nominal trajectory 5. Y axis of OSCS

Orbit elements, obtained by using the estimated state vector in the end of active interval, and elements of the passive orbit are compared in the table 6.

Table 6: Comparing the estimated orbit elements and standard ones

	Mean value	Root-mean-square
Minimal height (km)	-0.661	3.474
Maximal height (km)	3.620	11.335
Period (sec)	0.038	0.118
Declination (deg)	-0.00023	0.111

Prospects of using GPS and Glonass systems

The analysis of plots, presented at the Fig. 2--6, shows the trend of decreasing of accuracy. It is caused by the decreasing of a number of measurement sets, used for the orbit control at the active interval. The accuracy can be considerably increased by situating GPS and Glonass receivers onboard and transmitting their data, along with the measurements of range and range rate, to the processing center. Now these works are in preparations.

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