THE OPTIMISATION OF ATTITUDE PROFILES FOR SMART-1: A HIGHLY CONSTRAINED PROBLEM (THE TRUTH ABOUT ATTITUDE)

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1. ABSTRACT

This paper describes how the attitude of the 3-axis stabilized spacecraft SMART-1 is constructed.
SMART-1 is the first in a series of technology demonstration missions from ESA. Launched in September 2003, it is using an Electric Propulsion (EP) engine to spiral out of the Earth gravity field and to reach a scientific moon orbit. This is the first time that such an engine is used as main (and only) propulsion system in an ESA mission [1].
Several types of attitude profiles have to be commanded, including pointings to ground stations, EP-thrust direction, inertial directions and different payload scanning motions. All pointings have to fulfil specific constraints.
The first part of the paper describes how the attitude pointings are constructed and explains how to solve the constraint problem of high rates, Star Tracker blindings, Sun illuminations of Solar Arrays and S/C heat. The second part shows how to connect different pointings by generic smooth slews to achieve a final continuous attitude profile.

2. INTRODUCTION

The SMART-1 AOCS permits to follow any attitude profile commanded by the ground. The attitude control is based on reaction wheels as actuators for all nominal modes and 1N hydrazine thrusters for reaction wheel desaturation and for rate reduction in case of contingencies. Two autonomous star trackers, 5 attitude rate sensors (gyroscopes) and 3 coarse sun sensors are utilized as attitude sensors.

The EP engine is mounted on a two axes articulation mechanism (EPMEC) attached to the S/C. The advantage of this propulsion system is that by accelerating ions in an electric field the specific impulse can be improved by a factor of up to 10 compared to chemical propulsion. SMART-1 has by August 2004 successfully operated its EP engine for more than 3000 hours. While the engine is firing, a control loop of the AOCS aims to reduce the total S/C angular momentum by rotating the EP engine with respect to the S/C-body. The control loop reaches a steady state when the EPMEC-articulation is such that the thrust direction points to the S/C centre of mass. In this steady state the thrust direction is about 4 degrees away from its mid-position, in which the thrust vector is aligned with the S/C *Z*-axis. Typical excursions from the steady state rotation angles are of the order of 1 degree. As it is important that the thrust vector is aligned with an optimised inertial direction the S/C compensates for these articulation angles i.e. while in EP firing mode the commanded attitude is the one of the EP engine and the S/C-body is tilted relative to the commanded attitude according to the current articulation angles.

The Solar Arrays (SA) are mounted along the S/C *Y*-axis, and can be rotated infinitely, a fact that proved to be crucial for the SMART-1 LEOP. Because of the high power demands of the EP-engine the off pointing of the SA normal from the sun direction must stay within a few degrees.

Fig.1: The SMART-1 Spacecraft

The Star Trackers (ST) are mounted on the +/-*Y*-faces with their boresights about 40 degrees away from the +/-*Y*-axis. As a consequence the sun will never be in the field of view (FOV) of a ST as long as the SA panels are perpendicular to the Sun-direction (S). However it is possible for a ST to be blinded by the Earth or the Moon. Simultaneous blinding of both STs must be avoided, since the attitude rate sensors do not permit the outage of both STs for more than one hour.

Apart from constraining the attitude, the STs also impose a constraint on the S/C rate: To guarantee tracking the angular SC rate is limited to 0.15 degrees/s. Due to a limitation in the torque that can be exerted on the reaction wheels the angular acceleration of the S/C is limited to 0.0005 degree/s^2.

The attitude types needed include pointing directions from the S/C to targets on Earth, Moon, Sun, ground stations, inertial targets and EP directions. Besides this numerous science and commissioning pointings are done.

The paper is split into two parts. This first part describes the construction of the attitude profiles, when certain pointings have to be followed (either scientific or EP-pointing). The second part describes how the different pointing types are joined with slews.

3. CONSTRUCTION OF POINTING PROFILES

For SMART-1 all pointing profiles (scientific and EP-pointing) have in common that a certain axis in spacecraft frame $P_{sc}(t)$ must be aligned with a given time-dependent inertial direction $P(t)$. At an instant in time t, all attitudes fulfilling this requirement form a one-parametric set where the open parameter is the rotation angle *Alpha(t)* around this axis. To define this angle a reference frame for its evolution is needed. This is accomplished by parallel translation i.e. the integration of the kinematics equation of motion [2], with the rate around this axis set to zero. The choice of this rotation angle such that all constraints are fulfilled is subject of this section. For SMART-1 this was implemented as a three-step-process. In the first step an ideal-attitude-profile is defined in which the spacecraft Y-axis is put perpendicular to the Sun. In the second step the profile is modified as to fulfil constraints on angular rate and acceleration. In a third step the remaining geometric constraints such as ST-double-blindings are incorporated.

3.1 Ideal-Attitude-Profile

The orientation of a 3-axis stabilized S/C with solar arrays mounted parallel to the S/C Y-direction is calculated by exploiting the fact that the cross product of two vectors is orthogonal to them. Let S be the S/C-to-Sun-direction in inertial frame, and P an arbitrary target unit vector (non parallel to S), then

$$X_{sc} = P_{in}, \quad Y_{sc} = \frac{X_{sc} \times S}{|X_{sc} \times S|}, \quad Z_{sc} = X_{sc} \times Y_{sc}, \quad A = (X_{sc}, Y_{sc}, Z_{sc}) \tag{1}$$

is an attitude transformation matrix for a S/C attitude such that P is parallel to X_{sc} and S is orthogonal to Y_{sc}. This construction only works while S and P are not parallel (in this case the size of the solution set in infinite). Instead of X_{sc}, any S/C axis orthogonal to Y_{sc} can be chosen. For a S/C axis not orthogonal to the Y_{sc}-direction, the simple application of the cross product must be replaced by a more general method:

Instead of solving the problem Y_{in} perpendicular to S, and P_{in} , and Y_{in} being a unit vector, i.e.

$$\langle Y_{in}, S \rangle = 0, \quad \langle Y_{in}, P_{in} \rangle = 0, \quad \langle Y_{in}, Y_{in} \rangle = 1 \tag{2}$$

the problem Y_{in} perpendicular to S, Y_{in} on a cone around P_{in} and Y_{in} being a unit vector, i.e.

$$\langle Y_{in}, S \rangle = 0, \quad \langle Y_{in}, P_{in} \rangle = a = \langle Y_{sc}, P_{sc} \rangle, \quad \langle Y_{in}, Y_{in} \rangle = 1 \tag{3}$$

must be solved. The resulting set of attitude solutions therefore contains 0,1 or 2 elements.

While the Y_{in} direction defined by the cross product will always be perpendicular to the Sun-direction S, in general there is no such solution when the angle in-between P_{in} and S is smaller than the angle β in-between P_{sc} and the S/C X-Z-plane. In this case a closest optimum is defined, resulting in a Sun-depointing of the Y_{sc} direction by an angle smaller than β.

In fact even the case where P_{sc} lies in the X-Z-plane there is a singularity. This itself is not a problem, since the probability if hitting these conditions is zero. The problem that arises is, that this is an essential singularity in the sense that the limit values for approaches of $P_{in}(t)$ from different sides are different:

$$\lim_{t->to,t<to} P_{in}(t) \times S = -\lim_{t->to,t>to} P_{in}(t) \times S,$$

where $P_{in}(to)=S$. In fact they are anti-parallel, resulting in attitudes, which are 180 degrees apart. For paths $P_{in}(t)$ that do not hit S but get very close the result is similar, i.e. near S a high angular excursion is made by the attitude profile. In case the time interval is too small, this means that the S/C rates are too high.

To avoid this situation and to find a solution to this problem, the angle function *Alpha(t)* is changed:

3.2 Limiting Angular Rates and Accelerations

An automatic tool was developed to keep the angular rate and acceleration of the S/C within limits: The problem that needs to be solved, is to guarantee the performance of the EP engine, which has high power demands, i.e. off-pointing of the SA normal from the sun direction shall be minimized, while the S/C rates are below the limit.

The idea is, to edit the *Rate(t)* function (derivative of *Angle(t)*) around the time interval where the rate limit is broken in the following way: Cut off the rates when they are above the limit, and push them up in a slightly bigger interval around the cut, so that outside this bigger interval the attitude is the same. In essence this means that instead of rotating the S/C too fast at a certain point, start the fast rotation earlier, go to the limit (not above), and stay at the limit a bit longer so that the same angle around the pointing axis is done.
The best and most practical approach proofed to be the insertion of a trapezoid into the *Rate(t)* with the following properties:

Req.3.1. The maximal rate shall be smaller then the limit

Req.3.2. The maximal slope of the rate shall be smaller then the limit

Req.3.3. Angle (area below the graph) shall be unchanged (rate limiting case) or 180 Degrees apart (transition case). The second option means that no roll manoeuvre (swing-over) is made. This makes sense if the excursion of the angle in the ideal profile is close to 180 Degrees.

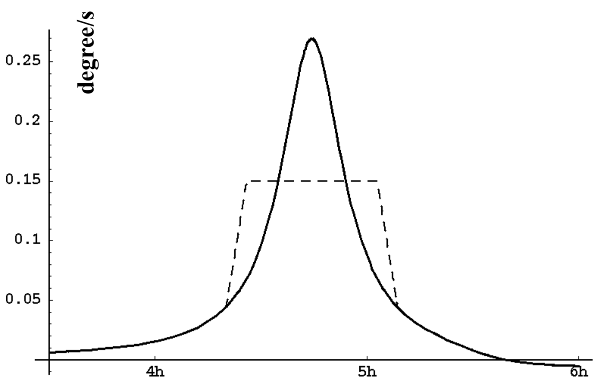

Fig.2: Angular rate around the pointing axis for the rate limiting case vs. the ideal case.

Problems that arose are:

1. The SAs will not be perpendicular to the Sun-direction anymore. This is a trade off that cannot be avoided.
2. Although the real singularity is never hit, one will come arbitrary close, which can cause numerical problems.
3. In the case when S is close to the pointing direction plane (local plane in which the pointing direction lies) it will not make sense to do a 180 degree swing-over, since SMART-1 can rotate the SA infinitely. In this case the transition trapezoid is the best solution (Fig.3). This is avoiding the swing-over and does a transition to the inverse ideal solution 180 degrees apart.
4. The pointing direction is only given in a finite time interval (EP). In general no full trapezoid can be constructed and the best approximation is done (the initial or final attitude may then be different).
5. Two very close singularities my cause an overlap of the respective trapezoids.
6. Large number of trapezoids needs to be handled.

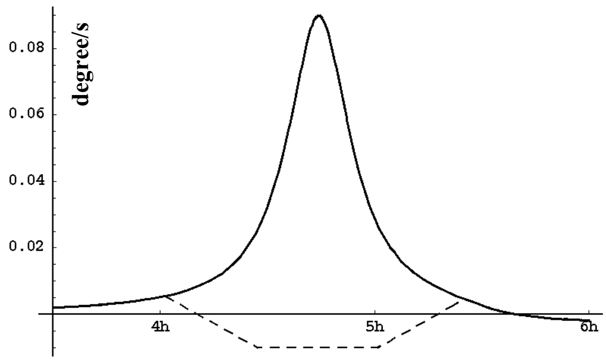

Fig.3: Angular rate around the pointing axis for the transition case (where no swing-over is done) vs. the ideal case.

3.3 Incorporating Star Tracker Blindings

The profiles created using the procedure described in the previous two subsections did not violate any further constraints for the majority of cases. However, once per month the orbital geometry was such that the flip-over occurred when the spacecraft was in-between the Earth and the Moon and both ST were blinded simultaneous. These rare cases had to be analysed individually.

Analysis of the situation was performed using plots of the type shown in Fig. 4. The plot shows for a period of time and all possible rotation angles around the pointing direction which constraints are violated. Any two attitude profiles following the pointing *P(t)* only differ by a rotation angle around the pointing direction. By plotting this rotation angle versus time relative to some reference profile an attitude profile following *P(t)* can be visualised (see Fig. 4). The reference profile was defined such (1) that it followed the pointing *P(t)* and (2) that it had no rates around the pointed S/C-axis P_{sc}. Using such a profile as reference has the advantage that the slope in this plot is proportional to the S/C rate around the pointed S/C axis P_{sc}.

Fig. 4. shows an example for an EP-pointing profile in early October 2003, when the thrusting was performed in direction of the S/C velocity vector relative to Earth. Here the baseline profile with the *Y*-axis perpendicular to the Sun-direction would cause a double blinding at the end of a flip-over. In this case there are two possibilities for constructing an attitude profile, which fulfils all constraints.

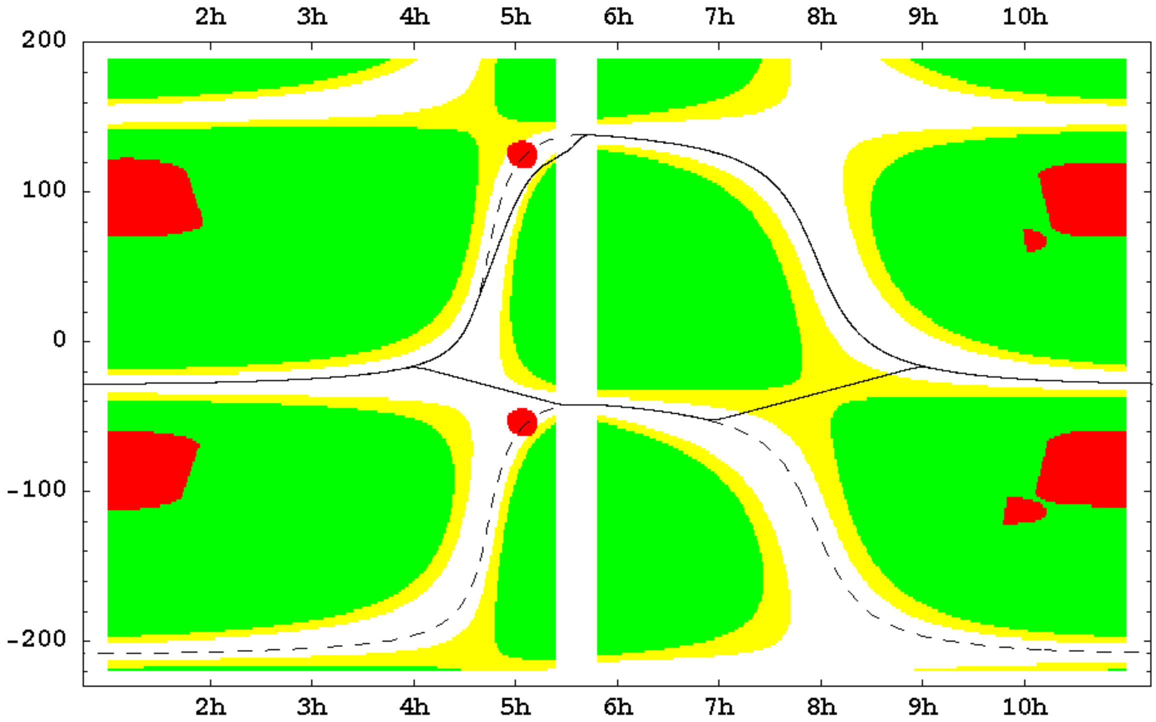

Fig.4: For 2003/10/04 1h-11h the constraint violations are shown for any possible rotation angle (in degrees) using a colour-code. Light and dark grey areas correspond to incidence angles on the *Y*-face by more than 6 and 10 degrees respectively. The white strip between 5h and 6h is due to an eclipse. The dark patches correspond to ST-double-blindings. Also shown are the ideal attitude profiles (dashed) and two possible solutions avoiding the ST-double-blindings (solid).

First, one can do an excursion from the baseline-profile around the time when the double-blinding would occur. From Fig. 4 it can be seen that this is possible because there is a gap between the area of the double blinding and the area of incidence of more than 10degree on the *Y*-face (in LEOP and the Earth-spiral-phase, thermal and power constraints imposed that incidence angles of more than 10 degrees on the S/C *Y*-faces are not allowed. Incidence of more than 6 degrees could be tolerated for 120 minutes. After a thermal analysis these constraints were even tightened).

The second possibility is to make a transition from one ideal solution to the other solution. This transition between the two different solutions is only possible here because the Sun direction is close to the orbital plane and because the pointing direction is along the S/C velocity vector. The velocity is twice per orbit almost aligned with the Sun direction (once its parallel and once anti-parallel). In this case it is possible to rotate the spacecraft relative to one solution of the ideal-profile to the other solution without exceeding the limit of the incidence angle on the *Y*-face.

4. SLEWS

4.1 Purpose

All the different pointings have to be joined by "slews" to achieve a final continuous profile. The slew motion itself has no specific pointing requirement between the initial and final attitude but must satisfy the following constraints:

- Req.4.1. The slew shall be continuous in attitude and rate
- Req.4.2. The *Y*-axis (SA) shall be as perpendicular to Sun direction as possible
- Req.4.3. –*X* side illumination shall be avoided or minimized
- Req.4.4. S/C rate shall be below limits
- Req.4.5. RW levels and torques shall be below limits

The first three requirements are satisfied by the way the slew is constructed whereas appropriate choice of slew duration solves the last two. Since this duration is found by an iterative procedure it is for performance reasons an advantage to describe the slew with one analytic function.

4.2 Slew Motion Construction

The initial and final attitude and derivative is expressed as 1-3-2 Euler angles and the slew motion is constructed by polynomial fitting of each angle individually. This ensures that the motion is smooth and analytically described and that the boundary conditions of attitude and rate are met (Req. 4.1). The SA and *–X* illumination requirements are furthermore fulfilled by calculating the initial and final Euler angles relative to the following inertial reference frame:

$$\begin{aligned} x_{ref} &= Sun\text{-}direction(t_{beg}) \\ z_{ref} &= \frac{x_{ref} \times y_{sc}(t_{beg})}{\left| x_{ref} \times y_{sc}(t_{beg}) \right|} \\ y_{ref} &= z_{ref} \times x_{ref} \end{aligned} \tag{4}$$

The initial and final attitudes and derivatives are described as 1-3-2 Euler rotations relative to this frame.

$$\begin{aligned} A_{xzy}(\phi,\theta,\psi) &= A_y(\psi) A_z(\theta) A_x(\phi) \\ \dot{A}_{xzy}(\phi,\theta,\psi) &= A_{xzy}(\phi,\theta,\psi,\dot{\phi},\dot{\theta},\dot{\psi}) \end{aligned} \tag{5}$$

The simplest fitting of each Euler angle is done using a 3rd order polynomial. So for each angle *v* the following should be solved for the coefficients a_i:

$$\begin{aligned} & f(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\ & f(t_{ini}) = v_{ini} \wedge \dot{f}(t_{ini}) = \dot{v}_{ini} \wedge \\ & f(t_{fin}) = v_{fin} \wedge \dot{f}(t_{fin}) = \dot{v}_{fin} \end{aligned} \tag{6}$$

The time variable *t* should be transformed to e.g. the interval from 0 to 1 for numerical reasons.

By using 3 or 7 order polynomials for the fitting of each angle it is possible to get a more optimal torque profile and/or continuous torques at the cost of more complicated optimisation. However, the method explained here has been completely adequate for SMART-1, MEX and Rosetta.

4.3 Illumination Requirements

Before looking into the illumination limits, notice that it is possible to choose the initial and final Euler angles such that:

$$\begin{aligned} & \theta \in [-\pi/2, \pi/2], (\Leftarrow \theta = -ArcSin[A_{2,1}]) \\ & \psi \in [-\pi, \pi] \end{aligned} \tag{7}$$

Note also that in the short time of a slew it is possible to write the sun direction vector in S/C frame as

$$A_{xzy}(\phi,\theta,\psi) \cdot x = (Cos(\psi)Cos(\theta), -Sin(\theta), Z) \tag{8}$$

And from Eqn. 8 it follows that the Sun will not illuminate *–X* if θ belongs to the interval [-Pi/2, Pi/2]. But from the polynomial fitting it is then clear that if the initial and final attitude does not illuminate the *–X*, then the slew will also avoid it (neglecting the derivative of θ), and if one or both of the initial/final attitudes illuminates *–X* then the slew will generally go the way with the smallest illumination even if it is a longer way. The derivative of θ can of course force the *–X* from shadow into illumination, but this is unlikely because of the low rates of the S/C and the presented method is an efficient way of avoiding the Sun on -X (Req. 4.3) except in the most specially constructed cases.

The requirement that the SAs (*Y*-axis) are as perpendicular to the sun as possible is also fulfilled. Because this means that the sun vector in S/C frame shall have the smallest possible *Y*-component. From Eqn. 8 this means that θ shall be close to zero. From the polynomial fitting and Eqn. 7 this is clearly fulfilled (Req. 4.2).

4.4 Time Optimisation

After the slew motion has been constructed (to fulfil Req. 4.1-4.3), it is checked for last requirements of S/C rate and RW levels and torques (Req. 4.4-4.5). If the check fails the slew is made longer and recalculated whereas it is made shorter in case all checks are fine. This is done in a loop until it has converged to the optimal solution.

The check of the rate is trivial but for the RW checks there are two possibilities. 1) If the initial RW levels are known, then the entire RW level and torque profile during the slew can be calculated and a “direct check” can be performed. 2) If the initial RW levels are not known, but an estimate of the maximum total angular momentum of the entire S/C is available then a “worst case” check can be performed over the entire slew profile. This is done by stepping through the slew profile and calculating the worst possible RW level and torque for each RW at each time.

5. SMART-1 EXPERIENCES

SMART-1 needs a continuous supply of a continuous attitude profile. This gave Flight Dynamics a wide control over the attitude profile and many problems can be solved via ground command with a simple interface to the S/C. The spiral orbit also causes the repetition of many of the tasks with slightly different conditions. For example a regular change (each orbit) between Earth pointing for communication to EP pointing for thrusting is needed for a major part of the mission.

For SMART-1 it has been very useful to have the "worst case" slew optimisation available. Reason being, that approximately 10 days of command profiles are uplinked each time and the EP makes it difficult to predict the RW levels accurately so far in the future. On the other hand it has been quite easy to estimate the maximum build-up of momentum based on telemetry data from the previous period, and this is all what is needed for the calculation of the worst case optimised slews.

The three steps approach for construction of the attitude profiles themselves proved to be very successful. First two steps being automatic and tightly structured for often occurring problems. The third step interactive for rarely violated constraints that do not allow any simple automatisation. So far all the pointings needed for the optimised EP-manoeuvres could always be performed.

While SMART-1 is in EP mode, the commanded attitude is the attitude of the EP engine. This means that in general the SA will not be perpendicular to the Sun-direction S, if the cross product is used to calculate the attitude. Assuming a fixed position of the EPMEC, one can instead solve the problem using the general ideal profile (see section 3). Since the main part of the deviation of the $-Z$ axis of the thrust direction ($-Z_{EP}$) is caused by the compensation of the centre of mass, a constant Z_{EP} direction in the S/C frame can be assumed to compensate for the resulting Sun illumination of the Y-faces. This approach proved to be necessary for thermal reasons of the STs. It improved the redundancy significantly, since otherwise one ST would be regularly over-heated and could not be used. In effect the slight incidence of the sun on the Y-sides, where the STs are mounted could be reduced and the ST temperature was lowered by 5 degreesC.

References

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