RECONSTRUCTED BEAGLE 2 TRAJECTORY

Rüdiger Cramm(1), José Maria de Juana Gamo(2), Rainer Bauske(3)

(1) *ESA, ESOC, OPS-GFT, Robert-Bosch-Str. 5,D-64293 Darmstadt, Germany*
E-mail: Ruediger.Cramm@esa.int
(2) *GMV SA, ESOC, OPS-GFT, Robert-Bosch-Str. 5,D-64293 Darmstadt, Germany*
E-mail: Jose.Maria.De.Juana.Gamo@esa.int
(3) *TERMA GmbH, ESOC, OPS-GFI, Robert-Bosch-Str. 5,D-64293 Darmstadt, Germany*
E-mail: Rainer.Bauske@esa.int

ABSTRACT

The scientific landing probe Beagle2 was released by the Mars Express spacecraft six days before arrival at planet Mars. In this paper the lander's trajectory from release until atmospheric entry is reconstructed. Observational data for the initial state vector and the release momentum are considered. The solar radiation pressure force is taken into account by using a flat plate model. From the reconstructed trajectory, various impact point parameters at atmospheric entry are derived and compared with the nominal target values. It is shown how individual uncertainties in the propagation model map onto the B-plane at atmospheric entry and onto the planetary surface. The uncertainty in the initial state vector at release is identified to be the dominating source in the uncertainty of the lander's arrival location.

1. INTRODUCTION

The Mars Express spacecraft was launched on 2003-06-02 with destination Mars. It carried a planetary lander (Beagle2) on board, which was released from the spacecraft on 2003-12-19, six days before arrival at Mars. The release took place along the spacecraft +*z* axis by a spring ejection mechanism causing a Δv and a rotational movement on the lander. After release, communication of Beagle2 with neither the spacecraft nor with antennas on Earth was possible until landing. Since Beagle2 had no actuators for orbit or attitude control on board, it travelled passively until atmospheric entry. Therefore a precise targeting and a proper ejection was important. Three days before release the spacecraft was manoeuvred to fine target the trajectory. After Beagle2 release, the Mars Express spacecraft was retargeted off the collision course to prepare for the Mars orbit insertion on 2003-12-25. The planned contact with Beagle2 after landing could never be established.

The reconstruction of the lander's trajectory is split into two parts. The first part covers the time span from release until atmospheric entry. The second part spans from entry until landing on the planet. The atmospheric entry point is defined as the point in time when the spacecraft crosses 120 km altitude above Mars. It took Beagle2 more than 138 hours to travel the first part of the trajectory, while the second part through the atmosphere lasts only approximately five and a half minutes. This paper focuses mainly on the reconstruction of the first part of the trajectory, to our best knowledge, using available data and observations. For this part of the trajectory an error analysis is carried out and uncertainties are mapped onto the B-plane at entry and onto the planet's surface, applying an existing atmosphere model. Reference [1] deals in more detail with the second part of the trajectory (i.e. after atmospheric entry) and can be understood as a continuation of this paper.

The B-plane is defined as the instantaneous plane perpendicular to the incoming hyperbolic velocity vector ($\vec{V}_{\text{inf}}$) containing the centre of Mars. It is spun up by the R and T axis of the following RST system: the orthogonal system has its origin in the centre of Mars. The unit vector along the S-axis is aligned with $\vec{V}_{\text{inf}}$

$$\vec{S} \parallel \vec{V}_{\text{inf}} \tag{1}$$

The T-axis is computed from

$$\vec{T} = \vec{S} \times \vec{N} \tag{2}$$

with $\vec{N}$ being the unit vector in the direction of the Martian rotational axis at epoch J2000. The R-axis is finally obtained by

$$\vec{R} = \vec{S} \times \vec{T} \tag{3}$$

Thus the T-axis is contained in the Mars equatorial plane and each point contained in the B-plane has the S-coordinate identical to zero.

Epochs in this paper are given in barycentric dynamical time (TDB) throughout. For convenience the days relative to the arrival are expressed as A-n (read A

.

minus n) where n denotes the days relative to arrival on 2003-12-25. The Beagle2 release took place at A-6.

2. TARGETING FOR LANDER EJECTION

With a trajectory correction manoeuvre (TCM) on 2003-10-11 the spacecraft was put on collision course with Mars to target for the lander ejection, which was due more than two months later. Nine days before Mars arrival, i.e. at A-9, a fine-targeting manoeuvre was planned and executed to compensate for perturbing accelerations occuring since the TCM (e.g. reaction wheel off-loadings, spacecraft safe mode at A-23, etc). In order to optimise the A-9 fine targeting manoeuvre the orbit was determined using tracking data until A-11 inclusive. Additionally a prediction of about 2 cm/s as net Δv of a reaction wheel off-loading at A-10 was considered. The resulting optimised manoeuvre parameters are given in Table 1 in the Earth mean equator at epoch J2000 reference system. This manoeuvre targeted the lander to nominal B-plane parameters at atmospheric entry and a landing site, for which the relevant values are given in Table 2 and Table 3, respectively. Note that the S-component of a point contained in the B-plane is by definition zero. The Cartesian landing site coordinates are expressed in the Mars rotating frame as specified in [3].

Table 1: Commanded fine-target manoeuvre

Execution time 2003-12-16 08:06:24.184		
Δv magnitude	=	0.340206 *m/s*
Right ascension	=	182.0662 *deg*
Declination	=	-0.2783 *deg*

Table2: Targeted B-plane impact point

Entry time 2003-12-25 02:51:25.989		
B-plane R	=	- 1193.0 *km*
B-plane T	=	+ 6917.8 *km*

Table3: Cartesian landing site coordinates.

landing X	=	-42.9 *km*
landing Y	=	3322.6 *km*
landing Z	=	682.1 *km*
Longitude	=	90.74 *deg*
Latitude	=	11.60 *deg*

From the fine targeting manoeuvre to Beagle2 release no further thrusting, e.g. reaction wheel off loading, was performed. Thus, with this A-9 manoeuvre, the spacecraft was operated for the last time in terms of orbit control. Each further correction on the Beagle2 ejection state could only be controlled by the ejection time and spacecraft attitude.

3. DATA USED FOR THE TRAJECTORY RECONSTRUCTION

For the trajectory integration a RungeKutta 7(8) algorithm with automatic step size control was applied. During the almost six days journey, accelerations due to the point masses of the Sun, the planets and the Earth's moon, and due to solar radiation pressure are considered. For the first integration arc from release until entering the Martian sphere of influence at about 2003-12-22 18:49 the Sun is the centre of integration. After entering the sphere of influence the orbit propagator uses Mars as centre of integration and spherical harmonics of the Martian gravity field up to 24x24 are taken into account. For the propagation from atmospheric entry to landing point an atmosphere model as specified in [2] is applied. Further details can be found [1].

The following propagation parameters and uncertainties need to be known for the Beagle2 orbit integration and for the error analysis

- Initial spacecraft state vector immediately before lander release
- Release Delta V magnitude
- Release Delta V direction
- Solar radiation pressure acting on Beagle2

In the following subsections these parameters are discussed in more detail. The commanded epoch 08:32:14.184 (central time) at A-6 is used as epoch for the lander ejection. Since the expected release process lasts only 0.117 seconds [2], the release is treated as impulsive. In this paper uncertainties of the Martian atmosphere are not considered. Ref. [1] tackles this problem in more detail.

3.1 Initial spacecraft state vector

To achieve a precise spacecraft state vector at Beagle2 release, a dedicated spacecraft orbit determination is performed. This orbit determination takes into account all available tracking data from 2003/11/10 (after the trajectory correction manoeuvre) until 2003/12/19 08:32:14, i.e. lander release. The determined state vector directly before lander release and the covariance matrix will be used for the error analysis. Without giving here the full matrix, its diagonal elements' information is that the 1σ position uncertainty is less than 3 km and the 1σ velocity uncertainty is less than 9 mm/s. The main contribution to the uncertainty stems from the inertial z-component, perpendicular to the plane containing the Earth equator.

3.2 Release Delta V magnitude

Following the formulae from [2] on the ejection impulse implied by the spring device, a nominal Δv of 0.2919 m/s for Beagle2 and 0.018104 m/s for the MEX spacecraft were expected. The Δv on Beagle2 was not observed directly, but the NASA DSN 70 meter antenna in Canberra observed the effect of the spring release on the Mars Express spacecraft. The ground station recorded two way S-Band Doppler data over the release. Comparing the observed frequency shift over the release with the expected frequency shift, and assuming no spacecraft attitude error, an overperformance in terms of Δv on the spacecraft of (1.3 ± 0.5) % (1σ) could be noticed [5]. Following the principle *actio equals reactio*, the same overperformance is translated into a resultant Δv on Beagle2 of (0.2957 ± 0.0015) m/s.

3.3 Release Delta V direction

The lander is released along the spacecraft +z axis; for its direction the nominal (commanded) release attitude is considered. The 1σ uncertainty is 0.14 deg in any direction, according to [2]. An error of the s/c attitude at ejection has been neglected, because it can be conservatively estimated to be less than 10% of the 0.14 deg direction uncertainty.

3.4 Solar Radiation Pressure on Beagle2

During the entire trajectory the Sun radiation falls onto the lander from behind (solar aspect angle of about 124 deg). A simple flat plate model with the surface area exposed orthogonal to the Sun is considered. The surface area is computed by integrating the projected visible (from the Sun) surfaces of Beagle2 onto the plane perpendicular to the Sun direction. These surfaces are basically a flat bottom part and a lateral part shaped like a truncated cone (frustum). The latter is not entirely visible from the Sun. Geometric properties of these parts are well known [2]. The resulting equivalent flat plate area is 0.39755m^2.

Optical properties of the material are taken from [4] and are used to compute the solar radiation pressure coefficient to be applied in the model. It is assumed that the incident radiation is either absorbed, reflected specularly, reflected diffusely or some combination of these. With an absorptivity coefficient of 0.14 [4] and the assumption that half of the reflected radiation is reflected specularly and the other half diffusely, the resulting solar radiation pressure coefficient results in 1.33444. The used mass of Beagle2 is 68.86 kg.[2].

The 1σ uncertainty is taken to be 25% of the total force. Taking into account that the solar radiation pressure factor should be a value between 1 (if all radiation were absorbed) and 2 (if all radiation were reflected specularly), the 3σ values shall cover conservatively the extreme cases.

4. ERROR ANALYSIS

When propagating the lander's trajectory, the question arises how uncertainties of the propagation parameters affect the location of the entry point. We represent the uncertainty of the lander's trajectory as three-sigma error ellipses in the B-plane at atmospheric entry. First we build the state transition matrix $\Phi(t_0,t_1)$:

$$\left(\Phi(t_0,t_1)\right)_{ij} = \frac{\partial b_i(t_1)}{\partial x_j(t_0)} \tag{4}$$

This 3x6 matrix contains the partial derivatives of the three B-plane coordinates (b_i) at time of entry (t_1) with respect to the six state vector components (x_j) at time of lander release (t_0). Apart from the initial state vector, the trajectory integration depends also on other propagation parameters (q_j) which are, in detail, the solar radiation pressure, the release Δv magnitude, and the ΔV direction (i.e. right asc. and declination). Therefore also the sensitivity matrix $S(t_1)$ needs to be considered. This 3x4 matrix relates the changes in these parameters (q_j) to changes in the B-plane parameters:

$$\left(S(t_1)\right)_{ij} = \frac{\partial b_i(t_1)}{\partial q_j} \tag{5}$$

Numerical values for the both matrices, Φ and S, have been obtained by using a symmetric difference quotient approximation.

These matrices Φ and S now allow mapping of the uncertainties in propagation parameters to the entry time. The uncertainties in initial state vector (δx_i) are represented by the 6x6 state vector covariance matrix P at t_0

$$\left(P_{SV}(t_0)\right)_{ij} = E\{\delta x_i \delta x_j\} \tag{6}$$

As discussed in section 3.1, the covariance matrix used was output of an orbit determination.

The 4x4 matrix P_Q contains the uncertainties in the remaining propagation parameters. Its diagonal elements are the square of the 1σ uncertainty:

$$\left(P_Q\right)_{ii} = \sigma_{ii}^2 \tag{7}$$

where for the individual σ_{ii} the 1σ uncertainties as discussed in sections 3.2 to 3.4 have been applied. It is assumed that these propagation parameters are not correlated with each other, i.e. the off-diagonal elements are zero.

The 3x3 covariance matrix P_B at t_1 of the three B-plane parameters is then given by

$$\begin{aligned} P_B &= \begin{bmatrix} \Phi & S \end{bmatrix} \begin{bmatrix} P_{SV} & 0 \\ 0 & P_Q \end{bmatrix} \begin{bmatrix} \Phi^T \\ S^T \end{bmatrix} \\ &= \Phi\, P_{SV}\, \Phi^T + S\, P_Q\, S^T \end{aligned} \tag{8}$$

omitting in the representation the dependence of t_0 and t_1 from now on. No correlation between the spacecraft state vector errors and the other propagation parameter uncertainties for the Beagle2 trajectory integration is assumed.

To obtain the error ellipse in the B-plane, the matrix P_B is reduced to a 2x2 matrix R_B. That means R_B contains only the elements relating to the R and T components of P_B. The off-diagonal elements of this reduced 2x2 matrix are usually non-zero and therefore an orthogonal transformation C is applied to obtain the diagonal matrix

$$C\, R_B\, C^T = \begin{pmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_b^2 \end{pmatrix} \tag{9}$$

This gives the eigenvalues $\sigma_a{}^2$ and $\sigma_b{}^2$, so that $3\sigma_a$ and $3\sigma_b$ specify the length of the semi-major and semi-minor axis of the 3σ error ellipse. The eigenvectors contained in matrix C give the corresponding directions of the axis in the R-T plane.

Analogous to the described approach, the uncertainties can be mapped to the landing site: for t_1 the landing time is used and for the coordinates b_i in matrices Φ and S the landing site coordinates are applied.

5. RESULTS

Using the data described in section 3 for the trajectory reconstruction, the results displayed in Table 4 are obtained. This table compares the reconstructed figures to the nominal ones; the so-called entry angle is the flight path angle at point of entry in the rotating frame. Applying the error analysis based on the uncertainties described in section 3, a three-sigma error ellipse for the impact point is obtained with a semi major axis of 18.0 km and a semi minor axis of 2.5 km. The angle of the semi-major axis with respect to the T-axis is -74.7 deg (angle positive towards the R-axis). The fact that the individual uncertainties (initial state vector, solar radiation pressure, release Δv magnitude and direction) are not correlated with each other, makes a more detailed analysis possible to resolve the contribution of each individual uncertainty to the total three-sigma error ellipse. The characteristics of the error ellipse for each uncertainty are displayed in Table 5. To specify the orientation of the ellipse within the B-plane two angles are given, one for the semi major and another one for the semi minor axis: the angles are with respect to the T-axis, positive towards the positive R-axis.

Table4: Comparison of targeted and reconstructed entry parameters and landing site

	NOMINAL	*RECONSTRUCTED*
Entry time on 2003-12-25	02:51:25.989	02:51:21.900
Entry angle *(deg)*	-16.50	-16.63
Entry longitude *(deg)* Entry latitude *(deg)*	83.13 11.21	82.96 11.14
B-plane R *(km)* B-plane T *(km)*	- 1193.0 6917.8	- 1185.1 6914.9
<u>Landing Site</u> X *(km)* Y *(km)* Z *(km)* Longitude *(deg)* Latitude *(deg)*	 - 42.9 3322.6 682.1 90.74 11.60	 - 29.2 3323.6 678.1 90.50 11.53

Table 5: Three sigma error ellipses for the individual uncertainties.

Uncertainty	***Semi minor axis*** ***Semi major axis***	***Orientation angle***
Only state vector	1.56 km 17.99 km	15.20° -74.80°
Only release Δv direction	0.10 km 0.11 km	-9.60° 80.40°
Only release Δv magnitude	1.16 km	-9.12°
Only SRP	1.59 km	15.27°
All errors	2.46 km 18.00 km	15.35° -74.65°

The individual uncertainties, mapped onto the B-plane, are visualized in Fig. 1. Note that both the uncertainty of the solar radiation pressure and the release Δv magnitude only have a one-dimensional effect in the B-plane, i.e. their error ellipses have the minor axis equal to zero. In Fig. 1 only a small part of the contribution of the state vector uncertainty can be seen. As its error ellipse is much larger than the ones for the other uncertainties, it cannot be fully displayed in this plot.

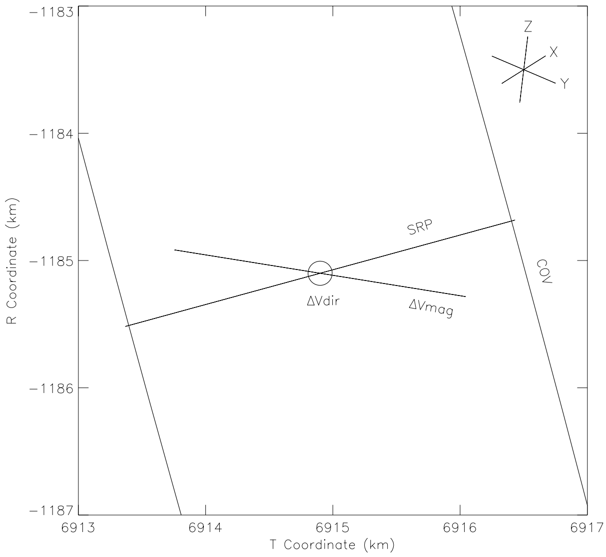

Figure 1: Three-sigma error ellipses for individual uncertainties: SRP (solar radiation pressure), ΔVmag (release ΔV magnitude), ΔVdir (release ΔV direction) and COV (initial state vector covariance information).

All uncertainty ellipses in Fig. 1 are centred on the reconstructed impact point. The three directions in the upper right corner of the figure indicate how a pure (uncorrelated) perturbation in either the inertial x, y, or z direction (Earth mean equator at epoch J2000) maps onto the B-plane. The three displayed lines in the B-plane are caused by perturbations of equal size along one of the three inertial directions at lander release.

A B-plane plot of the total three-sigma error ellipse centred on the reconstructed impact point is shown in Fig. 2. This Figure also shows, according to [2], the allowed entry angle corridor of -17.5 deg to -15.5 deg. The mark of the nominal impact point (diamond) intersects the line of nominal entry angle at –16.5 deg.

Mapping the uncertainties onto the planetary surface, i.e. continuing the trajectory propagation through the atmosphere and assuming a nominal lander descent phase (see also [1]), a three-sigma error ellipse with a semi major axis of 27.6 km and a semi-minor axis of 2.7 km is computed and displayed in Fig. 3. A step of 0.1 deg in either longitude or latitude corresponds to about 6 km on the surface. The given planetocentric longitude is positive towards East. Note that for this mapping no atmospheric uncertainties have been considered.

6. DISCUSSION

The shape and size of the resulting error ellipses are driven by the uncertainties of the used propagation parameters, i.e. the uncertainties in state vector, the solar radiation pressure model, and the ejection Δv. The

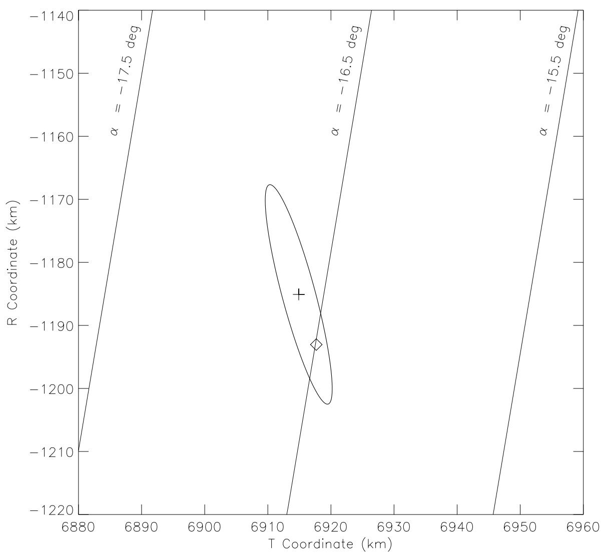

Figure 2: Reconstructed and nominal B-plane impact point with three-sigma error ellipse and allowed entry angle corridor.

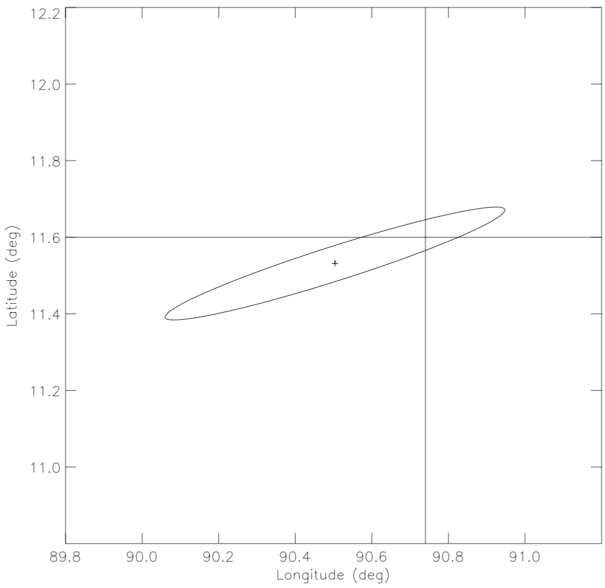

Figure3: Reconstructed landing site and mapped three-sigma uncertainties onto the planetary surface. The nominal landing longitude and latitude are displayed.

state vector and its uncertainty are the result of a dedicated orbit determination. The correctness of these was confirmed a posteriori with subsequent orbit determinations leading to the very successful spacecraft Mars orbit insertion. Moreover cross verification of orbit determinations done at ESOC and JPL showed consistent results during the Mars approach phase [5] which lasted several weeks. Conservative uncertainties are used for the solar radiation pressure force. The ejection mechanism is modelled according to [2], making use of observational evidence of an overperformance.

The nominal target trajectory was fixed with the fine targeting manoeuvre at A-9. An orbit determination with tracking data until A-11 and a predicted Δv of a reaction wheel off-loading at A-10 (about 2 cm/s) was used to design this manoeuvre. Unavoidable mismodellings in the orbit determination and orbit propagation, mainly due to non-perfect performance of the manoeuvres (wheel off-loading and fine targeting), caused the initial state vector for the lander trajectory starting at A-6 to be different from the target trajectory. The reconstructed Beagle2 trajectory however can be considered to be “close” to the targeted trajectory. The B-plane target point is contained in the 3σ error ellipses of the reconstructed trajectory. The error ellipse lies well within the allowed entry angle corridor of -15.5 to -17.5 degrees [2].

The analysis of the impact of individual uncertainties in the propagation model reveals that the uncertainty in initial state vector is by far the dominating source in the position uncertainty at atmospheric entry. This uncertainty was reduced as much as possible by performing a dedicated orbit determination using all available tracking data prior to Beagle2 release. The second largest contribution stems from solar radiation pressure where a conservative uncertainty of 25% for one sigma was assumed. A more detailed analysis of the solar radiation pressure yields that the force at the time of lander delivery acts mainly in the inertial x-y plane with equal components along the x and y axis. Considering the mapping of the inertial directions onto the B-plane according to Fig. 1, the direction of the solar radiation pressure force is reflected by the small R component and the larger T component in the B-plane plot of Fig. 1. It is coincidence that the size of the three-sigma solar radiation pressure uncertainty equals approximately the size of the minor axis of the state vector error ellipse. The spacecraft attitude error is neglected for the error analysis for two reasons. On one hand the spacecraft attitude uncertainty is approximately one order of magnitude smaller than the considered Δv direction error. On the other hand the release Δv direction uncertainty contributes only very little to the total error ellipse.

The landing site of the reconstructed trajectory is displaced by approximately 14 km with respect to the target. The target point is, as for the B-plane, plot well within the three sigma error ellipse.

References

1 Bauske, R., Dependence of the Beagle2 Trajectory on the Mars Atmosphere, ISSFD 2004.

2 Beagle2 ICD, MEX lander delivery module : Beagle2-ESOC interface, Astrium, BEA2.ICD.00005.S.MMS.

3 Seidelmann et al., Report of the IAU/IAG working group on cartographic coordinates and rotational elements of the planets and satellites: 2000, Celestial Mechanics and Dynamical Astronomy 82, 83-110, 2002.

4 Beagle2 PID-B, BGL2-LUX-ID-13, issue 4, Leicester University, 15-06-2000.

5 T. Morley, Mars Express Navigation for Beagle2 Separation and Mars Orbit Insertion, ISSFD 2004.