PRECISE RELATIVE ORBIT ESTIMATION OF INSAT MISSIONS

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ABSTRACT

The increase in the use of geostationary satellite services calls for the number of geostationary satellites to be placed close to each other in the orbit leading to the colocation of satellites. Present trend in space communication development calls for keeping a number of clustered satellites in a specified formation to synthesize a large telecommunications system. To operate these satellites without the risk of collision or any other undesired mutual interference, the Inter Satellite Distance (ISD) between these satellites should be maintained within certain limits. Hence it is necessary to explore the feasibility of relative orbit determination as accurately as possible. This paper presents the method of precise relative orbit estimation by means of ground based tracking applicable to Indian Space Research Organisation's (ISRO's) co-located INSAT missions using weighted least squares technique. Mathematical modeling of relative orbit determination along with sample sets of results using INSAT co-located satellite's real data and its comparison with the ISRO's regular operational orbit determination for individual satellites is presented.

1. INTRODUCTION

Due to the increase in the need of geostationary satellites, it has become necessary to place a number of satellites to place at the same longitudinal position of the orbit and operate. Onboard inter-satellite tracking equipment would be desirable for better achievable orbit accuracy compared to the most single ground tracking system.

In the absence of such equipment, one can make use of individual ground tracking data of those satellites and obtain differential measurements proposed by Kawase [1] to determine the relative orbit. A good amount of work has been carried out by several authors for the relative orbit determination of closely located geostationary satellites by means of ground-based differential tracking data. They adopted either sequential or batch estimation technique. This paper presents the method of precise relative orbit estimation by means of ground based tracking applicable to Indian Space Research Organisation's (ISRO's) INSAT missions using weighted least squares batch estimation technique.

This concept has been realized through INSAT-2C which is colocated with INSAT-2B at 93.5 deg, Metsat-I/Kalpana-I with INSAT-3C at 74 deg and INSAT-2E with INSAT-3B at 83 deg east longitude. The colocation strategy is based on eccentricity and inclination separation and they are tracked by ground antenna.

This is a new conceptualization for getting accurate relative orbit estimation in addition to regular orbit determination ODP (ISRO's operational Orbit Determination Program), which provides the individual orbit estimation for each satellite. The brief description of the ODP software methodology is given in [2]. This paper describes the mathematical modeling of relative orbit determination along with sample sets of results using live satellite data. This problem has almost identical treatment with that of terminal rendezvous in near circular orbits for relative motion. Basically Hill's equations of relative motion are integrated by Cowell's method. The observations are modeled. Partial derivatives are computed through increment method. The state parameters are refined using weighted least squares technique and iterative differential correction process to obtain definitive relative parameters precisely.

The relative state can also be obtained from the estimated state parameters of individual satellites. Since the estimated individual state parameters have certain limited accuracy, the relative state obtained by differencing these state parameters will have further inaccuracy.

In the absence of onboard inter-satellite tracking equipment the differential measurements from the individual ground tracking data are constructed. The ground based tracking observations of range and angles are considered for estimating the relative orbit. Differential range and angle measurements between the satellites are used in this method. By means of differential measurements the common error to each measurement for each satellite is minimized from which an improved accuracy of relative orbit determination is expected. Definitive state parameters obtained from regular orbit determination of colocated individual satellites is an initial guess for the relative orbit estimation process. This paper presents some cases of INSAT missions from the data collected during

February 1996 to July 2004. Theoretical formulation of 'relative observations generation' from the tracking data of individual colocated satellites and also the 'relative observations prediction' are described along with the estimation procedure. The orbit generated is based on special perturbation method. The system of equations of motion (Hill's equations) are integrated by second sum method. The dominant perturbative forces are considered for relative position computation. Basing on the method described above, a software for relative orbit determination known as RELOD was developed and operationalized for INSAT missions.

2. RELATIVE ORBIT DETERMINATION SYSTEM

The Relative Orbit Determination System consists of total three modules namely 1.*'Relod_Obs_Gen'* to generate relative observations for relative orbit determination using batch least squares method; 2.*'Relod'* to determine the relative orbit of main and sub satellite using batch least squares method. 3.*'Relod_Obs_Pred'* to predict the inter satellite distance (ISD) between main and sub satellite, relative position and velocity for a given duration. The descriptions of *Relod_obs_gen* and *Relod_obs_pred* are also described in the following sections

3. EQUATIONS OF RELATIVE MOTION

The relative motion of two geostationary satellites closely placed can be well described in an orbit-reference frame by the set of second order non linear coupled differential equations described by Kawase [3] and Kechichian [4] shown in Eqn 1.

Let x- axis is along the radial direction from the earth, y is along the target orbit path and z is orbit normal.

$$
\begin{aligned}
&\ddot{x} - 2n\,\dot{y} - 3n^2 x - \frac{3n^2}{2r_r}\left(y^2 + z^2 - 2x^2\right) = f_x \\
&\ddot{y} + 2n\,\dot{x} - \frac{3n^2 xy}{r_r} = f_y \\
&\ddot{z} + n^2 z\left(1 - 3\frac{x}{r_r}\right) = f_z
\end{aligned}
\qquad (1)
$$

Here r_r is the radius of the reference orbit, n is angular velocity of the sub satellite and f_x , f_y , f_z are the acceleration components. In relative motion theory, x,y,z are considered to be small compared to r_r. This allows considering up to second order terms in the equations of relative motion is good enough. Even if no perturbing effects were considered, the exact description of this relative motion is given by the set of coupled nonlinear second order differential equations. These equations can only be solved through numerical integration.

4. PERTURBATIONS

In this present study perturbation due to the solar radiation pressure (SRP) is considered. The other perturbations namely lunar-solar gravity and the earth non-sphericity have no effect as they act in the same way on the satellites [3].

4.1 SRP Perturbation

The perturbation components in x, y, z directions due to solar radiation pressure (SRP) as described by Carlini and Pastor [5] are

$$
\vec{S} = \begin{Bmatrix} S_x \\ S_y \\ S_z \end{Bmatrix} = \frac{P.k}{|\vec{R}_s|}(AM_1 - AM_2)\begin{Bmatrix} X_s \\ Y_s \\ Z_s \end{Bmatrix} \qquad (2)
$$

Where P is the solar radiation pressure at mean Earth-to-Sun distance, *k* the correction factor that accounts for the real distance and $\vec{R}_s = \{X_s, Y_s, Z_s\}^T$ the sun's position vector in the orbit frame. $AM_1 = (A_1/m_1)(1+€_1)$ is effective area to the mass ratio of the main satellite and . $AM_2 = (A_2/m_2)(1+€_2)$ is effective area to the mass ratio of the sub satellite (A is the area exposed to the Sun, m is the spacecraft mass, and € the reflection coefficient of the satellite surface to the solar energy).

Now, the system of Eqns. 1 with SRP perturbations can be written as:

$$
\begin{aligned}
&\ddot{x} - 2n\,\dot{y} - 3n^2 x - \frac{3n^2}{2r_r}\left(y^2 + z^2 - 2x^2\right) = S_x \\
&\ddot{y} + 2n\,\dot{x} - \frac{3n^2 xy}{r_r} = S_y \\
&\ddot{z} + n^2 z\left(1 - 3\frac{x}{r_r}\right) = S_z
\end{aligned}
\qquad (3)
$$

Where Sx , S_y , S_z are given in equation (2). These coupled nonlinear second order differential equations are integrated numerically through Cowell's method.

5. COWELL'S METHOD

The chief advantage of special perturbation technique is its high accuracy. The perturbative acceleration acting on the satellite is modeled. The dominant force is solar radiation pressure. Other perturbative accelerations are not significant as most of the forces get averaged out. Numerical integration methods are customarily classified into two parts. Single step and multi step methods. Multi step methods normally need a starter. The single step methods that are considered are those of Runge Kutta or RK-Gill etc. Multi step methods are "Adams" or its variants. Exclusively for solving second order equations, double integration method is the recommended procedure. These methods are stable.

Gauss-Jackson-Merson's (GJM) 8^{th} order method is employed here. A brief description of the algorithm is presented by Ken Fox [6].

The basic equations to solve the equations

$$\ddot{x} = F(x, \dot{x}, t) \tag{4}$$

are

$$\begin{aligned} x_{i+1} &= x(t_i + h) = h^2(\nabla^{-2} x_i + \frac{1}{12} x_i + \frac{1}{12}\nabla \ddot{x}_i \\ &+ \frac{19}{240}\nabla^2 \ddot{x}_i + \frac{1}{40}\nabla^3 \ddot{x}_i + \frac{860}{12096}\nabla^4 \ddot{x}_i + ...), \\ \dot{x}_{i+1} &= \dot{x}(t_i + h) = h(\nabla^{-1} \ddot{x} + \frac{1}{2}\ddot{x}_i + \frac{5}{12}\nabla \ddot{x}_i + \frac{3}{8}\nabla^2 \ddot{x}_i \\ &+ \frac{251}{720}\nabla^3 \ddot{x}_i + \frac{95}{288}\nabla^4 x_i + \frac{19087}{60480}\nabla^5 \ddot{x}_i + ...) \end{aligned} \tag{5}$$

in standard backward difference notation.

The Gauss-Jackson corrector formulae are

$$\begin{aligned} x_{i+1} &= h^2(\nabla^{-2}\ddot{x}_i + \frac{1}{12}\ddot{x}_{i+1} - \frac{1}{240}\nabla^2\ddot{x}_{i+1} - \frac{1}{240}\nabla^3\ddot{x}_{i+1} \\ &- \frac{221}{60480}\nabla^4\ddot{x}_{i+1} - \frac{19}{60480}\nabla^5\ddot{x}_{i+1} - \frac{9829}{3628800}\nabla^6\ddot{x}_{i+1}) \\ \dot{x}_{i+1} &= h(\nabla^{-1}\ddot{x}_i + \frac{1}{2}\ddot{x}_{i+1} - \frac{1}{12}\nabla\ddot{x}_{i+1} - \frac{1}{24}\nabla^2\ddot{x}_{i+1} - \frac{19}{720}\nabla^3\ddot{x}_{i+1} \\ &- \frac{3}{160}\nabla^4\ddot{x}_{i+1} - \frac{863}{60480}\nabla^5\ddot{x}_{i+1} - \frac{275}{24192}\nabla^6\ddot{x}_{i+1}) \end{aligned} \tag{6}$$

6. ORBIT DETERMINATION PROCESS

The process of determining relative orbit using ground based tracking observations is described in this section. The orbit determination refines set of parameters by using smoothed tracking observations. In this present context, the parameters are relative state vector components. The system involves measurement modeling, trajectory generation and estimation. Trajectory generation is performed numerically as described in the previous sections. These relative state vector components are given in the body frame of target satellite. Therefore, it is required to convert the differential tracking observations such as differential range and differential angles into the components of relative state vector in the body frame of target satellite at the given observation time.

6.1 Measurement Modeling

The process of analytically relating the spacecraft tracking observations to the spacecraft state vector is referred to as observations modeling. These modeled observations are functions of spacecraft's position, velocity as well as specified model parameters such as tracking station location, timing errors etc.

Relative Orbit Determination is a process which maps the differential observations at the same or different time into a common fundamental set of six relative state parameters at some epoch time. This process requires certain coordinate transformations to transform the ground based observations to the body frame observations.

6.1.1 Observations conversion

In order to evaluate the difference between predicted measurements from the relative state and the observed measurements for estimation process, the tracking measurements from the ground station are to be converted into the body frame.

Conversion of ground based tracking measurements, such as range and angles into the body frame differential observations require the following transformations.

- Conversion from topocentric frame to inertial frame.
- Conversion from inertial frame to body frame.

6.2 Estimation

Given a vector of tracking measurements which depends on the instantaneous position and velocity of set of tracking data and of the orbital position, compute optimal estimate of relative position valid over the period during which tracking data is collected.

The general procedure for all definitive orbit computations is to set up some dynamical model of the orbit and use the observations to improve the orbit parameters of the model by the process of differential correction. Weighted Least Squares estimation process is applied. Observations are considered for some pre-selected time period and by differential correction of parameters of the model, the sum of the squares of the residual are minimized. The basic idea of least-squares estimation as applied to orbit determination is to find trajectory and the model parameters for which the square of the difference between the modeled observations and the actual measurements becomes as small as possible [7]. In this estimation process of "Weighted Least Squares" it is necessary to compute partial derivatives of observations with respect to model parameters. Brief description of method of estimation adopted in this present study (RELOD s/w) is as follows.

Given m observations and an orbit that is to be corrected using these observations consisting of n parameters,

$P_j = 1,...n$, and if M_o is the observed quantity, M_c is the corresponding computed quantity from the mathematical model, then each of the observed quantity can be considered a function of state parameters, the station coordinates and time and if necessary of other relevant parameters affecting the motion of the satellite. In this context, only the dependence of orbital parameters is of importance. Thus,

$$M_c = M_c\{P_j\} \quad (7)$$

By comparing the computed quantities with *m* observed quantities, the residuals are given by,

$$\Delta M_i = M_{oi} - M_{ci}\{P_j\}, \quad i = 1,2,......m. \quad (8)$$

The object is to obtain expressions in terms of known quantities. The residuals after the first iteration become:

$$\Delta M_i' = M_{oi} - M_{ci}\{P_j'\}, \quad i = 1,2,...m. \quad (9)$$

where,

$$P_j' = P_j + \Delta P_j .$$

Expanding by Taylor series and linearizing, the conditional equations become:

$$\Delta M_i' = \Delta M_i - \sum_{j=1}^{n} \frac{\partial M}{\partial P_j} \Delta P_j \quad , \quad i = 1, 2, ...m. \quad (10)$$

By multiplying weighting factors, for all *m* observed quantities the equations of condition can be written in matrix notation as:

$$W_1 R' = W_1 R - W_1 A \Delta P \quad (11)$$

where W_1 is the weighting matrix of mean measurement error (total expected error for the each measurement type due to both random noise and systematic errors) which is derived from the covariance matrix of observations by taking square root of the diagonal elements. R' is the column matrix of residuals after differential correction, R is the column matrix of residuals before differential correction, A is matrix of partial derivatives.

If everything is ideal R' should contain all zeros after just one iteration. The solution for ΔP would have been trivial if there were same observed quantities as that of parameters to be refined. In practice, 'm' is much greater than 'n' and the second order terms are not negligible, so that one iteration does not suffice. Therefore, weighted least square technique is adopted. The condition of the least square approach is:

$$[W_1 R']^T [W_1 R'] = minimum \quad (12)$$

,

that leads to the normal equations:

$$A^T W_1^T (W_1 A \Delta P - W_1 R) = 0 . \quad (13)$$

Letting

$$W = W_1^T W_1 ,$$

the equation yields:

$$\Delta P = (A^T W A)^{-1} A^T W R \quad (14)$$

.

is the correction to P for the current iteration. This process is to be carried out for a number of iterations till the solution converges. The method adopted in RELOD software for convergence and measurement rejection criteria are described in [8].

7. RELATIVE OBSERVATIONS GENERATION

The differential tracking observations of range, azimuth and elevation ($\delta\rho$, δAz, δEl) are derived from the range, azimuth and elevation (ρ , Az, El) of both the main and sub satellite tracking data. The tracking strategy is that both the satellites are tracked from the same antenna from a ground station. The first/main satellite is tracked for duration of five minutes with a sampling rate of ten or twenty seconds to get one ranging slot and after a gap of five minutes the second/sub satellite is tracked for the same duration (five minutes) with the same antenna to get another ranging slot from other satellite. This tracking pattern is repeated at every one hour. About two days of data is collected for relative orbit estimation process. The tracking data is preprocessed by 'Tracking Data Preprocessing Software (TDPP)' to generate selected smoothed observations files. 'Relod_Obs_Gen' software takes selected observations files of both (main and sub) satellites as input to generate relative observations file. Each ranging slot containing range, azimuth and elevation data is fitted with least squares curve fit method. From the fitted points of each ranging slot of both satellites, the differential/relative observations are generated.

Three sets (obs_1, obs_2 and obs_3) of five minutes durations available in twenty five minutes for every one hour. Here obs_1 and obs_3 represent main satellite; obs_2

represents for the sub satellite. The differential observations δobs are derived as follows:

The differential tracking observations of the sub satellite with respect of the main satellite are then given by

$\delta obs = obs_2 - ½ (obs_1 + obs_3)$ at the time of obs_2 .

8. ELATIVE OBSERVATIONS PREDICTION

Relative Observations Prediction software namely 'Relod_obs_pred' predicts the relative state of main and sub satellite for the given duration of time. The relative state is generated by integrating numerically the hills equations of motion given in Eqns. 3. Relative state is computed at an interval (seconds) given as input to the software.

The determined relative state obtained from RELOD software is used as input to predict the relative state. This software reads the definitive ephemeris of collocated satellites generated from ODP determined orbital elements. It generates predicted relative distance also known as Inter Satellite Distance (ISD) and relative velocity and their differences with respect to ODP determined parameters. Sample sets of results are shown in the form of figures.

9. RESULTS AND DISCUSSION

The precise relative orbit determination is carried out based on the method and assumptions described above. In total, ten different cases of colocated satellites INSAT-2E, INSAT-3B; INSAT-2B, INSAT-2C tracking data collected from February 1996 to July 2004 was taken for this present study. Relative observations were generated, relative orbits were estimated and predictions were carried out through relative orbit determination system. These results were compared with the relative orbits estimated through ODP. Table -1 gives the comparison of relative states obtained through RELOD and ODP. The prediction of Inter Satellite Distance (ISD) for a period of one week between colocated satellites and its comparison with ODP are shown in Fig. 1 and 2.

From the results obtained, we notice that the estimated ISD varies from 0.5 to 2.7 km with respect to ODP. From the figures shown (Fig. 1 and 2) to describe predicted ISD and its variation with respect to ODP, we notice that maximum ISD difference is about 5 to 6 km with respect to ODP after one week.

10. CONCLUSIONS

The process of Precise Relative Orbit Estimation and Relative Orbit Prediction of colocated INSAT missions in the absence of onboard inter satellite ranging through mission operational RELOD software using ground based tracking data was described. A number of test cases of INSAT colocated satellites relative orbit determinations carried out through RELOD software were demonstrated. Results were compared with the orbit estimations carried out through mission operational regular orbit determination ODP software Importance of relative orbit estimation was demonstrated in addition to regular orbit determination which estimates the states of individual satellites. Considerable differences were observed to consider for maintaining colocation when more and more satellites are to be colocated.

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Table - 1 Relative Position and Velocity Comparison

S. No.	Co-located S/C	Epoch	Diff (ODP-RELOD)		S. No.	Co-located S/C	Epoch	Diff (ODP-RELOD)	
			Position (km)	Velocity (m/s)				Position (km)	Velocity (m/s)
1	2E & 3B	2004-07-04 -00-30-00	-2.754	0.003	6	2C & 2B	1997-01-01 -18-00-00	1.683	0.226
2	2E & 3B	2003-08-28 -00-20-00	1.652	0.016	7	2C & 2B	1996-05-01 -16-30-00	0.565	-0.012
3	2E & 3B	2002-10-24 -00-20-00	0.493	0.052	8	2C & 2B	1996-04-03 -16-30-00	0.955	0.007
4	2C & 2B	1998-03-05 -23-52-00	-0.675	0.162	9	2C & 2B	1996-03-13 -15-30-00	1.044	0.015
5	2C & 2B	1997-07-05 -00-10-00	-2.074	-0.001	10	2C & 2B	1996-02-29 -00-00-00	0.655	0.021

*2B, 2C, 2E and 3B refer INSAT-2B, INSAT-2C, INSAT-2E and INSAT-3B respectively.

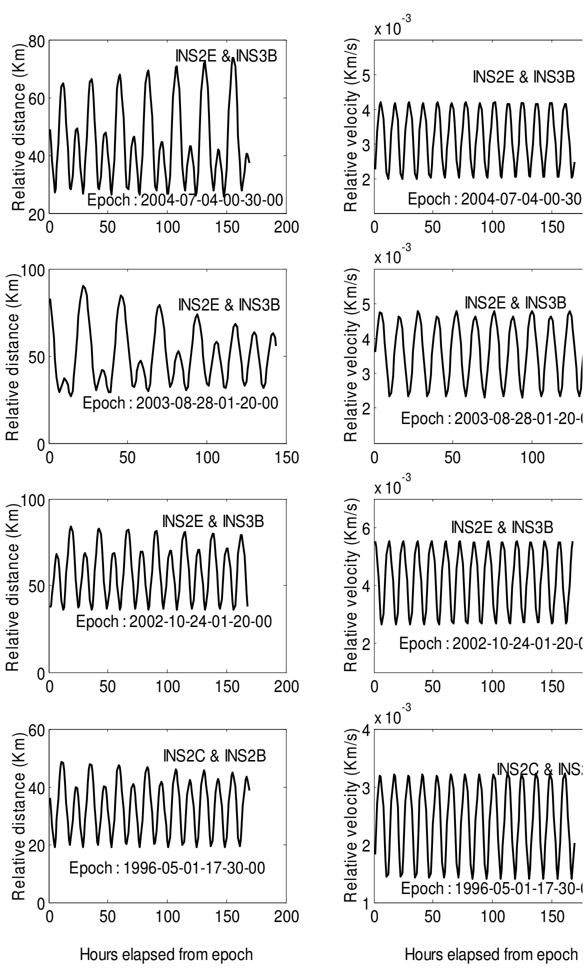

Fig. 1 Relative distance and velocity

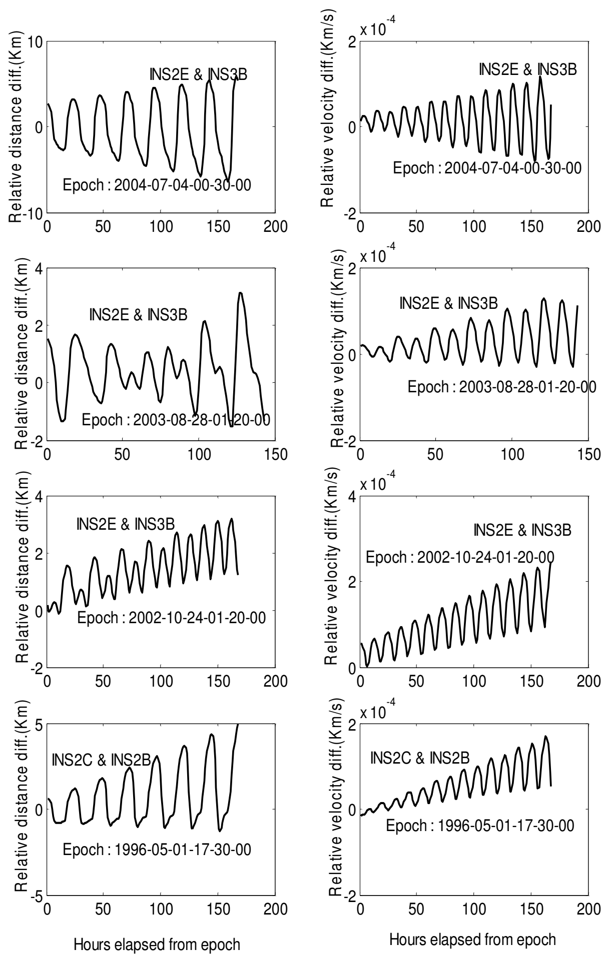

Fig. 2 Differences of relative distance and velocity between ODP and RELOD