ON THE MOON-TO-EARTH TRAJECTORIES WITH GRAVITATIONAL ESCAPE FROM THE MOON ATTRACTION

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ABSTRACT

The Moon-to-Earth trajectories of "detour" type are found and studied in frame of the Moon-Earth-Sun-particle system. These trajectories use a passive flight to the Earth from an initial elliptic selenocentric orbit with a high aposelenium and differ from usual ones of a direct flight to the Earth using an initial hyperbolic selenocentric orbit. A qualitative theoretical analysis of these trajectories is performed. The Earth perturbation increases the selenocentric energy from a negative value first to zero and then to a positive one and therefore leads to a passive escape of the particle motion from the Moon attraction near the translunar libration point L_2. This results in the particle flight to a distance of ~1.5 million km from the Earth where the Sun gravitation decreases the particle orbit perigee distance to a small value, that leads to the particle approach the Earth vicinity in ~100 days of the flight. A set of the Moon-to-Earth "detour" trajectories for the flight to the Earth-altitude of ~50 km for the atmospheric reentry is defined by a numerical method. The start from both the low orbit of the Moon satellite and the Moon surface is considered. Characteristics of these trajectories are presented. They are compared with the usual trajectories of the direct flight. The "detour" Moon-to-Earth trajectories with initial elliptic orbit and gravitational escape from the Moon attraction are shown to result in essential economy of energy relative to the usual ones with initial hyperbolic orbit. A more exact control system of navigation and correction is required for the Moon-to-the Earth "detour" flight of spacecraft.

1. INTRODUCTION

Investigations of space trajectories for flights from the near-Moon vicinity to the Earth are important for both Celestial Mechanics and Astronautics. Usual trajectories (see, e.g., [1]) for the Moon-to-Earth direct space flights within the Earth's sphere of influence with respect to the Sun are well studied. In this case, perturbations caused by the Sun are small, and the model of the restricted three-body problem (Moon-Earth with a particle of negligible mass) is used in fact. Trajectories of this type were used for space flights from the Moon (in both the USA project of the Apollo manned flight and the Soviet project for robotic capture of the lunar matter and its delivery to the Earth [2]). These trajectories are characterized by small (several days) flight time and by the fact that the departure of the particle from the Moon occurs along a hyperbola. Recently [3-7, et al.], a new class of trajectories with the Earth–to–Moon indirect detour space flight was discovered in the framework of the four-body system (Earth-Moon-Sun-particle). These trajectories use first the space flights towards the Sun (or away from the Sun) beyond the Earth's sphere of influence, and only afterwards, space flights towards the Moon. These space flights seem to be similar to bielliptic ones proposed by Sternfeld [8-10]. But they differ in dynamics. Here, the perigee distance rises due to the Sun gravitation. In addition, now the particle approaches the Moon along an elliptic trajectory, i.e., the capture by the Moon takes place. Thus, for the spacecraft capture to the lunar satellite orbit or for its landing onto the Moon, these detour space flights are more profitable than direct or bielliptic ones. An idea arises to employ this detour scheme for the Moon-to-Earth space flights [4]. The present paper describes main results of numerical and analytical studies in the problem shown. A family of trajectories for passive space flight to the Earth from an elliptic orbit of a lunar satellite has been constructed, and characteristics of these trajectories are analyzed [11]. In addition, the effects of gravitational perturbations resulted in the formation of these trajectories, particularly, in both the particle's gravitational escape from the lunar attraction and passive decreasing the perigee distance of the particle orbit (approximately from the value of the lunar orbit radius to almost zero), which makes possible the particle passive flights to the Earth, have been analyzed.

2. MOON-TO-EARTH DETOUR TRAJECTORIES

2.1 Algorithm of Calculations

As a result of the analysis and with taking into account the experience of the Earth-to-Moon trajectories studies [7], a numerical algorithm has been developed that has allowed us to find a family of detour trajectories for space flights to the Earth from elliptic orbits of the lunar satellite. These trajectories correspond to the spacecraft start from both the Moon's surface and the low orbit of the lunar satellite for several positions of the Moon on its orbit. The spacecraft trajectories have been determined by integration (using the method described in [12]) of the equations for the particle motion. These equations are written in the Cartesian nonrotating geocentric-equatorial coordinate system OXYZ in the attraction field of the Earth (with taking into account its main harmonic c_{20}), the Moon, and the Sun with the high-precision determination of the Moon and Sun coordinates, which is based on the DE403 JPL ephemerides. The particle motion in the selenocentric coordinate system MXYZ is also determined.

2.2 Some Numerical Characteristics of the Moon-to-Earth Detour Flights

Characteristics of a typical detour trajectory are presented in Figs 1-3.

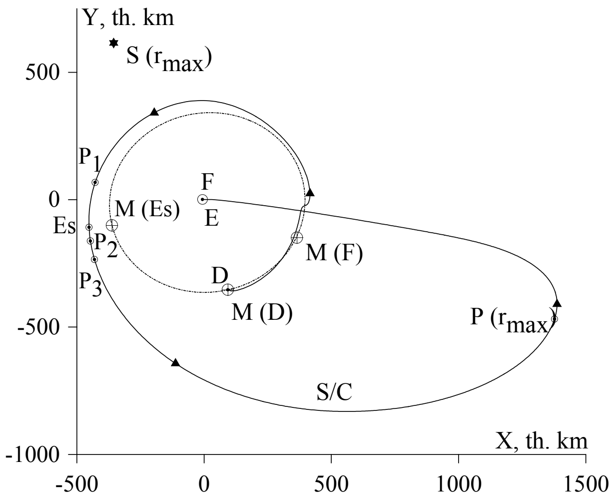

Fig. 1. The XY view for the Moon-to-Earth geocentric trajectory of detour flight

The solid curve in Fig. 1 presents geocentric motion of a spacecraft, and the dot-and-dash line shows the lunar orbit M. At the point D, the spacecraft flies away from the Moon on May 11, 2001 (for the position of the Moon near the apogee), from the perilune of an initial elliptic orbit with the perilune altitude $H_{\pi 0}$ = 100 km and semimajor axis a_0 = 38 455 km. This orbit is close to the final orbit of the Earth-to-Moon space flight, which was presented in [7]. All the following motion of the particle is passive (without taking into account possible corrections). Under the effect of the Earth's gravitation, evolution of the selenocentric orbit and an increase in the selenocentric energy

$$E_s = V^2/2 - \mu_M/\rho = -\mu_M/2a_s \qquad (1)$$

occur. In Eqn. 1 V and ρ are the selenocentric velocity of the particle and its distance from the Moon, respectively, a_s is semimajor axis of the particle orbit, and μ_M ($\approx$ 4902 km^3 s^{-2}) is the lunar gravitational parameter. At the point P_1 in the space flight time $\Delta t \approx$ 19 days, the energy is $E_s \approx$ - 0.031 km^3 s^{-2}, $a_s \approx 79 \cdot 10^3$ km, and $\rho \approx 76 \cdot 10^3$ km. At the point Es for $\Delta t \approx$ 20.6 days and $\rho \approx 91.85 \cdot 10^3$ km in the region of the translunar libration point L_2, there is the escape from the lunar attraction, i.e., E_s = 0 here, and the orbit is parabolic with the zero velocity "at infinity", V_∞ = 0. Further, the particle moves from the Moon along a hyperbola. At the point P_2 for $\Delta t \approx$ 21.1 days and $\rho \approx 101 \cdot 10^3$ km, the energy is $E_s \approx$ 0.011 km^2 s^{-2}, V_∞ = 0.15 km s^{-1}. At the point P_3 for $\Delta t \approx$ 21.9 days and $\rho \approx 120.2 \cdot 10^3$ km, the energy becomes equal to $E_s \approx$ 0.031 km^2 s^{-2}, and V_∞ = 0.25 km s^{-1}. Then, the spacecraft flies away from both the lunar orbit and the Earth and reaches in $\Delta t \approx$ 70 days the maximal distance $r_{max} \approx 1470 \cdot 10^3$ km from the Earth. At that moment, the point S (r_{max}) determines the direction to the Sun. By the effect of the Sun gravitation, the perigee is gradually lowered, and for $\Delta t \approx$ 113 days at the point F, the spacecraft approaches the Earth E having the perigee's osculating altitude H_π = 50 km.

Figs 2 and 3 show the evolution of the spacecraft detour motion with respect to the Moon at the initial part of the space flight where there is the escape from the lunar attraction.

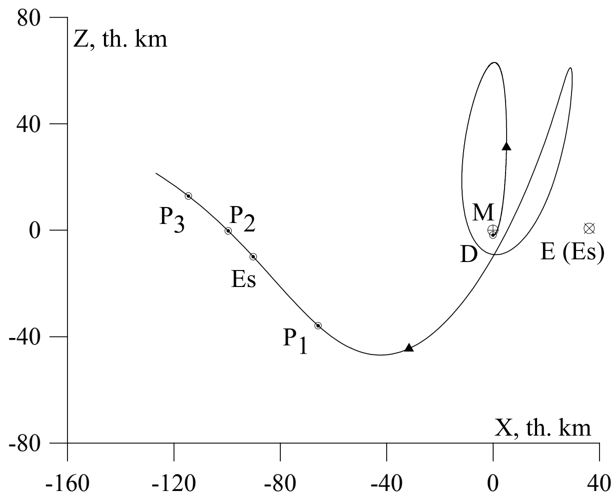

Fig. 2. The XZ view for the Moon-to-Earth selenocentric trajectory of detour type at initial part of the flight

Fig. 2 gives selenocentric trajectory in the XZ plane. The point E (Es) determines the direction to the Earth at the moment of the escape from the lunar gravitational attraction. The initial (at the point D) spacecraft velocity is $V_0^+ \approx$ 2282 m s^{-1}. For leaving a circular lunar-satellite orbit with the altitude of 100 km and velocity $V_0^- \approx$ 1633 m s^{-1}, the velocity increment is $\Delta V_0 \approx$ 649 m s^{-1}. For the usual direct space flight scheme and the minimal departure energy, $V_\infty \approx 0.8$ km s^{-1}, the flight time T ≈ 5.5 days, we have the initial velocity V_0^+ of about 2443 m s^{-1}, the velocity impulse ΔV_0 of about 810 m s^{-1}, that is at about 161 m s^{-1} more than for the case of detour space flight. For a case when spacecraft leaves the Moon's surface, the detour trajectory (with a_0 = 38455 km again) has approximately the same characteristics as for the indicated case of the start from the lunar satellite orbit. The decrease in the velocity increment is equal to about 156 m s^{-1} in this case.

Fig. 3 gives the selenocentric energy constant $2E_s$ versus the time for the initial part D P_1 Es P_2 P_3 of the motion with escape from the lunar attraction. Here and below, on Fig. 4, the time t is counted off from the Julian date 2451898.5, that is 20.12.2000.0.

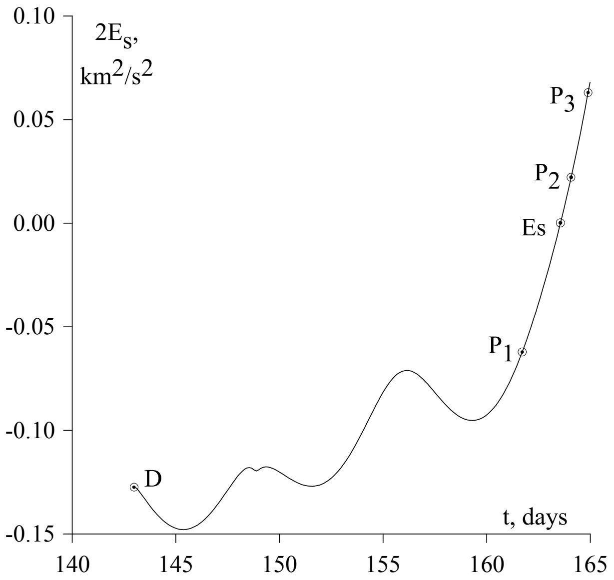

Fig. 3. Selenocentric energy versus the time for initial part of the Moon-to-Earth detour flight with escape from the lunar attraction

3. EARTH GRAVITY EFFECT ON THE PARTICLE ESCAPE

We now qualitatively analyze the gravitational effects on the formation of the detour trajectory. First, we estimate an increase $\Delta E_s = - E_{s0}$ of selenocentric energy (1) from the negative value E_{s0} for the initial elliptic orbit to the zero energy which can be caused by the Earth gravity during the particle selenocentric motion on the arc D Es from the initial state D to the escape point Es. On the base of the orbit evolution theory [13] and assuming that the particle orbit eccentricity is e ≈ 1, the mean energy is $E_s \approx - \Delta E_s/2$, and taking into account the change in the Moon-Earth direction, we obtain [7]:

$$\Delta E_S \approx sign\beta\left(\frac{15}{2}\pi\mu_E\left(\frac{\mu_M}{a_M}\right)^3 n_M\beta\right)^{2/9} > 0. \quad (2)$$

In Eqn. 2, μ_E is the Earth's gravitational parameter, a_M is the semimajor axis of the Moon's orbit, n_M is the angular velocity of its orbital motion,

$$\beta = \cos^2\gamma\sin 2\alpha, \quad (3)$$

γ is the slope of the radius vector $\boldsymbol{r}_B$ for an external body (for the Earth, in this case) to the plane of the particle orbit, and α is the angle between the projection of the radius vector $\boldsymbol{r}_B$ onto this plane and the direction to the orbit perilune. For $\Delta E_s > 0$, it is necessary to have $\sin 2\alpha > 0$, $0<\alpha<\pi/2$ or $\pi<\alpha<3\pi/2$. Estimation by Eqns 2-3 gives $\Delta E_s = - E_{s0} \approx 0.096$ km^2 s^{-2}, $a_0 \approx 25{,}600$ km for middle value $\beta = 0.5$. This gives estimation for minimal value of semimajor axis a_0 for the initial elliptic selenocentric orbit in the Moon-to-Earth detour trajectory. This theoretical evaluation well fits the results for our numerical calculations of the Moon-to-Earth detour trajectories. E.g., for the Moon-to-Earth flight during a month from May 12, 2001, and for initial inclination i_0=90°, we received for the Moon-to-Earth calculated trajectories the minimal value of semimajor axis a_{0min}≈24,500-27,000 km for initial ascending node Ω_0=0; a_{0min}≈23,500-28,500 km for Ω_0 ≈ - 63.9°; a_{0min}≈24,000-28,000 km for Ω_0= -90°, see Fig.4.

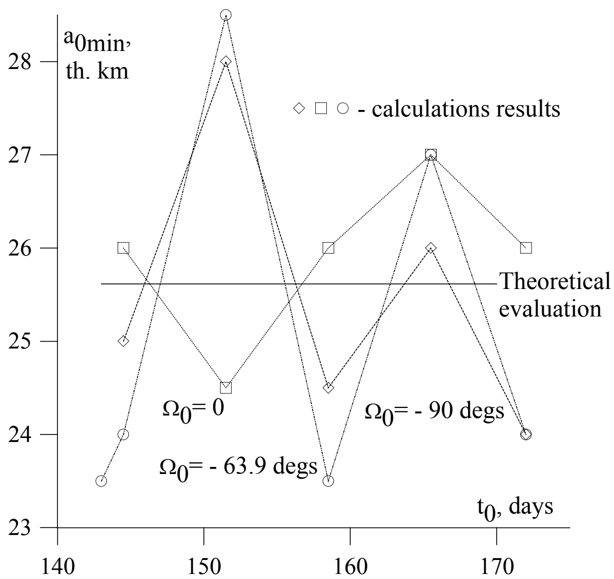

Fig.4. Minimal value of initial semimajor axis depending on the time of start from near-Moon elliptic selenocentric orbit for the Moon-to-Earth detour trajectories

We can see that, if the orientation of the particle initial orbit relative to the Earth is suitable and its negative energy is large enough, the Earth's gravitation provides a sufficient increase in the particle orbital energy and allows its passive escape from the lunar attraction.

4. EARTH GRAVITY EFFECT ON THE PARTICLE ACCELERATION TO HYPERBOLIC SELENOCENTRIC MOTION

Now we approximately analyze the acceleration of the particle motion with respect to the Moon from the zero energy to a positive one for a hyperbolic trajectory with velocity at "infinity" V_∞ which is equal to about 0.15 – 0.25 km s^{-1} on the subsequent short arc Es P_2 P_3 (even on the somewhat larger arc P_1 Es P_2 P_3 from the energy $E_s < 0$). This acceleration is qualitatively described by approximate model of the one-dimensional rectilinear particle's motion with the Earth, placed on the same line at a distance r_M beyond the Moon [7], see Fig. 5.

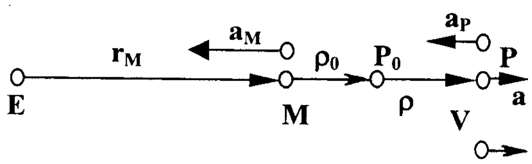

Fig. 5. A model for the particle selenocentric motion from the Moon

In this case $d\rho/dt > 0$, i.e., the particle moves away from the Moon. The Earth's perturbation $\delta a_E = \mu_E/r_M^2 - \mu_E/(r_M + \rho)^2 > 0$, it accelerates the particle motion. For this model, assuming that, approximately, r_M = const, we can integrate the equations for the perturbed motion of the particle:

$$E_S(\rho) - E_0 = (\mu_E / r_M^2)(\rho - \rho_0) + \mu_E / (r_M + \rho) -$$

$$- \mu_E / (r_M + \rho_0), \quad E_S(\rho_0) = E_0; \tag{4}$$

$$\rho(E_S) = B/2 + [B^2/4 + r_M B]^{1/2}, \tag{5a}$$

$$B = (E_S - E_0) r_M^2 / \mu_E + \rho_0^2 / (r_M + \rho_0); \tag{5b}$$

$$V^2(\rho) = 2E_S(\rho) + 2\mu_M / \rho; \tag{6a}$$

$$t(\rho) - t_0 = \int_{\rho_0}^{\rho} d\rho / V(\rho). \tag{6b}$$

Example. Let for the presented trajectory at the point Es of the gravitational escape the selenocentric energy be $E_s = E_0 = 0$, $\rho = \rho_0 = 91850$ km, $r_M = 376000$ km. Then, the model of Eqns. 4-6 gives $\rho \approx 102.5 \cdot 10^3$ km for V_∞ = 0.15 km/s (point P_2), $\rho \approx 120.4 \cdot 10^3$ km for V_∞ = 0.25 km/s (point P_3), and $\rho \approx 55 \cdot 10^3$ km for $E_s = -0.031$ km^2/s^2 (point P_1). We can see the qualitative correspondence with the numerical results presented above, especially for $E_s > 0$.

Thus, for the given class of the Moon-to-Earth detour space flights, the Earth's gravitation in the region of the translunar libration point L_2 allows increasing the selenocentric energy of the particle motion from the zero value to the positive one for a hyperbolic trajectory. Afterwards, the particle moves away from the Moon orbit, at a large geocentric distance.

5. SUN GRAVITY EFFECT ON DECREASE OF THE PARTICLE ORBIT PERIGEE DISTANCE

Next, we estimate the effect of the Sun gravitation on the variation Δr_π of the particle orbit perigee distance r_π on the final arc P_3 F of the space flight. We use the theory [13] of the orbit evolution for one orbital revolution of a planet's (the Earth', here) satellite due to an external body's (the Sun's, now) gravity perturbation assuming the Earth-Sun direction to be constant. Since the final geocentric distance $r_{\pi f}$ for the particle orbit perigee is very small ($r_{\pi f} = r_{\pi 0} + \Delta r_\pi \approx 0$), we assume that eccentricity $e \approx 1$ and take for r_π its mean value $r_\pi = (2r_{\pi f} - \Delta r_\pi)/2 \approx -\Delta r_\pi/2$. Thus, we have:

$$\Delta r_\pi \approx \text{sign}\beta((15/2)\pi(\mu_S/\mu_E)\beta)^2 a^7 / a_E^6 < 0. \tag{7}$$

Here, μs is the gravitational parameter of the Sun, a_E and a are the semimajor axes for the Earth's orbit and for the particle geocentric orbit, the value β is determined by Eqn. 3 with the Sun as the external body. For $\Delta r_\pi < 0$, it follows from Eqn. 7 that $\sin 2\alpha < 0$, $\pi/2 < \alpha < \pi$ or $3\pi/2 < \alpha < 2\pi$. Then, we estimate the desired value of the semimajor axis for the spacecraft orbit as

$$a \approx [|\Delta r_\pi| a_E^6 / ((15/2)\pi(\mu_S/\mu_E)\beta)^2]^{1/7}. \tag{8}$$

For estimation, we have assumed that $\Delta r_\pi = -500 \cdot 10^3$ km and $\beta = -0.5$. Then, according to Eqn. 8, the semimajor axis of the particle geocentric orbit at the final part of the flight is $a \approx 870 \cdot 10^3$ km and its apogee distance is $r_\alpha \approx 1500 \cdot 10^3$ km. If we take into account that the Earth-Sun direction is not constant in time, this changes the result only slightly. Thus, if the orientation of the particle orbit with regards to the Sun is suitable enough and the orbit apogee distance is large enough (of about $1.5 \cdot 10^6$ km), the particle perigee distance decreases from about the lunar-orbit radius to almost zero. This allows the particle's passive approach the Earth.

6. CONCLUSIONS

Reviewing the results of our analysis, we can see that gravitational perturbations of the Earth and the Sun make it possible for the particle beginning its motion from the selenocentric elliptic orbit to escape the motion from the lunar attraction, to transfer it to the Moon-to-Earth detour trajectory, and then to approach the Earth. This leads to noticeable decrease in the energy consumption for the Moon-to-Earth space flights. Such a conclusion is confirmed by both the numerical calculations of relevant trajectories and their theoretical analysis.

7. ACKNOWLEDGEMENTS

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